Control System Design
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Lecture – 15
Steps for performing control design (Part 1/2)

Hello, in the previous clip we took a look at Nyquist stability theory and how it enables to determine the stability of our closed loop system. And subsequently identified some of the drawbacks of the Nyquist plot despite its central importance in frequency domain based control design because, Nyquist plots or the final courts of appeal as for stability of the closed loop system is concerned. We discovered that Nyquist plots are not intuitive and therefore, we cannot easily anticipate how the Nyquist plot of controller times the plant would look, if you are given the Nyquist plots of either the controller or that of the plant.

And that became the motivation for us to search for a better canvas upon which to do our design. And the such candidate was the Bode plot where we saw that we plot the log of magnitude of the transfer function versus the log of frequency. And this allows transforming the product operation which is what is used when one has to plot the Nyquist plot into a summation operation firstly.

And secondly, we have the asymptotic multitude Bode plots to be essentially a bunch of straight lines whose slope change near the corner frequencies of the open loop system. Therefore, it becomes intuitively very easy for us not only to visualize the Bode plot of a plant or a controller, but also anticipate what the Bode plot of plant times the controller would look like because, it could be simply the sum of the Bode plots of that of that of the plant and of that controller.

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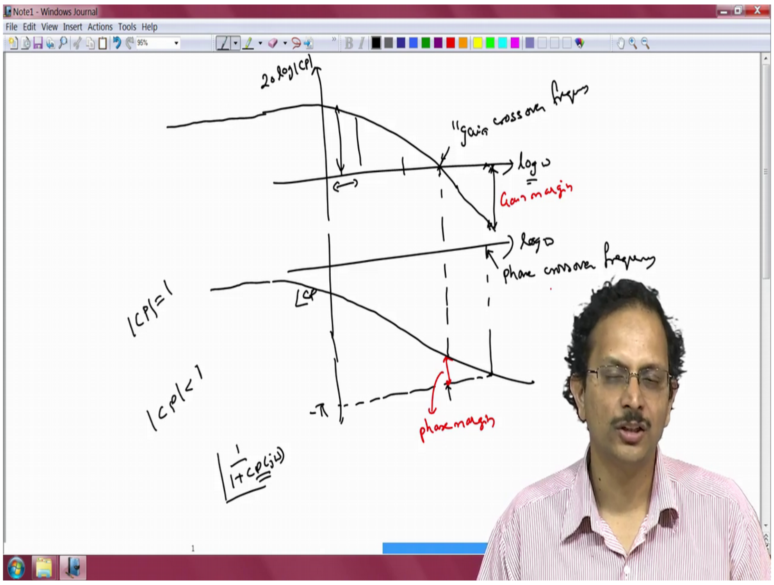

So, I have shown here the typical Bode plot that I drew towards the end of the previous clip for the plant times the controller. What is very convenient about this design tool is that it allows us to keep track of the magnitude of the open loop transfer function namely C times P. And, this magnitude allows us to determine whether we are able to meet our performance specifications or not.

So, if C times P is not large enough then we need to ensure that our controller gain is increased further. So, it becomes large enough and ensures that for example, the error in tracking which is given by 1 by 1 plus C times P of j omega is adequately small within the frequency range where this tracking needs to be performed within in other words within the frequency range of the reference signal. Likewise, if there is an output disturbance that needs to be rejected then the extend by which we reject the output disturbance is once again given by the same term 1 by 1 plus C times P.

So, by making so that C times P of j omega is adequately high in the frequency range where the disturbance output disturbance effects our system, we can make sure that our we are able to reject the disturbance by the desired amount. But all along the good thing about the Bode plot is that every time we make a change to the open loop gain or the phase of the controller. It will change the Bode plot and we can keep track of the new gain crossover frequency and the new phase margin and likewise a new crossover frequency and the new gain margin. And always make sure that both the gain margin and the phase margin are adequately large for us to have an adequately stable closed loop system on our hands.

So, the Bode plot therefore, is a nice tool that allows us to design for performance because the gains at the frequencies of interest to us are directly evident in the plot, at the same time keep track also of the stability because the phase margin and the gain margin is evident in the plot. So, our designs therefore, would be performed using the Bode plot. However, there is there are two issues that one needs to address one adopts the Bode plot as the tool for doing control design.

