Control System Design
Prof. G.R. Jayanth
Department of Instrumentation and Applied Physics
Indian Institute of Science, Bangalore

Lecture - 16
Steps for performing control design (Part 2/2)

Hello. In the previous clip we had started on doing control design and we had laid out the steps involved in doing control design. So, the first step was to convert time domain specifications on performance into close loop frequency domain specifications, and subsequently to convert close loop frequency domain specifications into open loop frequency domain specifications and execute the design so that these open loop specifications are met. And finally, go back to the close loop system and check whether the close loop specifications are met. And finally, check if the desired time domain specifications are met. So, these are the steps in control design.

So, we have to check both for close loop specifications and for time domain response, because there are approximations that are made in going from time domain to frequency domain and from close loop to open loop. We also discussed that in the interest of appealing to a broad audience, we cannot focus on any one kind of input that might be of interest to one specific community and we have to decide on a particular standard input.

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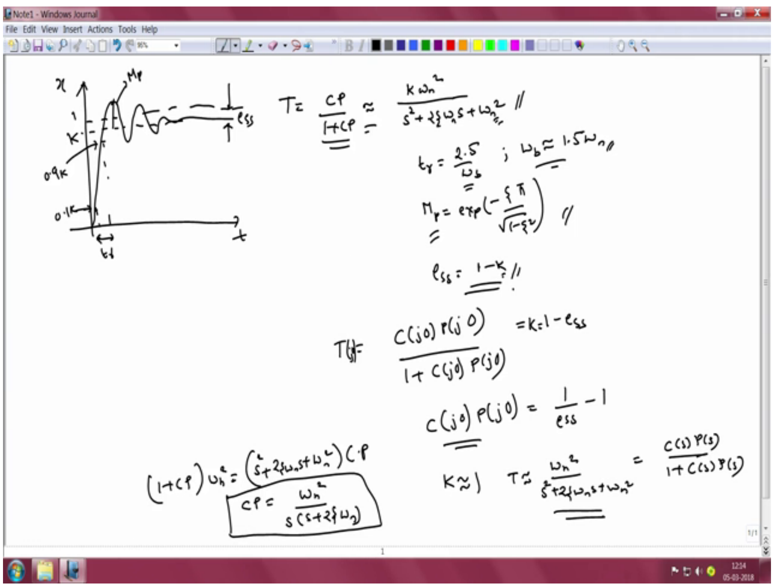

So, which shows the step input as the one as a reference input. And the response to a step input in the step response would be the one that we would be looking at to evaluate the performance and stability of our close loop system.

So, we assumed it as typical step response of a close loop system might look something like this, and we attach a few characteristics to the close loop response and 1 was the rise time which was the time it took for the response to rise from 10 percent of its steady state value to 90 percent of its steady state value. Then there was the peak overshoot and then finally, the steady state error. And then we were confronted with a dilemma about how to translate this time domain specifications into close loop frequency domain specifications, particularly because you are not yet designed the controller

Indeed the control is the one that has to give us the specifications. So, our transmission function T which is equal to C P by 1 plus CP is as at un designed, but yet we have to somehow come up with performance specifications in the closed loop frequency domain that give us these particular closed loop time domain specifications. To undertake this exercise before even having designed c, we invoked the advantages that would accrue from the motion of dominant close loop poles. So, we assume that our close loop system would have a pair of dominant close loop poles.

Regardless of whatever controller structure we might end up having at the end of the day, and with that assumption we could approximate our close loop system to be a second order system. So, this is only an approximate relationship in order to allow us to translate the time domain specifications into corresponding close loop frequency domain specifications subsequently. Now that we have assumed that our close loop system would look somewhat like a second order system though it might definitely not be a second order system in practice. We could then use standard relationships for the rise time the peak overshoot and steady state error for a second order system, and translate the corresponding time domain specifications to correct the parameter of our close loop system.

Namely the constant k which is given which is obtained from the specification of steady state error ESS the damping zeta which can be obtained from the peak overshoot using this equation. And finally, the constant omega n which can be obtained from the rise time specification and this rise time specification relates the rise time to the bandwidth of the closed loop system, and the bandwidth is related to the natural frequency omega n according to this particular equation. So, by using two together one can estimate omega n and fix all the parameters of its approximate model.

Since, we have decided to undertake design using bode plot we have to translate the close specifications into corresponding open loop specification. So, in other words we should come up with the correct performance requirements on the loop gain or the product of the controller and the plant transfer functions that give us this approximate close loop transfer function. So, the easiest to do is to come up with the value of the loop gain or the value of the controller transfer function times the plant transfer function at steady state for dc references. And, that is given by c of g of j 2 times p of j 2 we know that our transfer function is given by C of j 2 times p of j 2 divided by 1 plus C of j 2 time p of j 0.

