Control System Design
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Lecture – 10
Introduction to feedback control (Part 2/2)

Hello. In the previous clip, we were looking at the basic philosophy of feedback control and were attempting to come up with a prototype solution for the problem of inverting a plant. So, the solution that we came up with is shown in this schematic here.

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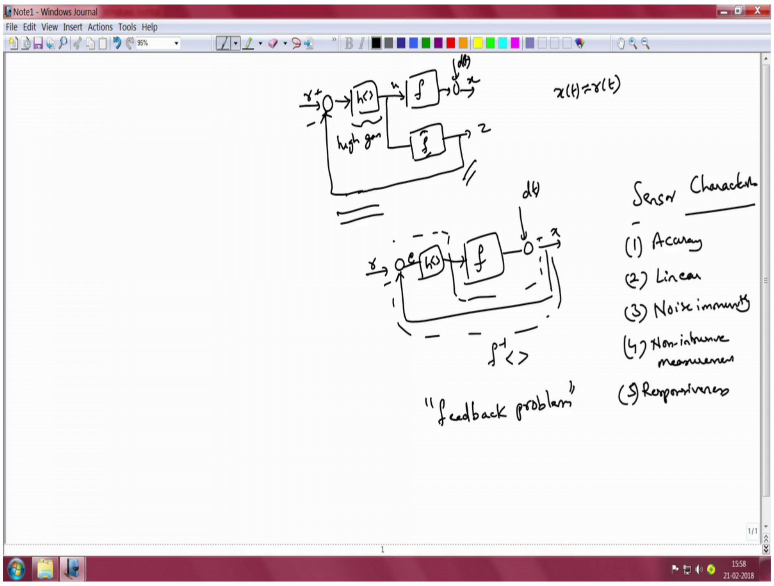

So, we create a copy of the plant which we call as f cap. And then we feedback the output of the copy, and modify the error between the reference and the output of the copy by means of a function h. And we discovered that this function has to be a high gain function, in order for us to be able to invert the plant approximately. And that is at the heart of what you want to do as control engineers. We want to get x of t to be approximately equal to r of t and for us to do that we have to find somewhere to find the f inverse of the plant.

Now, this solution is still an open loop solution to converting a plant in an approximate manner. It will work as long as we can create a good copy f cap of the plants dynamics. So, at the heart of this approach is our ability to model the plant very well, and thereby have f cap to be approximately equal to f. But there could be situations where one cannot model the plant very well. Or even if one models the plant at a certain point in time, one into drift and changes of parameters the plants model might change at a later point in time.

So, in such instances we would have the copy f cap to not be an exact replica of the plant and that in turn would mean that we will not be able to perfectly inverted, and thereby not get x of t to be equal to r of t. There is one other situation where this approach of open loop control cannot get us to ensure the x of t equal to r of t. And that is in the presence of disturbances that we cannot measure.

So, for instance if I have to consider an output disturbance d of t which affects my system. Suppose I cannot measure d of t, then there is no way for me to use that and compensate it is effect when I am creating the copy of the system and disturbance.

So, in both these situations so, in the first situation, we have some uncertainty in relation to the plant. In the second situation we have some uncertainty in relation to the environment in which the plant operates. So, in we have therefore, disturbances either input disturbances or output disturbances that we cannot measure that act on the plant then this very neat prototype solution phase.

However, we can save the day by not creating f cap, but by rather investing in a sensor and directly tapping the output of the plant itself. In other words, instead of f cap if we could choose to directly look at x and feed that back, compare that output with the reference and pass the error e through a high gain function h and the output of the high gain function to f. Then this network where which includes the sensor that is sensing the output and this high gain function together behave like a timbers and therefore, help to invert the plant. And this is at the heart of feedback control.

So, at the heart of feedback control is the assumption that we are able to measure the output of the plant, and then use that information compare it with the desired output of the plant. And then manipulate the error by means of a high gain function in order to minimize the error.

Now, the reason we employed the feedback problem the feedback control architecture as the last architecture was because, one needs to invest in a sensor. And sensors need to have several important characteristics for them to reliably read out the output of the plant. For instance, the sensor has to be accurate, it should be linear, it should have good noise immunity, it should be able to make non-intrusive measurement and it should be responsive.

