Control System Design
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Lecture - 17
General controllers (Part 1/3)

Hello. In the previous clip we looked at translating closed loop frequency domain specifications to open loop frequency domain specifications, and then marking the same on a bode plot. And now we are ready to undertake the design of the system so, that these specifications are met.

Now before we undertake formal design, I want to introduce to you some very commonly used and very popular control architectures, which are already available for us to satisfy these specifications. So, what I would do in this clip therefore, is to introduce to you a few of such very commonly used control architectures, because 1 can readily realize the designed low frequency performance, mid frequency performance and high frequency stability specifications by using these common controllers.

Before I introduce these controllers, I want to draw the bode plot of the plant and I have sketched the bode plot of the plant here. It is assumed that the plant that we are considering in this clip has a constant DC gain.

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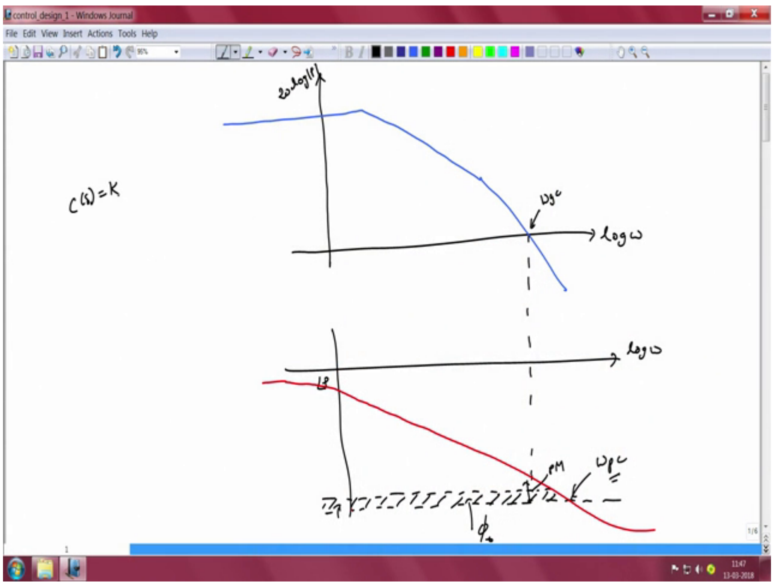

And then it rolls of the in the manner that I have shown here, it may have multiple corner frequencies. Although I have just drawn 2 or 3 in this schematic, it may have many more which are not shown by this schematic. And the phase response of this plant is supposed to look something like this by this red curve. So, it starts from a phase close to 0 and then starts to decay monotonically, it crosses over the certain frequency, which we know is the phase crossover frequency and then further continues to decrease at higher frequencies.

Likewise, the gain characteristic is assumed to crossover at gain cross over frequency omega gc, and the difference between the phase lag of the plant and minus pi at the gain crossover frequency is of course, the phase margin of our system the way it exist even before we have attempted to design any controller. Now since, we are interested in achieving a minimum extent of stability for our closed loop system, and we discovered that there is a direct correlation between the stability of our closed loop system and the phase margin, because the phase margin is related to the close loop damping.

And the closed loop damping is intern related to how far away our closed loop poles are from the imaginary axis, and therefore away from the threshold of instability. Thus, we would like to have a certain minimum phase margin to ensure that our close loop system always meets this particular damping requirement and I shall call that minimum phase margin as phi naught.

So, what we shall attempt to do when we are looking at how the different already available popular controllers affect our open loop bode plot is to make sure that, none of them will result in the bode plot having a phase margin less than this particular minimum value, which I shall call as phi naught. So, what we shall do next is to look at some of these common already available readily available control architectures, and see in what way they help to improve the performance of the closed loop system.

The first and the most simple controller is the proportional controller. So, the transfer function of a proportional controller is c of s is equal to k.