The first issue is that we note that this is the Bode plot of the open loop system whereas, the typical specifications that would have would be for the closed loop system. So, how to translate the closed loop specifications into open loop specifications, so what is one constraint that is one extra step that has to be undertaken. The second issue that needs to be addressed is that this is a frequency domain based design tool whereas, most of the specifications that are given to us as a engineers would be in the time domain. So, we would expect our transient response to vanish within certain duration of time to would expect our overshoots and ringing to be less than a certain percentage or we may have to have a steady state error less than a certain amount.

So, many of these are time domain specifications and therefore, one has to convert the time domain specifications into frequency domain specifications and then do the design. So, if one wants to adopt Bode plots as the chosen tool for doing control design one has to adopt the following steps. In the first step one has to convert the time domain specifications, which is what is assign to a control engineer into the corresponding frequency domain specifications on the closed loop system.

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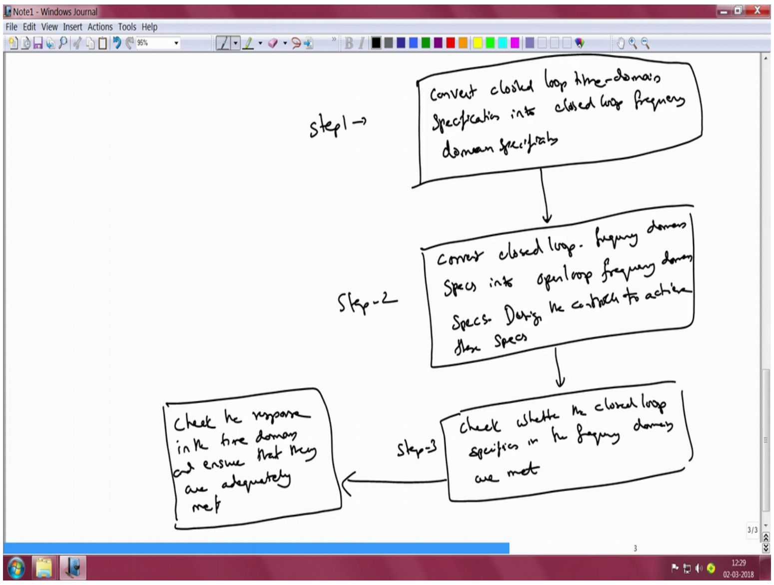

So, the first step 1: Step 1 is to convert closed loop time domain specifications, specifications into closed loop frequency domain specifications; so this is the first step. Now, that we have a closed loop frequency domain specifications we have to next perform this transformation of the closed loop specifications into its corresponding open loop specifications because, we are ultimately doing our control design in using Bode plots where we plot the open loop transfer functions. So, we have to have a conversion of closed loop frequency domain specifications into open loop frequency domain specifications convert closed loop frequency domain specs into open loop frequency domain specs.

Now, that we have the specifications in for the open loop system we can mark them out on our Bode plot and design our controller such that the open loop system namely the controller times the plant achieves these specifications, either in terms of performance requirements or in terms of stability requirements. Now, once our open loop system design is done in a such a manner that it satisfies our closed loop open loop specifications. Then we have to then work out the final closed loop transfer function and check whether the closed loop specifications are also met check whether the closed loop specifications in the frequency domain are met.

Now, there is one final step that one has to adopt in going from closed loop time domain specifications to closed loop frequency domain specifications we make some assumptions and approximations. And therefore, our assumptions and approximations might result in error between the actual specifications and the once that we might eventually achieve. Therefore, after completing the design in the frequency domain one has to look at the response of the system in the time domain to ascertain that the specifications in the time domain have all been adequately met.

So, the last step therefore, is to check the response in the time domain and ensure that they are adequately met. So, if not then one has to change the control structure little bit to ensure that they are met, so we have 4 steps in the first step we convert the closed loop time domain specs to closed loop frequency domain specs. In step two we convert closed loop frequency domain specs to open loop frequency domain specs and then we design the controller to achieve these specs.