In other words, the output of the sensor should change at a time scale that is much smaller than the time scale in which the output of the plant itself changes. So, all these characteristics should be present in the sensor that we invest to measure the output of the plant. And if we are able to find such a sensor, then we can implement model inversion by adopting the feedback root. And whenever we invest in a sensor and use it output to correct for the error between the reference and the actual output of the plant, then the problem is called the feedback problem.

So, although in this architecture we seem to have feedback in the model inversion. What we do see is that ultimately we are still processing all the signals inside a computer. It might look like a feedback path, but this feedback path is still inside the computer. We have not really tapped into the actual output of the plant. Hence this is still a filter problem. The first one is still a filter problem you see; however, were to directly look at the output of the plant and compare it with the reference and manipulate the error using the high gain function it is called the feedback problem.

So, we therefore, see the context in which feedbacks feedback becomes unavoidable. It is in a scenario where we either have uncertainty in the model f or this model changes with time in a manner that we cannot predict, or you have disturbances either input disturbance or output disturbances that affect your plant.

So, having discussed the situations in which feedback control is warranted and is unavoidable. Let us now take a look at the advantages of feedback control by means of a numerical example. So, this example is drawn from one of the very one of the very early applications of feedback control, that ultimately let to it is white spread use and development. So, to talk about this example we need to go to the twentieth century early parts of twentieth century during the time of the birth of telephonic.

Now, this was in late 1920’s. Now in order for telephonic to be a practical proposition, engineers had to use what were known as repeater amplifier in order to boost the power of the signal every few kilometers the problem though what is that these repeater amplifiers or non-linear, firstly. Secondly, they suffered from a lot of cross talk.

And thirdly one could not reliably expect a certain amplification factor given a particular amplifier. There used to be a lot of variation in the gains of the amplifier and this the input output relationship also tended to be non-linear and so on and so forth. If you had multiple amplifiers beside one another they would be crosstalk between them. So, this was the kind of these were the kind of amplifier that the electrical engineers was stuck with. They had research quite a bit into improving the characteristics of the repeater amplifier, but without too much success.

So, it was in this setting that an individual by name h s black came up with the idea of feedback control, as a technique whereby one can dramatically improve the amplifiers linearity. One can also minimize greatly the cross coupling the cross talk between amplifiers, and minimize the effect of variation in amplifier gain. So, exactly how this gets done we shall see by means of a numerical example.

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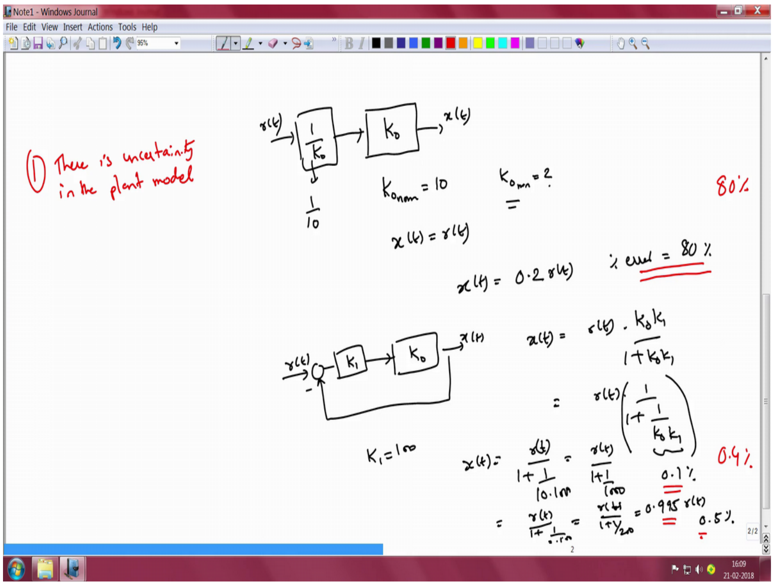

So, let us assume that h s black had amplifiers of a certain gain k naught. Now ordinarily one more expect is gain to be a constant and particular value, but the amplifier during h s black times were unreliable. So, nominally k naught was 10 let us say. But there could be quite a bit of variation between amplifiers. So, it could potentially reduce all the way to 2. So, it could reduce by a factor of phi, let us say.

Now, therefore, it is not possible for us to get the output of the amplifier to be exactly what we desire it to be when we are not really sure about the amplification factor of the amplifier. Suppose we knew it amplification factor precisely let us say we knew it to be exactly 10, then in order to get the output x of t to be equal to r of t, all we need to do is simple open loop inversion of the gain. So, we can just choose a network that attenuates the gain by a factor of 1 by k naught, and feed that to the amplifier in which case you would get x of t to be equal to r of t.