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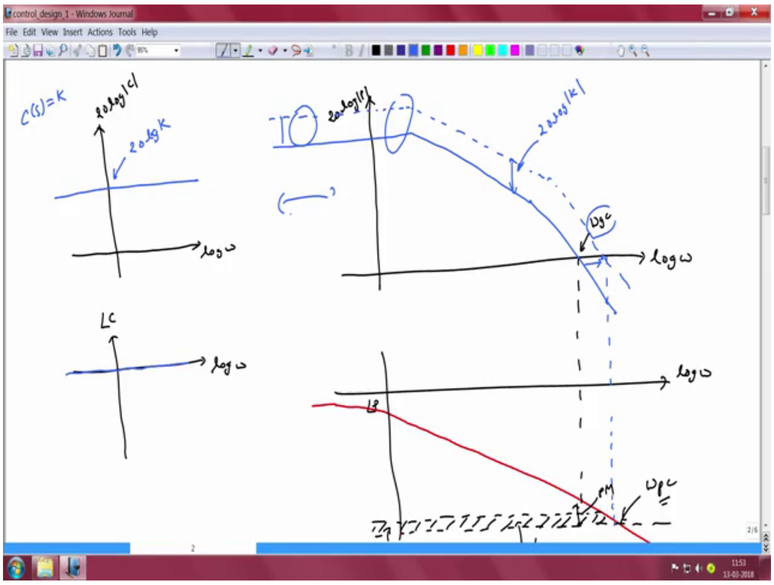

Now, if you want to plot bode plot of a proportional controller, then we would have the magnitude characteristic to be a simple constant, independent of frequency and the value would be 20 log of k and the phase of this controller c of s equal to k would be constant and equal to 0 at all frequencies. So, the phase characteristic would be 0 and the magnitude characteristic would be a constant. Now how would this change the bode plot of our open loop system? So, we have the open loop system to be a cascade of the controllers transfer function and the plant transfer function, the bode plot of the plant transfer functions has already been indicated on the right.

So, we just need to add these 2 to understand how the plant transfer function would appear. So, we would notice that the phase plot would remain unchanged because at all frequencies the phase of the controller will be equal to 0. So, the phase of c times p will simply be equal to the phase of p itself, whereas the gain of the open loop system c times would be increased the entire gain characteristic of the plant would be pushed up by a constant value equal to 20 log of k.

So, if I want to indicate that on the same graph the gain characteristic would look something like this, the gain characteristic of the plant has just got pushed up and the amount by which it has got pushed up is 20 log magnitude of k. Now, can we push up the gain characteristic indefinitely for the case of the general plant? Clearly it is not possible because if you see what has happened, when we increase the gain the gain crossover frequency omega gc has shifted to the right and at the new location where the gain crosses over. We see that the phase margin is not as large as it was before and in particular it has entered into this forbidden zone, where we wanted a minimum phase margin of phi naught. So, there is only so much increase in gain that is possible without destabilizing the close loop system.

Therefore, the utility of a proportional controller is rather limited, because in on account of the fact that you have only modest increase in gain possible in order to ensure that the close loop system is still going to be stable. We also end up having to achieve very low increase in gain in the low and the mid frequency regions. So although, we might want to have fairly high gains in these frequency ranges where stability is not really a concern, the proportional controller does not allow us to increase the gain indefinitely. Because any increasing in gain in that frequency range also results in an equal increasing gain in the high frequency range and could potentially destabilize our close loop system.

So, the utility of the proportional controller is rather limited and there are certain circumstances where a proportional controller can still be used, to get respectable performance. And it is in case of plants, which do not have a phase crossover at all. Now for instance 1 possible example are thermal systems such as furnaces and you know refrigerators and air conditioners and so on. Generally, these plants can be mod can be modeled fairly well using a first order transfer function. Now if the model of the plant is accurately captured by a first order transfer function, then we know that the phase characteristic of a first order system can go from 0 to minus 90 degrees.

So, it will never even reach minus 180 degrees. And therefore, this concern of phase crossover and instability and. So, 1 do not even come up. So, the proportional controller is suitable for first order systems such as thermal systems, but for most other cases on it is own a proportional controller is not very useful. Now, if we come back to the bode plot, we see that the proportional controller has not allowed us to increase the gain significantly in the low frequency range. And, one alternate technique that allows us to increase the gain in the low frequency range is the integral controller.

