Control System Design
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Lecture – 18
General controllers (Part 2/3)

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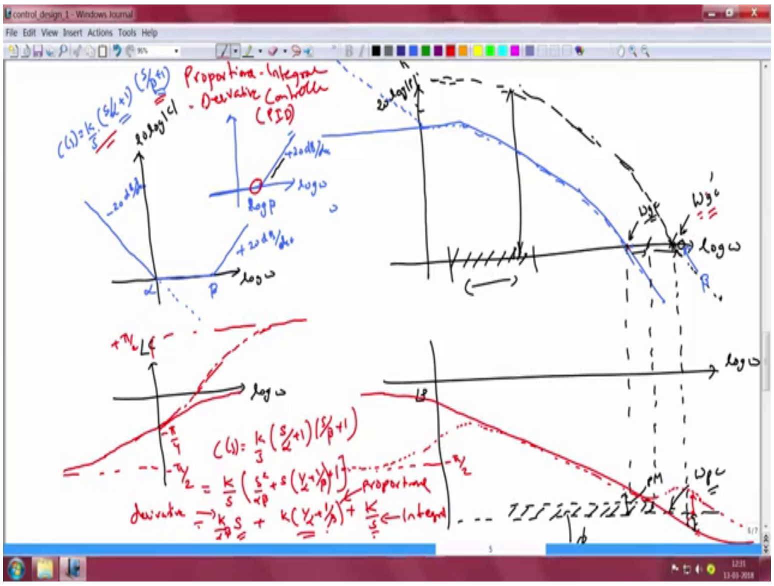

Now, with this increased gain we may have succeeded in meeting our mid frequency performance specifications in tracking references or rejecting disturbances. However, we have ended up paying a fatal price for increasing the loop gain in this frequency range. Why is that so? That is because a new gain crossover frequency is significantly to the right of the gain crossover frequency is a plant itself. And at that gain crossover frequency if one notice notices one finds that the phase is now greater than minus 180 degrees which means that our closed loop system is going to be unstable.

Now, with a pi controller there is nothing one can do if one bought to simply multiply the proportional the pi controller with this constant k 1, then one has to accept an unstable system on once hands. So, how do we solvate the situation? Remember and note that we have managed to meet the performance specifications, by making sure that the mid frequency gain is adequately high, but we are now paying price in terms of stability of the closed loop system. So, if we focus on what the root cause of the problem is it is that near the gain crossover frequency the phase has fallen below minus 180 degrees. So, what we need to do is to make sure that the phase of the overall open loop system is not below minus 180 degrees in this frequency range. How do we accomplish this objective?

To accomplish this objective let us say we were to add a 0 slightly to the right of the gain crossover frequency. So, let us say I add a 0 here, or in other words I multiply my controller with the transfer function s by beta plus 1, where beta is the location where I have added the 0. Let me first draw the bode plot of just this term alone s by beta plus 1. So, the magnitude plot if we were to draw the magnitude plot of s by beta plus 1, then up to omega equal to beta or if we are using the logarithmic scale, up to log omega equal to log beta the magnitude characteristics of this is 0 dB.

So, in other words, if we have chosen to add our 0 slightly to the right of our new gain crossover frequency, then this term here s by beta plus 1 does nothing to the already designed magnitude characteristics. Which means that does not affect our performance in any particular way, we are still able to track the reference or reject disturbances in the mid frequency range, exactly as we could do earlier. And beyond omega equal to beta, the magnitude characteristic increases a plus 20 decibels per decade. How about the phase characteristics?

The phase characteristics of this particular controller is that for very low frequencies the phase of the term s by beta plus 1 is close to 0, and at omega equal to beta or log omega being equal to log beta, the phase of s by beta plus 1 becomes plus 45 degrees or plus pi by 4. And then the phase approaches plus pi by 2 as omega tends to infinity. So, I shall be consistent and represent all angles in radians and therefore, reliable this as plus pi by 4.