Therefore if we have specified the steady state error, then we know what value of k we should have and we can show the therefore, should be equal to 1 minus ESS. So, t t of 2 should be equal t of j 2 should be equal to k which in turn should be equal to 1 minus ESS from this particular equation and from this we can work out that our the product of the controller transfer function namely C of j 2 evaluated at 2 frequency, times the plant transfer function evaluated at 2 frequency should be equal to 1 by ESS minus 1. So, converting the close loop specification at 2 frequency, to corresponding open loop specification on C of j 2 times p of j 2 was quite straight forward and this is what e test

Next we have to also translate the other specifications namely that of omega n and that of zeta, to corresponding open loop specifications. Now here once again we shall make an approximation we shall assume that in our a effort to translate the close loop specifications to open loop specifications our d c gain k is very close to 1. So, that may be a small difference between the actual value of a dc gain and one, but we shall assume that for all practical purposes it is equal to 1. And this is not an unreasonable approximation because generally we desire that at least for d c references most control systems show as close to 2 steady state error as it is possible.

So, if you make this approximation, then our transmission function approximately would look like omega n square divided by s square plus 2 zeta omega n s plus omega n square. Now if I go to write this as c of s times p of s divided by 1 plus c of s times p of s then I can undertake the algebra namely that 1 plus C times P times omega n square is equal to s square plus 2 zeta omega n s plus omega N square times CP by simplifying this expression, I would get C times P to be equal to omega n square divided by s times s plus 2 zeta omega n. So, the open loop transfer function which gives us this approximate close loop transfer function is given by this expression here.

Now, we can use this expression to sketch the approximate bode plot for our open loop system and use that to determine the gain cross over frequency and the corresponding phase margin.

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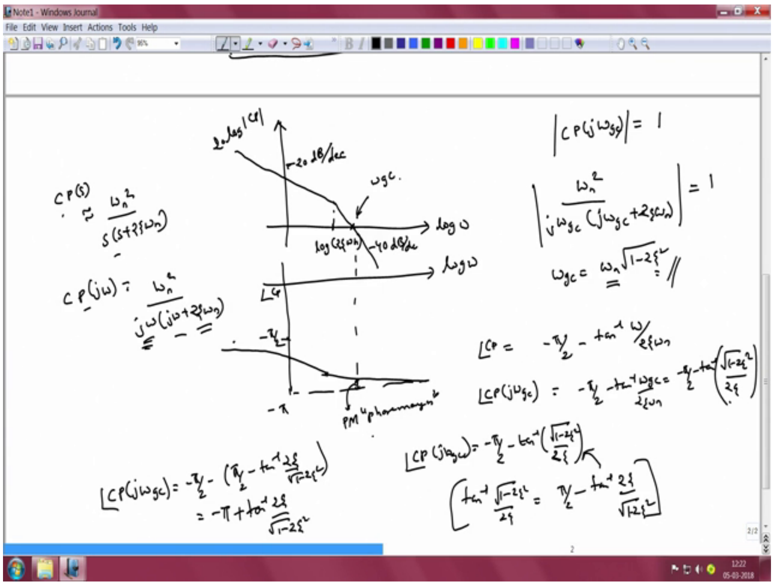

So, to remind you from the previous slide, we have the open loop approximate transfer function this is not the exact transfer function, because we have not yet designed the controller c. In order for us to get the approximate closed loop transfer function that we wrote out in the previous slide, the approximate open loop transfer function look should look something like this omega n square by s times s plus 2 zeta omega n.

So, to draw its bode plot we have to substitute s is equal to j omega, in which case we have we would have CP of j omega to be equal to omega n square divided by j omega times j omega plus 2 zeta omega n. We notice that there is one corner frequency at omega equal to 2 zeta omega n for this particular open loop system. So, this is the corner frequency and we have an integrator also as part of our open loop dynamics. So, our bode plot would therefore, have decaying characteristic and slope would be minus 20 decibels per decade up to the corner frequency minus 2 zeta omega n.

After which you would have the slope increase in its magnitude further, and it would reduce at minus 40 decibels per decade, because you would have another 20 decibels per decade contribution from this pole of the approximate open loop system. And this would get the magnitude characteristic to crossover at some particular frequency omega g c. How will the phase response look like approximate phase response, I want a emphasize that we have not design the controller.

So, we are not yet determine the exact transfer function c times p, all we are doing is coming up with an approximate transfer function that allow us to get the approximate closed loop transfer function, which in turn was useful in transferring the time domain closed loop specifications to frequency domain close loop specifications. So, the phase response for this transfer function c times p, we will start at minus 90 degrees because we have a integrator here. So, it will start at minus pi by 2 radians and then it will reduce and it asymptotically approach minus pi and it never crosses minus pi.