In the third step having design the controller and having known the plant transfer function we can now obtain the closed loop transfer function of the system. And we should check whether the frequency domain specifications on the closed loop transfer function have also been met, by our controller structure and that is once again because we have approximations in going from closed loop frequency domain to open loop frequency domain. So, having designed everything in the open loop frequency domain we go back to the closed loop frequency domain to check whether all the specifications have been met and if and if our approximations were reasonable.

But even after all of this there is there are approximations involved in going from closed loop time domain to close closed loop frequency domain. So, even if the closed loop frequency domain specifications are met there is no guarantee that the corresponding time domain specifications are met exactly. They will be met to a large extent, but perhaps there is want to be a scope for some difference between what was specified and what was achieved. So, one has to therefore, mandatorily check the response in the time domain and ensure that the specifications either in terms of disturbance rejection or reference tracking or stability are adequate met.

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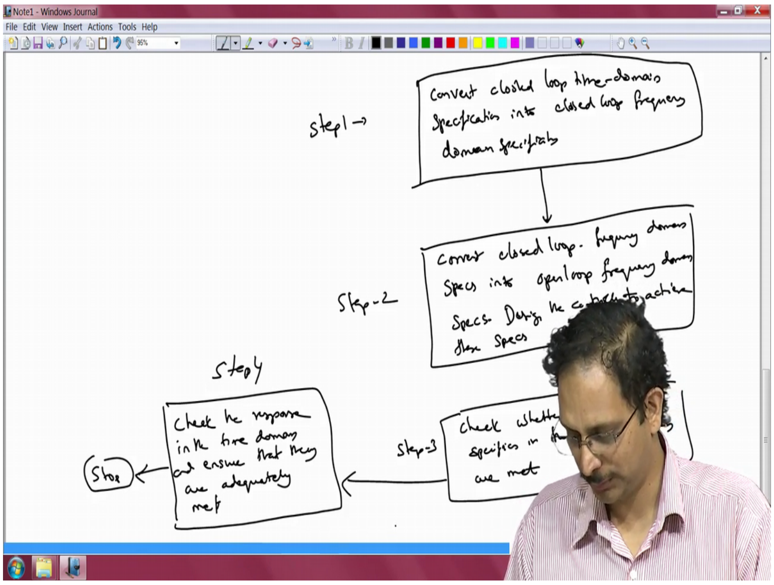

If they are all met then step four would mark the end of our design process and we can stop over control design. So, this is the flow for control design using Bode plots. So, let us now examine the different criteria and the techniques that one would adopt in following these different steps. As control engineers we are generally told to either track a certain reference or a certain set of references or reject a certain set of disturbances.

So, we will be specified the kind of time domain waveform that these references would have and the approximate waveforms of the disturbance also, based on the kind of disturbances that affect the plant that we are trying to control or the kind of references that our plant has to track. The first step since most of our design is being bone in frequency domain the first step is to be able to examine the frequency content of the either the references or the disturbances that we need to track.

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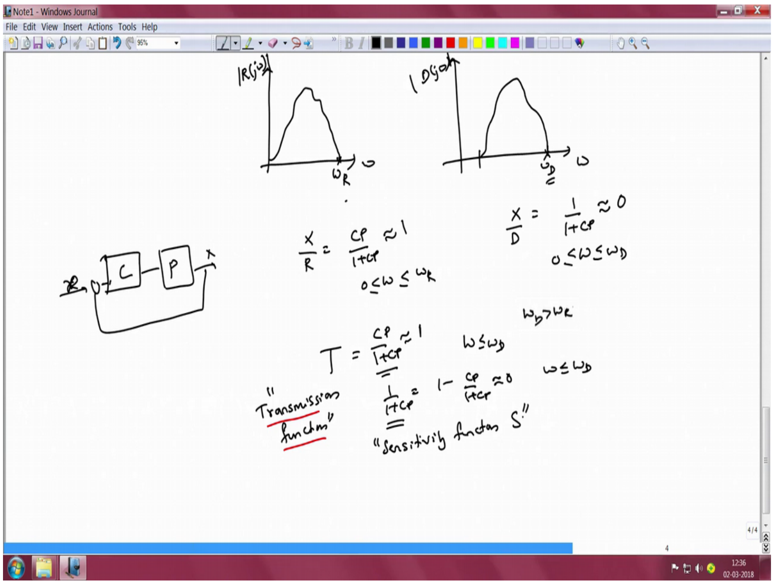

So, for instance I have ported here the magnitude of R of j omega as function of omega and let us say that this is a curve that look something like this. So, your reference has frequency content only up to a frequency omega capital R. It might have frequency content beyond omega R, but that might be so, low that we can ignore it without seriously affecting the performance for tracking our desired references. So, this might be the kind of frequency content for the reference.