Now, here the is where we have the clue to our solution. If you look at the magnitude characteristic for frequency is below beta, but just the neighborhood of beta we see that the magnitude characteristic has a magnitude of close to 0-degree dB, but the phase characteristic is not really 0. The phase characteristic is close to plus 45 degrees or plus pi by 4 radians. So, what we can achieve therefore by multiplying the already existing pi controller with the term of the kind s by beta plus 1; is that, we will be able to add a phase lead of plus 45 degrees in the vicinity of the gain crossover frequency omega g c prime. But as far as the magnitude characteristics at frequency is below omega g c prime are concerned it remains unaffected.

So, the magnitude characteristics will be exactly as it was before and as has been indicated by this dotted curve on the top, for frequencies up to omega g c prime. For frequencies beyond omega g c prime the magnitude characteristic will not roll off as it used to before. But it global, but it will roll off at a slope that is lesser by 20 decibels per decade. That is because the controller contributes to plus 20 decibels per decade increase in gain.

And the plant might already plant times the original pi controller might already have a certain rate at which the gain is dropping. The sum of these 2 will be the rate at which the new gain drops; it may be a little bit less than what it was for the pi controller cascaded with the plant alone.

But that is not of significance cross, because this gain is still be below 0 dB and therefore, not cause any special problems as far as the stability of the closed loop system is concerned. What is truly useful about this term s by beta plus 1 is what it does to the phase characteristics of our open loop system. We noted that the phase characteristics seen improvement of plus pi by 4 in the vicinity of that location where we have chosen to add a 0.

And since we have chosen to add the 0, in the vicinity of our gain crossover frequency the phase characteristic will shoot up by plus pi by 4 in the vicinity of the gain crossover frequency omega g c prime. And then will continue to shoot up, ultimately the phase characteristic will be better than that of the original characteristic by an angel of plus pi by 2.

Therefore, if one want to focus on the phase characteristic in the vicinity of the gain crossover frequency omega g c prime. One notices that with just the pi controller where we had k 1 times k by s times s plus 1 or s by alpha plus 1, one could only result in an unstable closed loop system, because the phase margin because the phase was greater than minus 180 degrees; however, because we are now added plus 45 degrees by introducing this term s by beta plus 1 in the controller, we see that the phase now is less than 180 degrees, phase lag is less than 180 degrees. And therefore, we have a finite a phase margin and therefore, our closed loop system has been rescued from instability and is now a stable closed loop system.

Therefore, the role of this term s by beta plus 1 in this gain is very clear. We chose to locate the 0 beta in the vicinity of the gain crossover frequency which qualifies as the high frequency range for our closed loops, because the gain crossover frequency marks the bandwidth of our closed loop system.

We chose to add it not with the intention of improving the gain characteristics of the open loop system. But rather it was chosen to be added in order to improve the phase characteristics and in particular it allows us to add a phase of plus 45 degrees near the gain crossover frequency, and thereby rescue a system that was on the verge of instability or was probably already unstable from becoming unstable, and it made the closed loop system stable by adding a phase lead of plus 45 degrees.

If you were to draw a bode plot of our newly synthesized controller, we see that at omega equal to alpha, we will have 1 0 and the purpose of the 0 as we discussed in the previous slide was to improve the mid frequency performance. It was intended to prevent the integrator from attenuating the plants characteristics, and thereby resulting low gain crossover frequency and sluggish response. Next we added a 0 at omega equal to beta, and the reason we added this 0 was because you wanted to rescue over closed loop system from instability, because this 0 enabled us to add plus 45 degrees phase in it is vicinity.

So, the magnitude characteristic will have an optic, it will increase at plus 20 decibels per decade starting from omega equal to beta. How about the phase characteristic of the overall controller? We see that in the vicinity of omega equal to beta, the controller adds a further phase lead of plus 45 degrees. So, the overall phase will go like this, and eventually will approach plus pi by 2. So, this solid curve here represents the phase characteristics of a pi controller, the dotted curve here represents the phase characteristics of the new controller which includes the 0.