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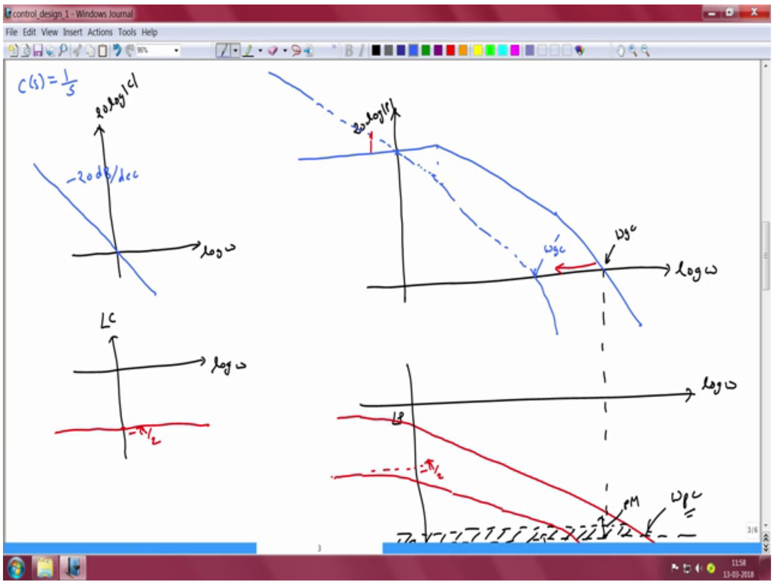

So, the transfer function of an integral controller is c of s is equal to 1 by s. If you want to draw the bode plot of an integrate an integral controller. We would have the magnitude characteristic to be a straight line of slope minus 20 decibels per decade and the phase of this transfer function when we set s is equal to j omega, we will see that the phase angle of c will be equal to minus 90 degrees, so minus pi by 2 at all frequencies.

So, even in this case the phase is constant, but it is not equal to 0, but instead it is equal to minus pi by 2. Now if you want to look at the overall open loop characteristics bode a plot of the open loop system, we have to add the bode plot of the controller with that of the bode plot of the plant. So, when we add these 2 we see that when log omega is equal to 0, then the gain of the controller is also 0. Therefore, when we add these 2 curves, we would get a curve that looks something like this. So, in the low frequency region we have this integrator characteristic resulting in this increasing magnitude characteristic with reducing frequency. And then this the new characteristic will have the same gain as that of the plant at log omega equal to 0 or in other words at omega equal to 1.

And then beyond that the gain of the controller has become less than 0 dB. Therefore, the gain of the open loop system will be less than that of the plant itself. And at the first corner frequency of the plant the role of the rate of decrease of gain will be increased, because of the presence of the plant pole and so on and so forth. So, the magnitude characteristic would look something like a dotted line shown on the right.

As far as the phase characteristic is concerned, we note that the phase characteristic of the integrator simply adds minus 90 degrees to the phase characteristic of the already existing plant. So, the phase characteristic of the open loop system is obtained by simply pushing down the phase characteristic of the plant by minus 90 degrees. So, the phase characteristic of the open loop system might look something like this.

And it starts at minus 90 degrees or minus pi by 2, and then it decays down as in a manner that is dictated by the phase characteristics of the plant itself. So, what are the advantages of an integral controller? If you see we notice that the integral controller has managed to increase the gain of the open loop system, in the low frequency range or in the frequency range where which are which is much less than the typical locations of the plants poles and zeros. And the smaller the frequency the higher is the gain contributed by the controller, indeed at omega equal to 0 you get infinite gain that is added by the controller. And hence you can get perfect tracking of DC references as a consequence of the fact that an integral controller has infinite gain at omega equal to 0.