And likewise in case of disturbance we may not be able to specify the exact waveform of the disturbance, but we may still be able to pass some judgment on the frequency range of this disturbance. So, we may once again be able to identify the frequency content and in particular we will be able to identify the frequency omega D, beyond which the frequency content or the spectrum of the disturbance would be vanishingly small.

So, what we need to do as control engineers is to make sure that up to omega R the transfer function that relates the output to the reference or which is given by CP by 1 plus CP should be close to 1 and if that is done. So, this should be true for the frequencies going from 0 all the way to omega R, if this is a ensured then we would be able to track the reference with adequate amount of accuracy.

Similarly, as control engineers if the frequency content of our disturbance is significant only up to frequency omega D, we need to make sure that the transfer function that relates the disturbance to the output namely X by D which is going to be equal to 1 by 1 plus CP should be close to 0. It need not be exactly 0, but it should be adequately small for us to reject the disturbances adequately well. So, this is what we need to do, now if you are given a system that is afflicted by both a disturbance as well as it has to track preferences, then P note which of these two numbers omega R or omega D is larger. And make sure that our transfer function CP by 1 plus CP is closed to 1 up to that particular frequency, because let us assume that omega D is the larger of the two in other words if omega D greater than omega R.

If CP by 1 plus CP is close to 1 up to omega D in other words for omega up to omega D then we would see that 1 by 2 plus CP which is simply given by 1 minus CP by 1 plus CP would be close to 0 up to frequencies omega D. Therefore, in our attempt to get CP by 1 plus CP to be equal to 1 all the way up to omega D, we are also ensuring that we are able to reject disturbances adequately well up to that frequency. So, we pick the greater of the two frequencies and try to make sure that our transfer function CP by 1 plus CP is close to 1 up to that frequency. Hence fourth let us coin a particular name for this transfer functions CP by 1 plus CP, we shall call it the transmission function T.

There is a good reason why T is called the transmission function because if one were to look at the feedback block diagram where you would have C and P and the output X and the reference R. That we would see that the extend by which the reference R is transmitted through the closed loop system is given by this transfer function CP by 1 plus CP and hence the name transmission function. Likewise there is a special name for the transfer function 1 by 1 plus CP and it is called the sensitivity function yes. So, we want the sensitivity function to be adequately low in the frequency range where we want to reject disturbances adequately well and we want the transmissions function to be as close to unity as is necessary.

In order to make sure that we track the references up to the desired frequency range omega R. Now, the exact spectrum of the reference for the disturbance is application specific however, this course is intended for a broad audience which could be people from electrical engineering backgrounds or mechanical engineering backgrounds or chemical engineering backgrounds and so on and so forth. And we cannot graph the exact spectrum of the references that are of interest all of these and we cannot decide on only one particular application that might be of interest to one community of engineers.

So, rather than deciding on tracking some general reference whose frequency content is shown in this slide we should decide on a standard reference that we would track, and examine the performance of the closed loop system in its ability to track this standard reference. So, what would be a good candidate for such a standard reference, we have impulse inputs we have step inputs we have triangular inputs we have sinusoidal inputs or some of the commonly used standard inputs. And a movement start would reveal that among these candidates the step input is a good candidate for being a standard input for us to look at the response of our closed loop system.