But the problem is this works very well when we know the gain very well, but if for the particular amplifier we are working with the gain is actually 2, but not 10, and when we are attenuating it we have assume that the gain is nominally equal to 10 and therefore, the attenuation factor is 1 by 10, then you would have a huge error. So, you would end up attenuating the output by a factor of 80 percent. So, it is a huge error. It is only in that one case when k naught was exactly equal to 10 would the error be 0 even a slight variation in k naught that would be early large errors in tracking a certain reference.

Here is where we can demonstrate advantage of feedback control. Let us see instead of adopting open loop control. We take the output x of t, feed it back and compare it with the reference r of t. And the error we manipulate by means of a high gain function. In this case I shall choose a high gain function to have a certain waving k 1.

And the output of that high gain function shall be effect to the plant. Now we can show by suitable algebraic manipulation. Here that x of t can be related to r of t by the equation x of t is equal to r of t times k naught k 1 divided by 1 plus k naught k 1 ; which I shall rewrite as r of t times 1 by 1 plus 1 by k naught k 1.

So, you notice that if the term k naught k 1 is very, very large, then 1 by k naught k 1 will be a very small number. So, even if the exact gain k naught might be uncertain, as long as you make sure that 1 by k naught k 1 is always a small number by choosing an appropriately high value for k 1, then we would get x of t to be almost approximately equal to r of t. Because in this case this factor here would be very close to 1.

So, for instance if you choose k 1, the gain of our high gain controller to be equal to 100. Then we see that x of t would be equal to r of t divided by 1 plus 1 by 10 times 100 in the nominal case. So, that is equal to r of t by 1 plus 1 by 1000. So, what we see therefore is that the percentage error in tracking the reference r of t in this in the nominal configuration would be about 0.1 percent. Now what happens when the plant gain were to be different? Let us say it was 2 instead of 10, in which case you would have x of t to be equal to r of t divided by 1 plus 1 by 2 times 100; and x equal to r of t by r of t by 1 plus 1 by 200.

So, we see therefore, that this is equal to 0.995 approximately times r of t. So, what is the percentage error? The percentage error now is nearly 0.5 percent. Now let us compare the performance of our open loop technique and the close loop technique. With our open loop technique, when we employed an amplifier of gain to, but designed our controller assuming at the amplifier would have a gain of 10 the ended up with 80 percent error.

So, when the amplifier actually had a gain of 10 the error was 0 percent, but otherwise gain was too it was immense the error was unacceptably large. On the other hand, when we employee feedback control, we see that when the amplifier gain was actually 10 the error was quite small just 0.1 percent, but on the gain on the amplifier gain was 2 which is substantially different from the nominal value. The difference has not been as dramatic the difference and performance has not been as dramatic as for the open loop case

Now, the error is just the little bit more just 0.5 percent. So, the difference in performance between the nominal case and the case of the low gain is just 0.4 percent with feedback control; whereas for the open loop control it is as large as 80 percent. So, you can clearly see therefore, that feedback control with the help of high gain access in greatly minimizing the sensitivity of the systems output to variation in the gain of the plant.

So, the first reason therefore, for employing feedback control is to use it when there is uncertainty in the plant model. And this particular numerical example clearly demonstrates the dramatic improvement that would approve as a result of using feedback control, in the presence of uncertainty regarding the plants model. What our applications warrant the use of feedback control? Let us now take another problem that blacks amplifier or afflicted with and that was the problem of disturbance.

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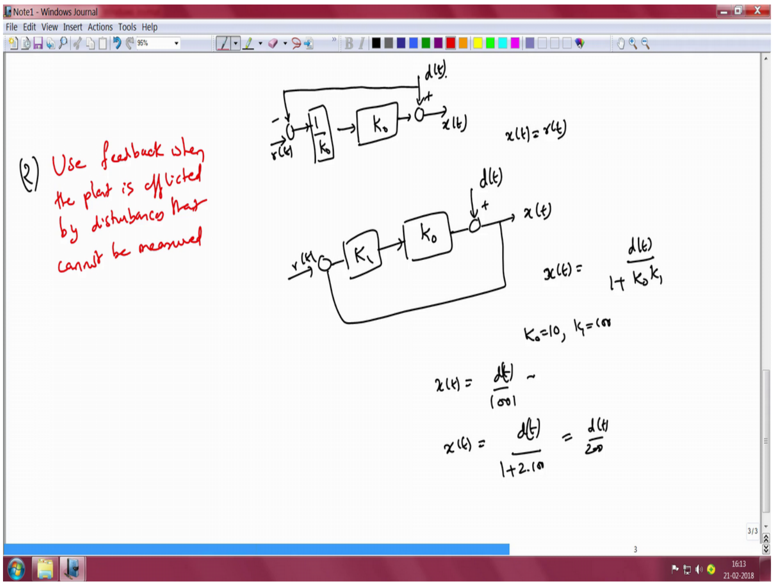