So in general therefore, the integral controller results in very high gains in the neighborhood of frequency omega equal to 0 or therefore, in the low frequency range and is therefore, ideal for tracking references or rejecting disturbances in the low frequency range. Again I want to remind you that the notion of low frequency is relative to the typical gain crossover frequency of the plant or the typical locations of the poles and the zeros of the plant. But what is its disadvantage? We notice that as a result of the magnitude characteristic of the integrator being less than 0 dB beyond omega equal to 1, it attenuates the gain of the plant and then causes the gain crossover frequency to reduce from what it was earlier for the original plant to a new location omega g c prime, which is much less than what it was before.

And since the closed loop band width is directly related to the gain crossover frequency, we note that accompanying this reduction in gain crossover frequency we would also have a reduction and closed loop bandwidth and the reduction in the closed loop bandwidth mean that we will not be able to track fast changing signals in the manner that we were able to do before. So, this is the price that one pays for simply going with a proportional controller.

While we are able to achieve high gain in the low frequency region and thereby achieve very good performance in tracking references and rejecting disturbances and so on. In that frequency range, the disadvantage is that we are reducing the closed loop bandwidth of our control system. And therefore, are not able to track references that we could track reasonably well earlier.

Now, why has this happened? So, on one hand this controller is better than the proportional controller, because at least in 1 frequency range namely the low frequency range we are able to approach the ideal of control namely to have very high loop gains and thereby achieve good tracking of references. But in the mid frequency range and the high frequency range the price that we are paying is that the gain has reduced significantly and our closed loop bandwidth. Therefore, has also reduced. The reason this is happened of course, is evident if we returned to the bode plot of the controller.

So, we see that this controller has a gain that is less than 0 dB beyond omega equal to 1 more generally, if we choose a controller of the form c of s is equal to k by s it would have a gain less than 0 dB beyond omega equal to k. Now attenuating the gain of an open loop system is in some sense running against the basic motto and philosophy of control engineering. We want to have as higher gain as possible because high gain is what allows to get reference is to be tracked and disturbances to be rejected.

Attenuation of the gain is counter to our objective as control engineers, and this is precisely what is preventing this controller from being able to track fast changing references. So, what can be done to address this issue? What we can do if we return to the bode plot of the controller again, is to note that if this decreasing characteristic were not to be there at least if it was not attenuating the gain of the plant, but the controller neither added gain nor attenuated the gain. Then at least we would have a bandwidth for the closed loop system that would be comparable to that of the plant itself. How do we accomplish this objective? We accomplish this by placing a 0 at the location where the gain of the controller crosses over. So, namely at omega equal to k for this general transfer function c of s is equal to k by s.

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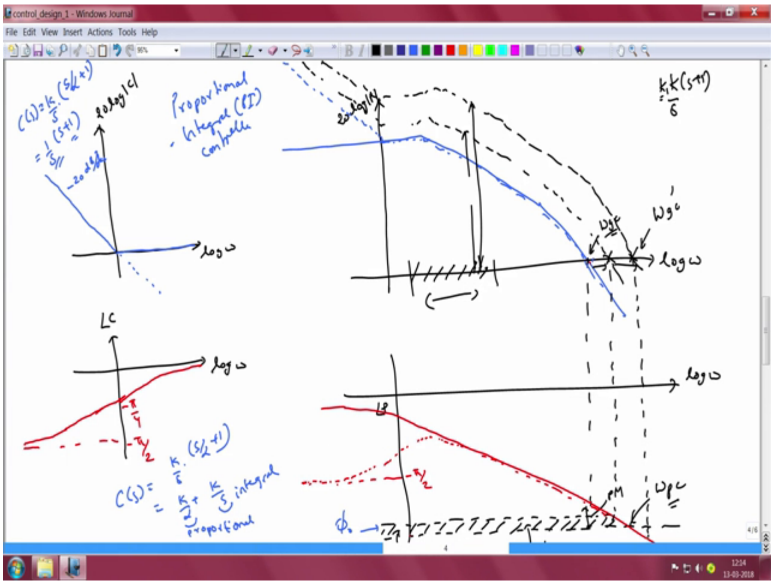

More generally, if we want to locate the 0 at a place where in the neighborhood of where the magnitude characteristics of the integrator crosses over, we would have a controller of the kind c of s is equal to k by s times, because we have chosen to add a 0, we would have a term of the kind s by alpha plus 1. Now in the particular case that we are considering where k is equal to 1 and alpha is equal to 1, we would have the controller to be 1 by s times s plus 1.