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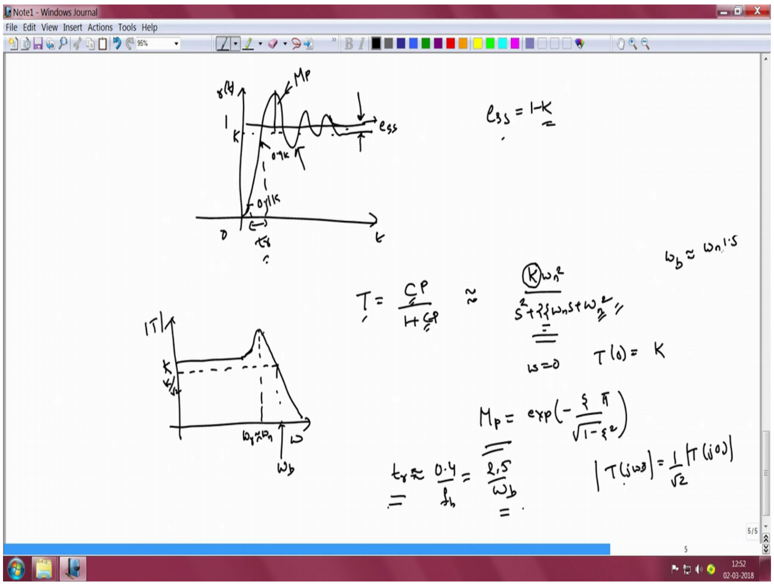

And that is because it reveals clearly the transient response of our closed loop system. Since, the input does not change after its initial step change from 0 to 1; we can clearly see the transient response of our system. This on the other hand is more difficult to do with sinusoidal inputs or triangular inputs or other inputs the changed with time because the transient response gets added on to the variation of the signal itself in time, and that makes it difficult for us to clearly visualize the transient part of the response from the signal itself.

So, a step signal is a good candidate for looking at a transient response, but also it is a good candidate for looking at the steady state behavior. If my closed loop system has a certain steady state error to constant references that would be revealed by a step input.

This in contrast is not revealed for instance by a an impulse input which once again does not change after it is, after it is initially applied around T equal to 0.

But, then the an impulse response does not reveal information about the steady state error of the closed loop system to constant references. Therefore, it is of used for us to adopt the steps input as the common reference input against which we would check the performance of our closed loop system. Now, as control engineers we might have a certain specifications in terms of the step response of the close loop system that need to be satisfied. If we look at the typical step response which might look as I have done in the figure here, we can mark out some of these specifications one would be the time that it would take for the response to rise up close to its steady state value and this is called the rise time.

So, formally the rise time is defined as the time it takes for the closed loop system to rise from 10 percent of it steady state value let us say its steady state values is K if its steady state value is K the time it takes for the response. So, rise from 10 percent of K to 90 percent of K 0.9 of K this time is called the rise time and depending on how fast we want our response to be we would want to choose this rise time to be adequately low.

Another characteristic of the step response that would be of interest to us is the error that exists after a steady input has been provided and that is captured by the steady state error ess. And it is evident that T if the steady state value of our response to a step input is K and the steady state error ess is given by 1 minus K. And as control engineers we want the steady state error typically to be quite small, especially if you are interested in tracking slow varying references the frequency is close to 0.

A third characteristic of the step response that would be of interest to us is the amount of ringing that we have here and the extend by which the response overshoots beyond its steady state value. In fact, these two are related the more is the number of the more is the extent of overshoot beyond its steady state value the larger will be the number of oscillations before its settles down.

So, we need not have to separately specified the number of oscillations and the percentage of the overshoot we can simply specify the percentage overshoot which we call as MP. And make sure that our closed loop system has this percentage overshoot to be neither too small nor too large.

So, there are other characteristics that you can identify for example, you can look at the settling time of the closed loop system which would be the time that it would take for the response to be within a certain band in the neighborhood of a steady state response. So, other such characteristics can also be um identified for the closed loop response, but for now we shall limit ourselves to three characteristics as a desirable characteristics for our closed loop response, namely the rise time the peak overshoot and the steady state error.

Now that we have these three time domain specifications on performance we now have to design a closed loop system which allows us to achieve these particular specifications. To do that as we said if we have to do it using Bode plots we have to first transform these time domain specifications into corresponding frequency domain specifications. These frequency domain specifications have to be for the closed loop system and here is where we meet our first road block.