So, let us once again model our amplifier as a gain k naught, but let us also assume that the electronics injected some noise d of t. This could be from a neighboring amplifier for instance. And the total the output of amplifier plus d of t was a actual output of the amplifier. Now if we could measure d of t, then what could be done is we can first of course, use open loop model inversion and then we subtract d of t from our reference.

If we do this, you will see that x of t will be exactly equal to r of t. This will work in a scenario where we can measure d of t, and we can we know precisely the value of k naught, but in black times neither of these what in the engineers hands. D of t was some signal that corresponded to some voltage signal that was appearing as the, at the input of a neighboring amplifier. And the engineers had no control over what it was. Likewise, the amplifier they were working with itself had again that was uncertain.

So, for both these reasons use open loop strategy of canceling a measurable disturbance could not be implemented. Let us see how once again feedback helps to address this problem. If one were to employee feedback, then you have this amplifier k naught and which is suffering as a result of the output disturbance d of t. We take the output of the amplifier feed it back compare it with the reference and the error is manipulated by high gain function in this case I shall choose a simple proportional gain k 1. And then the amplifier error is feedback to the amplifier.

So, if this was the configuration that one word to adopt, then one can show that the relationship between x of t and d of t is the x of t would be equal to d of t divided by 1 plus k naught times k 1. Now what this allows us to show is that in the nominal case when k naught is equal to 10 and k 1 is equal to 100. The x of t will be equal to d of t divided by 1001, or this is what this indicate is that we are able to attenuate it the disturbance by more than 3 orders of magnitude.

Even if the plant had uncertainty, even if the gain k naught for that particular amplifier was not ten, but rather it was too, we would still have x of t to be equal to d of t divided by 1 plus 2 times 100. So, which is equal to d of t by 200. So, it still attenuates the disturbance where factor of 200. So, there is huge attenuation in the effect of disturbance that is brought about by the use of feedback control. Especially, when you have uncertainty in the plant model and an inability to measure the disturbance and directly compensated using open loop techniques.

So, the second reason for using feedback is in a scenario when the plant is afflicted by disturbances that cannot be measure. So, when either you have disturbance that cannot be measured, or when the plant has uncertainties that we have no choice, but to employee feedback based solution to the problem of model inversion and getting x of t to be equal to r of t.

What other scenarios would feedback control the employed? So, the third problem of the amplifier the push pull repeater amplifiers during the time of h s black was nonlinearity.

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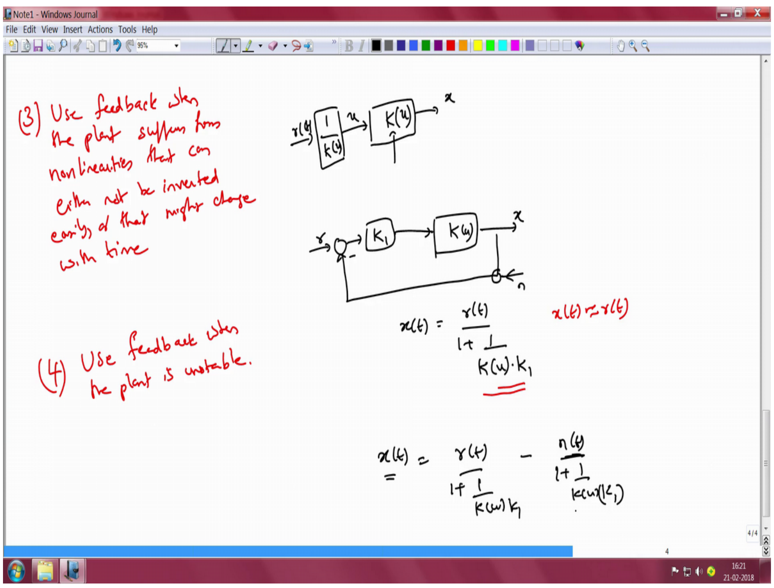

So, in this next example let us consider that the amplifier that we are dealing with is non-linear. So, it has again k and input u and output x, but the gain is not independent of the input, but rather it is a function of the input. Let us assume for the moment for the sake of simplicity with it is a static function of the input. In other words, there are no dynamic terms related to u dot and u double dot and so on and so forth.