What will the magnitude characteristic of this kind of a transfer function look like? For very low frequencies we note that this term s plus 1 does not add either gain or phase. So, the frequency response would be dominated by that of a integrator. So, asymptotic bode plot would be a straight line of slope minus 20 dB per decade, up to the corner frequency omega equal to 1 radian per second. Now at a corner frequency omega equal to 1 instead of attenuating the gain instead of that gain of a controller becoming negative as it happened in case of the integrator, the gain of this new transfer function k by s times s by alpha plus 1 will flatten out

So, it will coincide with the x axis or in other words the gain will be 0 dB. What about the phase characteristics? We know that at very low frequencies this term s plus 1 or s by alpha plus 1 contribute significantly to the phase the phase of this term will be close to 0. So, the overall phase will be dominated by that of the integrator. So, at very low frequencies is the phase of this transfer function will be close to minus pi by 2 or minus

90 degrees, but as we approach omega equal to 1 radian per second, the phase will gradually increase and that omega equal to 1 radian per second the term s plus 1 will contribute to a phase lead of plus 45 degrees. Therefore, the phase of the overall controller will start at minus 90 degrees and at omega equal to 1 it will go to minus 45 degrees or minus pi by 4 and as omega tends to infinity we see that the phase tends to 0 the overall phase of this transfer function tends to 0.

So, this is the approximate phase characteristic of this controller. How will this modify the overall bode plot of our open loop system namely that of the plant times the controller? So, we see that up to omega equal to 1, we would have the integrating characteristic add at one to that of the plant. So, bode plot would look something like this.

Now had we continued with an integrator, there would be an attenuation in the gain beyond omega equal to 1. But because your chosen a transfer function of this kind we see that this transfer function does not attenuate the gain of the plant indeed it neither amplifies nor attenuates, because the gain here is 0 dB or the gain is simply equal to 1. So, beyond the first corner frequency of the controller, the controller ceases to modify the plants characteristics. So, the overall characteristics will magnitude characteristics will of simply coincide with that of the plant itself.

And the gain crossover frequency with therefore, be still, but of the plant itself. What about the phase characteristic? We see that at low frequencies the phase is close to minus pi by 2. So, the overall phase of the controller times the plant will start at close to minus pi by 2 if the plants phase itself is starting close to 0 degrees.

So, this is where it will start. And then as the frequencies increased we would have an increase in phase due to the controller phase characteristics. So, the phase will start to increase and for frequencies well beyond omega equal to 1 of the location of the 0 of a controller, we see that a net phase of the controller has gone to 0. Therefore, the overall phase of the open loop system will be simply equal to the phase of the plant itself.

Therefore beyond a certain frequency, the phase characteristics will start to coincide with a phase characteristic of the plant itself. So, it starts at minus pi by 2. And then, gradually increases in the low frequency range, and then it starts to go inside with the phase characteristic of the plant itself. Now what is the advantage of this controller? When you look at the integral controller and compare what we have accomplished, you see that we have managed to retain the advantages of the integral controller, because this controller also ensures that the gain in a low frequency range is quite high and just like an integrator, it has infinite gain at omega equal to 0. And therefore, unable you to track DC references and reject DC disturbances perfectly.

But if you come to the mid frequency and the high frequency ranges. So, when we once again talk of mid frequency and high frequency, it is with reference to the gain crossover frequency of the open loop plant. So, we see that in the mid frequency range, which is the range in which the plants dynamics start to become prominent. We see that the gain of the overall system simply coincides with the gain of the plant is not an attenuated version of the gain of the plant with the consequence that the gain crossover frequency is exactly what it was for simple for the plant itself.