So, the new controller has the structure C of s equal to k by s times s by alpha plus 1 times s by beta plus 1. And if I were to expand out these terms, I would have it to be of the form k by s times s square by alpha beta plus s times 1 by alpha plus 1 by beta plus 1. Which when simplified would give us k by alpha beta times s plus k times 1 by alpha plus 1 by beta plus k by s. We notice that we have a term here that has s unit. So, it is a derivative term with a certain constant multiplying the term, that is added to a term that is a constant, k times 1 by alpha plus 1 by beta. And this in turn is added to a term that has an integrator in it. Because we have a differentiator term a proportional term and an integrator term in this; this kind of a controller is called a Proportional Integral Derivative controller or PID controller for short.

So, this here is the derivative term, this here is a proportional term and this here is a integral term. This controller is called the proportional integral derivative controller or PID. A few comments about the PID controller are in order. One thing that you must note is that the PID controller enables us to achieve our control and stability objectives over the entire frequency range of interest to us. The integral term allows us to achieve high gains in the low frequency range or the frequency range where the plants dynamics have not gained yet. And therefore, enables us to track references and reject disturbances very well in that frequency range.

If we come to the proportional term, the proportional term allows us to achieve reasonably good performance in the mid frequency rage, or in the frequency range where you have the plants dynamics being apparent, but still the frequencies are much less than the green crossover frequency of the open loop system. In this frequency range, stability is a console, because any attempt to increase the gain indefinitely in this frequency range can result in very large gain crossover frequencies and thereby result in unstable closed loop systems. The proportional term however, allows us to achieve modest improvements in performance in this frequency range.

When we come to the derivative term, we see that this derivative term helps us improve the performance in the frequency range or in the vicinity of the gain crossover frequency and beyond. Now in this frequency range the gain is already closed to 0 dB. So, one cannot even talk of any respectable control performance in terms of tracking references or rejecting disturbances.

The gain is so low that we will not be able to accomplish these objectives to any appreciable extent. However, this is the frequency range where we would have the gain crossing over and a phase crossing over; so, stability is of crucial concern. And the derivative term allows us to add phase lead in this frequency range, and thereby ensure that even if you have increase the gain a little bit higher in the mid frequency range or closed loop system would end up becoming unstable.

So, the derivative term takes care of our stability requirements, the proportional term takes care of our mid frequency, performance requirements the integral term takes care of our low frequency performance requirements. Given the versatility of this controller, in that each of these terms address concerns in one or another frequency range of interest to us, made by the mid frequency low frequency or the high frequency ranges. This is a very versatile controller and indeed a significant fraction of controllers found in industry are of the PID type. There is another point I wish to make in the connect in the context of PID controllers.

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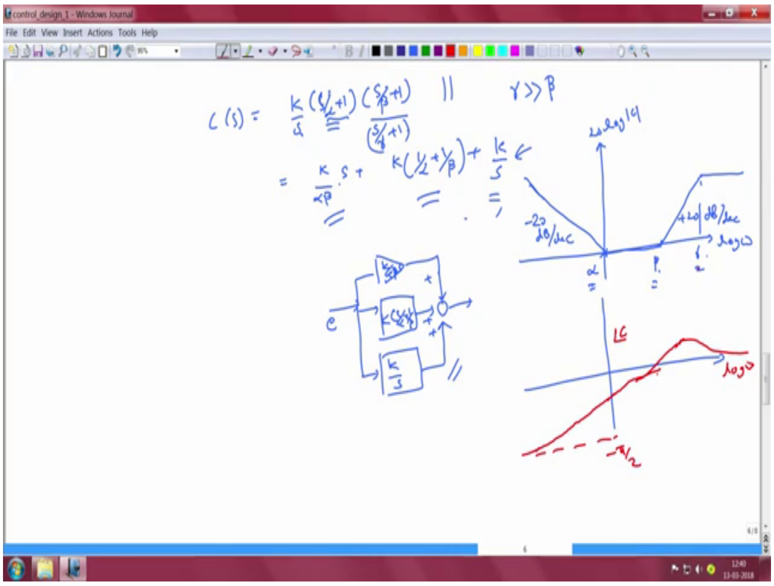