The closed loop system transfer function is given by T is equal to C times P by 1 plus C times P. Now we have to design the controller, so that our closed loop transmission function has a step response that look similar to what has been given here. But we have not yet designed our controller and therefore, our transmission function is not yet known to us given the fact that our transmission function is not yet known to us. We are now at this strange situation where we want to be able to convert the time domain specifications to frequency domain close loop specifications, but we do not know the structure of T.

So, how do we resolve this dilemma, here is where we adopt the notion of dominant poles? Although we might not have designed C and therefore, we do not know the exact structure of T. We assume that the closed loop transfer function T has a pair of dominant poles which are the once that predominantly determine the response of the closed loop system to step inputs. There might be other poles which will coming place on account of the particular structures that the plant and the controller might have. But regardless of the comptroller and the plant transfer functions we assume that our transmission function can be approximated fairly well as a second order system with a pair of complex conjugate poles.

So, in other words we assume that we can write T approximately as K times omega n square where K is a steady state value of the response of the transfer function to a step input divided by S square plus 2 zeta omega n S plus omega n square. So, even though our controller design has to wait until we are done with it using the Bode plots we assume that regardless of what the controller structures might finally, turn out to be we would be able to represent T as predominantly a second order system though for sure there will be other terms, but those terms can be neglected in the in steadying the response to the closed loop system.

Now, that we have approximate model for our closed loop system we can use the time domain specifications to fix the parameters of our closed loop system. So, for instance we have already identified the dc gain of our transmission function to be equal to K because at omega equal to 0 we want when we set S is equal to j omega and set omega equal to 0. So, when we set S is equal to j omega and evaluate it at omega equal to 0 the we would have T of 0 to be equal to K. So, this would be the magnitude of the transmission function at omega equal to 0 and therefore, represents the steady state value of the step response of our closed loop system and that should be equal to the value K that we have already identified from the time domain response.

So, with at least has we two more parameters namely omega n and zeta that need to be identified. If we look at the peak overshoot one can obtain the step response of a second order system which has a transfer function of this particular form and therefore, derive the dependence of the overshoot MP on the parameters of this second order model. It can be shown that MP has this particular structure we would have MP to be equal to exponential of n minus zeta times pi divided by square root of 1 minus zeta square. Therefore, if in the time domain response we have specified MP then we can obtain zeta from this particular equation and therefore, have a specification for the approximate second order closed loop response of our system.

What that lives us with is to the fix the parameter omega n and that we can obtain from the rise time of our closed loop system. So, if we have specified a certain rise time t r we can show that this rise time t r is approximately equal to 04 divided by f b. The f b is the bandwidth of our closed loop system which in turn is equal to 2.5 by omega b where omega b represents the bandwidth of the closed loop system in angular frequency. Now, the motion of a bandwidth is based on the value of the frequency at which the magnitude of T of j omega is equal to 1 by root 2 times its value at omega equal to 0 in other words magnitude of T of j 0.

Now, it can be shown that for a second order system a system that has a transfer function that looks something like this. The frequency at which the magnitude of the transfer function becomes 1 by root 2 times its magnitude at omega equal to 0 which is given by omega b is approximately equal to omega n times 1.5. Therefore, from the rise time specification we can use this equation to obtain omega b and from this equation that relates omega b and omega n we can obtain the frequency omega n.

Now, we have identified all the parameters of our approximate transmission function. So, if we were to graph the approximate transmission function T the magnitude of T as function of omega would look something like this. At very low frequencies its gain is K and as we approach omega n the magnitude will increase and then at the resonant frequency the magnitude will be at its maximum value and when zeta is small this resonant frequency as we discussed is approximately equal to omega n and then it will finally drop off.

The frequency at which the magnitude becomes 1 by root 2 times the steady state value another words the magnitude becomes K by root 2 is what we define as the bandwidth omega b. Now, this equation relates approximately the bandwidth of a second order system with low damping to the natural frequency omega n and this equation relates the rise time to omega b.

So, if in the time domain we have a certain specification on the rise time or in other words the speed with which of the quickness with which we want our closed loop system to track references. Then we can convert that omega b and from that obtain omega n and from that we are able to obtain the approximate transmission function which can subsequently be converted to the corresponding open loop specifications this we shall look at in the next presentation.

Thank you.