So, the control another attenuates nor amplifies the gain. And therefore, does not do anything to improve the performance in the mid frequency and high frequency ranges. This contrast with the performance of an integral controller, because integral controller actually attenuates the gain reduces the gain crossover frequency. And thereby prevents us from tracking references that could otherwise that could otherwise have been track reasonably well given the gain characteristics of the plant itself.

So, this controller we can expand and write it as c of s is equal to k by s times s pi alpha plus 1, and when expanded it will it will appear as k by alpha plus k by s. Now this can be viewed as a summation of 2 terms this is a proportional term and this is an integral term. So, this is a proportional term and this a integral term, and because this controller is a summation of a proportional controller and an integral controller this is called a PI or a Proportional Integral controller proportional integral or bracket PI controller. So, the purpose of a PI controller is to achieve good low frequency performance by enabling us to increase the loop gain in the low frequency region.

And achieve modest mid frequency performance, because we have managed to restore the magnitude characteristics or to that of the plant itself. More generally, if we choose to locate the 0 not exactly at the place where the magnitude characteristics of the controller crossover it will still allow us to achieve it may allow us to achieve slightly higher gains than what is possible by the plant itself. But the possible improvement is not significant is not as spectacular as it is in the low frequency range. Now suppose we return to the PI controller and let us say we have done the best that we can with a PI controller. So, if we return to the bode plot of the PI controller we have the magnitude characteristics resulting in the same gain cross over frequency as that of the plant. And therefore, the phase margin is exactly that of the plant itself.

And the phase margin of the plant is actually more than a little bit more than the value phi naught, that we set for ourselves as a minimum phase margin that we have to maintain in the course of our design. So, what do allows us to do is to increase the gain a little bit further. So, that the magnitude characteristics we look something like this. What is the maximum possible improvement in gain that is that can be achieved? The maximum possible improvement is one for which the phase margin becomes exactly equal to phi naught. Now if we increase the gain further then the phase margin will drop below phi naught and we are violating our specification that we have set for the minimum phase margin necessary.

So, we are therefore, able to increase the gain a little bit. So, our controller can look like k by s times s plus 1. Where k is at little bit of increment in gain that can be achieved in on account of the fact that there is a small difference between the phase margin of the plant and the minimum phase margin that we have set for ourselves. And this is a best that we can do with a PI controller we cannot increase the gain any further.

But suppose our requirements for either rejecting a disturbance or tracking a reference demands that we need slightly higher gain, especially in the mid frequency range. So, in the frequency range where we begin to see the plants dynamics, but we have not yet crossed the plants bode plot has not yet cross over. So, that is what we define as the mid frequency range.

In this bode plot I have plotted this mid frequency range. And suppose we have disturbances whose frequency contents in this range or references whose frequency content is in these range that need to be tracked. And suppose the gain that we are able to achieve using a PI controller namely this much magnitude is inadequate for us to track the references with a desired extent of accuracy or reject disturbances by the desired amount then what can we do. So, one possible way forward to address this requirement of higher gain in order to track the references or disturbances in this frequency range a little bit better is to further increase the gain by multiplying this with a constant k 1. Now if you able to do that our bode plot would look something like this. We have increase the gain further and therefore, our gain crossover frequency omega gc has shifted further to the right.

So, the gain crossover frequency of the plant is shown here, this becomes the gain crossover frequency of the plant times a PI controller. Now this becomes the gain crossover frequency of a plant times the PI controller times this gain k 1. With this it may be possible for us to get adequately high loop gain or C times P at in this frequency range for us to be able to track references or reject disturbances whose frequency content is in this frequency range.

But then in our attempt to accomplish this control objective of rejecting is disturbances or tracking this references adequately. Well, you may notice that we have paid. In fact, a fatal price, because at a new gain crossover frequency omega gc prime a 1 inspects the phase, one notices that the phase of the open loop system has now become greater than minus 180 degrees.