So, this is going to be the phase response and at the gain cross over frequency, we can evaluate the phase of this transfer function and the difference between minus phi and this phase is by definition the phase margin. Now what we are trying to see is whether we can relate the phase margin and the gain crossover frequency to the parameters omega n and zeta of our closed loop system.

So, to determine the gain crossover frequency by definition we have to set magnitude of CP of j omega g c where omega g c is the gain crossover frequency by definition there should be equal to 1. If this is equal to 1 then of course, 20 log of magnitude of this would be equal to 2 dB. And then would we have a gain crossing over the frequency axis in the bode plot. So, if this has to be true then substituting it in the expression for C times P we would have omega n square by j omega g c times j omega g c plus 2 zeta omega n magnitude should be equal to 1. So, you can solve this particular algebraic equation for omega g c and you would get omega g c to be equal to omega n square root of 1 minus 2 zeta square.

So, we see that if we have identified omega n and this can be obtained from the rise time specification in the time domain step response of our closed loop system. And if you have determined zeta which can be obtained from the peak overshoot specification, we can determine therefore, the gain crossover frequency of our open loop system to be equal to approximately omega n times square root of 1 minus 2 zeta square. We can now that we have identified the exact gain crossover frequency. You can also estimate the phase margin because the phase of this open loop system approximate open loop system would be given by the phase of the integrator.

And a phase lag of integrator is a constant at all frequencies and that is equal to minus pi by 2. And the phase lag ordered by the other pole at s is equal to minus 2 zeta omega n, and that phase lag will be equal to minus tan inverse omega by 2 zeta omega n where omega is a frequency at which we are evaluating the phase of that transfer function. Now we know that we have to evaluated at omega g c in order for us to extract the phase margin. So, the phase of C times P is given by this expression and the phase of C times P at omega g c is give. Therefore, given by minus pi by 2 minus tan inverse omega g c by 2 zeta omega n and that in turn is equal to minus pi by 2 minus tan inverse square root of 1 minus 2 zeta square divided by 2 zeta.

I get this expression by plugging in the expression for omega g c from here. So, the angle of C times P at j omega g c is equal to minus pi by 2 minus tan inverse of square root of 1 minus 2 zeta square divided by 2 zeta. And I use the identity that tan inverse of square root of 1 minus 2 zeta square by 2 zeta is nothing but pi by 2 minus tan inverse of 2 zeta by square root of 1 minus 2 zeta square. So, the standard identity from trigonometry that, I am employing and if I want to do that and plug this identity into this particular equation.

I would have angle of CP at j omega g c to be equal to minus pi by 2 minus pi by 2 minus tan inverse 2 zeta by square root of 1 minus 2 zeta square. And that is equal to minus pi plus tan inverse 2 zeta by square root of 1 minus 2 zeta square.

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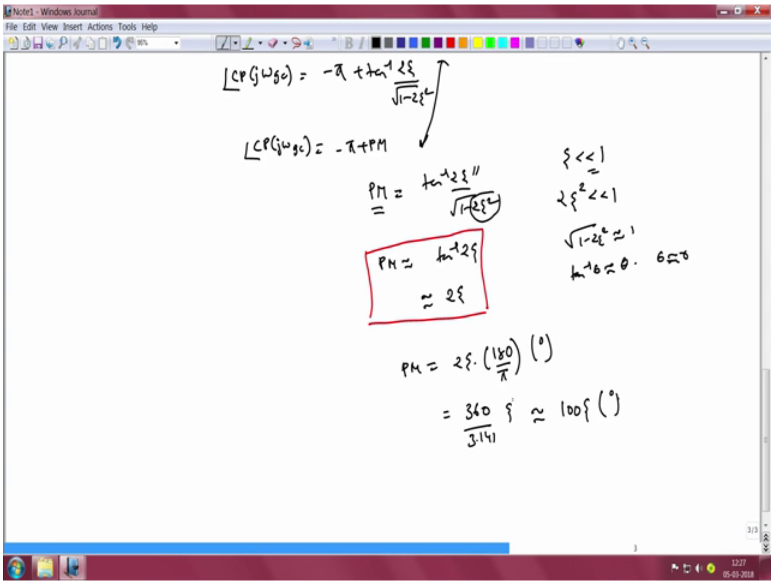

From the previous slide we have angle of CP at the gain crossover frequency to be equal to minus pi plus tan inverse 2 zeta by square root of 1 minus 2 zeta square and by definition of phase margin. We know that the angle of CP at j omega c omega g c is by definition equal to minus pi plus p m, where p m is the phase margin of our open loop system.