So, this is the structure of the amplifier. Once again if one could characterize this gain very well experimentally. Then one could equally well come up with an open loop model inversion. So, one could design an attenuator with a gain of 1 by k times u and then get x of t to be equal to r of t. But suppose the nonlinearity is such that one can not characterize it effectively, or even if one word to characterize, it at a certain point in time there might be drift in the parameters of your amplifier. And therefore, this nonlinearity is no longer what it was at the time of characterization.

Then our simple open loop control strategy breaks down. Once again feedback control can be shown to result in significant improvement in performance. So, let us see how that happens. Let us say we have this non-linear amplifier k of u. We have the output x once again I take the output, compare it with the reference or the error is effect to a high gain k 1. And then output of that effect to the repeater amplifier. In this case, I can show that x of t is related to r of t by the term x of t equal to r of t divided by 1 plus 1 by k of u times k 1.

So, what you see again? Is that if this term in the denominator k of u times k 1 is made small; by choosing an adequately high value for k 1, then our x of t will be approximately equal to r of t by 1. The other term being negligible in comparison with unity and therefore, x of t will be approximately equal to r of t.

So, the third reason for employing feedback control is in a scenario when you have nonlinearities afflicting the plant that cannot be modeled properly, or that may be changing with time. So, use feedback when the plant suffers from nonlinearities that can either not be inverted easily or that might change with time. So, this is another scenario associated with uncertainty of the plant, which would warrant the use of feedback control.

Now there is one final circumstance where feedback control needs to be employed mandatorily. That is not revealed by the particular example that we considered here namely that of repeater amplifiers. And that is when one is trying to control unstable systems. One example of an unstable system is an inverted pendulum. If you were to get a pendulum to stand upright that any small perturbation would cause it to fall down.

So, it is not possible using open loop controllers to get it to stand upright for all future time. One has to employee feedback control to stabilize unstable systems. So, the 4th scenario where feedback control is mandatory is when you are dealing with unstable systems. Use feedback when the plant is unstable.

So, these are the 4 conditions under which feedback control is mandatory. And for other problems in other words when there are situations where you can identify the plant very well, where you can identify the disturbance very well, when you can invert the nonlinearities and you will do not have instability in your plant dynamics, then feedback is optional. And often one might argue that it may not even be necessary. In fact, the use of feedback is not recommended when it is possible to employee open loop based control strategies in order to invert the plant. And that is because when one uses feedback one has to mandatorily employee a sensor.

Sensor as I said need to satisfy a range of characteristics before they can reliably read out the output of the plant. And it may or may not be possible to find such sensors. But despite our effort in finding such sensors, it may still happen that our sensor would be sensitive to measurement noise.

So, regardless of the quality of the sensor, the output of the sensor will also have a component related to noise. This could be because of electromagnetic interference or because of some other coupling capacitive coupling with neighboring wires line noise and so on and so forth.

Now because you have this noise which is not supposed to be there, but you cannot avoid on account of having used a physical sensor to measure the output, what will happen is a x of t will no longer change only in response to r of t, but will also respond to n. In particular, we can write down the x of t would be equal to in this case r of t by 1 plus 1 by k of u times k 1. In the case of a non-linear plant minus n of t by 1 plus 1 by k of u times k 1.

So, what it essentially does is that, it allows all of the noise to affect the response x of t. Because if we choose k, k of u times k 1 to be a large number, than this 1 by k naught of u times k 1 would be a very small number in which case the entire measurement noise n of t that is injected by your sensor will affect the output x of t, and this is clearly undesirable.

So, one would therefore, be wise to investigate whether the problem that they are confronted with can be solved using open loop model inversion techniques that we talked about in the previous clip. If that is not possible on account of uncertainties that is the only condition under which one has to mandatorily employee feedback. So, you have uncertainty in the plant model or is or the plant is afflicted by disturbances or the plant is unstable. It is in these scenarios that one has to mandatory employee feedback control.

Thank you.