In this in these lectures we have represented PID controllers in this particular form. K by s times s by alpha plus 1 times s by beta plus 1; where this 0 alpha is intended to improve the mid frequency performance and 0 beta is intended to improve the high frequency phase characteristics and thereby result in a stable closed loop system.

Now, we expanded this out as k by alpha beta times s plus k times 1 by alpha plus 1 by beta plus k by s. Now typically PID controllers are represented in this particular form and not in this form. And that is because as far as implementation of a PID controller is concerned, it is easy to implement this particular form. So, one can employ operational amplifiers and achieve band limited differentiation, and thereby realize this term k by alpha beta times s. And then one can use a simple proportional amplifier and realize the term k times 1 by alpha plus 1 by beta. Likewise, one can use another op amp based circuit to realize an integrator as well.

So, we would have k by s also that can be realized. So, this controller would essentially take the error as the input and pass it through a differentiation block, a proportional block and an integration block, add the outputs of all these blocks and the overall output of this addition is what is fed to the plant.

So, since the realization of a PID controller is done by using separate proportional integral and derivative blocks, it is generally represented in this particular manner. However, from the point of view of analysis, from the point of view of understanding what each of these terms the proportional term, the integral term, the derivative term, do to the overall loop gain, it is best to represent a PID controller in the form that I have written out here. Because this allows us to clearly see what each of these terms help with in terms of performance. Integrator helps with low frequency performance, this 0 at alpha helps with mid frequency performance. And a 0 at beta helps with high frequency stability considerations.

There is one other point that I wish to make in connection with the PID controller. If you notice the structure of the controller as it has been written here. We see that we have a second degree polynomial in the numerator of the controller and simply s in the denominator or a simply a first degree polynomial in the denominator; which means that this is not a causal transfer function, or in other words it cannot be physically realized. So, what is done? To avoid this issue is to cascade this controller with a term of the kind 1 by s by gamma plus 1; where the pole gamma has been added at a frequency much greater than beta. This term has been added; primarily, because we want to have a causal implementation or a causal realization of this controller.

Otherwise, it plays no role whatsoever in terms of improving the performance of this controller in any of the frequency ranges of interest to us. But for completeness sake, let me just draw the bode plot of this controller that includes this denominator term s by gamma plus 1 as well.

So, the magnitude characteristics start at start like that of an integrator. So, it reduces at minus 20 dB per decade, at omega equal to alpha it flattens out; because we have added a

0 at omega equal to alpha at. Omega equal to beta it starts to increase at plus 20 decibel per decade. And at omega equal to gamma it once again flattens out.

So, this is the magnitude characteristic. Here I have used omega and log of omega interchangeably. So, when I mark these points as alpha beta and gamma. I mean that log of omega at this point is log of alpha, flag of omega here is log of beta and log of omega there is log of gamma. So, this reduction is minus 20 decibels and the rise, here is plus 20 decibels per decade. How about the phase characteristic? The phase characteristic of this controller will have the integrator characteristic at low frequencies.

So, at low frequencies it will start at minus pi by 2. And then it starts to increase and it is going to be close to minus pi by 4 near the 0. And then it will increase further it will approach 0, at omega equal to beta there will be a further increase in phase. And as omega tends to infinity we would have the phase lead of the 0 at beta be cancelled by the phase lag added by this term by the pool identity omega equal to gamma. Therefore, the phase will come to close asymptotically approach 0 again. So, this is the angle of c as function of log of omega.