So, comp by comparing these 2 equations, we note that the phase margin is given by tan inverse 2 zeta by square root of 1 minus 2 zeta square. Therefore, we note that if you have to specify the damping coefficient for our closed loop systems zeta, which we in turn extract from the desired peak overshoot m p we are effectively specifying the phase margin of our system. And a couple of clips back we discussed that the phase margin is a measure of stability of our closed loop system it is. And it is the maximum permissible phase lag that our controller can potentially add before destabilizing the system.

So therefore, specifying the closed loop damping or in other words equivalently the maximum overshoot in the time domain is equivalent to specifying a desired stability margin for your close loop system. Now we can simplify the expression for the phase margin further by noting that if the damping coefficient zeta is much less than 1, then in the denominator of this function tan inverse 2 zeta by square root of 1 minus 2 zeta square the term 2 zeta square would be much less than 1. And what that would mean is that square root of 1 minus 2 zeta square would approximately be equal to 1 so, that the phase margin would be approximately equal to tan inverse 2 zeta.

And if zeta is much less than 1, we know that tan inverse of theta approximately equal to theta in the limit theta is close to 2. Therefore, we can make a additional approximation that the phase margin is approximately equal to just 2 zeta. So, this fairly simple expression allows us to very quickly compute the phase margin of our close loop system in radian, if we have specified the closed loop damping coefficient zeta. Now what is often done when we do control design is to represent the angles not in radians, but rather in degrees.

So, if 1 more to represent this in degrees, then 1 would have the phase margin to be equal to 2 zeta times 180 by pi this is in degrees. And we note that 2 times 180 is 360 and the value of pi is 3.141. And therefore, the phase margin is going to be equal to 360 by 3.141 times zeta. This can be simplified a little bit further to make it easy for us to remember quickly what the relationship between the phase margin and zeta is going to be, by noting that 360 by three 0.141 is approximately equal to 100.

So, therefore, the phase margin is approximately equal to 100 time zeta, when the phase margin is represented in degrees. So therefore, we have now managed to successfully convert the close loop frequency domain specifications into corresponding open loop frequency domain specifications.

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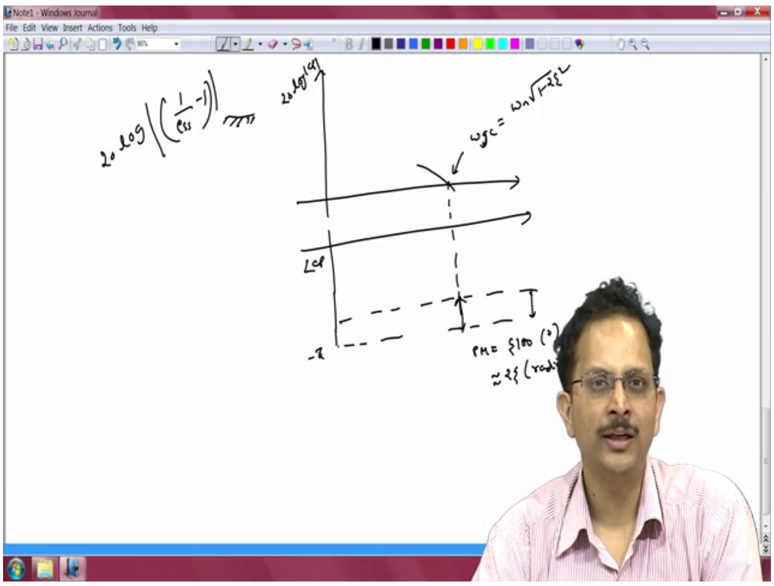

In particular I have sketched here the bode plot of our open loop system. We note that our omega g c can be obtained if we have specified the rise time and the damping coefficient as omega g c equal to omega n square root of 1 minus 2 zeta square, and the phase margin which is the difference between minus pi. And the phase of the plant to be related to zeta as phase margin is equal to zeta times 100, when the phase margin is represented in degrees or is approximately equal to 2 times zeta, when the phase margin is represented in radians.

And finally, our steady state error specification ESS helps us to determine the necessary gain for our open loop system in the vicinity of omega equal to 2 and that gain has to be equal to 1 by ESS minus 1 this is in linear scale to represented in the bode plot it has to be 20 log of magnitude of 1 by ESS minus 1. So, we have now successfully transferred the closed loop frequency domain specifications into corresponding open loop frequency domain specifications, and if you notice here we have made approximation on a log from time domain to closed loop frequency domain and from closed loop frequency domain to open loop frequency domain conversions.

Therefore when we do the design using bode plots, it is imperative for us to convert the design open loop transfer function into the corresponding close loop transfer function, to see if our close loop frequency domain specs have been met correctly. And subsequently to plot the step response of our close loop system, and see whether the time domain specifications have been met by the design closed loop transfer function.

We shall, in the next clip take a look at a few standard controllers that will allow us to design our open loop system to meet the specifications of the kind that we have outlined in this clip.

Thank you.