Plasma Physics and Applications

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Week – 02

Lecture 08: More Aspects of Debye Shielding

Hello dear students. In today's lecture, we will try to understand some more aspects of Debye shielding and we will try to answer some questions related to the assumptions that we have made in deriving the Debye potential. So, if you remember the basic assumption that we made while obtaining an expression for the Debye's potential is that we assumed the number density of let us say electron to be varying according to the Maxwell distribution which is E phi by k B T where phi is the potential and the charge density that results inside the Debye sphere is basically due to the difference of number of electrons and number of ions. So, this is the charge density. So, number of electrons are larger within the Debye sphere because they are attracted towards the positive charge and all of them are within this sphere whose radius is lambda d. Now, here while we were doing this we assumed the number of ions to be equal to N infinity and we assumed that the temperature of the ion was never considered that means that there is no distribution of ions as such and that is why we have cancelled this N infinity outright and we were left with only this E phi by k B T which also means that electrons are being heavier they are not moving or this potential is not influencing the ions they will constitute a constant background all this.

Now, we will take a situation in which number of electrons and the ions both of them are distributed according to their respective temperatures let us say T i and T e and then we will try to find out what is the expression for lambda d and how this expression of lambda d will be a function of the temperature of both the species T e and T i and we will look at two different cases in which the number of ions or if the temperature of ions becomes very large in comparison to the temperature of electrons and vice versa. So, this is going to be an objective of today's discussion and then subsequently we will derive or we will actually solve a numerical problem and which will actually conclude the entire discussion of the device shielding or device potential and everything. Looking back the device length lambda d signifies the distance over which a particular electric field or charges presence will be made invisible after that particular distance and lambda d is if you compare lambda d to the length scale of the plasma system it should be very

very small in comparison to this. This is probably the second plasma criteria that we have obtained and the third one is using lambda d we derived a parameter called as N d we call this parameter as plasma parameter.

1. $n_e = n_\infty \exp\left(\frac{e\phi}{k_B T}\right)$

$\rho = q(n_e - n_i)$

n_∞

T_i

$\frac{e\phi}{k_B T}$

λ_D

1) $\omega_p \tau_c \gg 1$

2) $\lambda_D \ll L$

3) $N_D \gg 1$

Plasma parameters

T_i, T_e

$\lambda_D (T_i, T_e)$

$T_i \gg T_e$

$T_e \gg T_i$

$$\frac{d^2 x}{dr^2} - \frac{x}{\lambda_D^2} = 0$$

$$x = r\phi$$

It tells you how many number of electrons how many number of charged species are present within the device sphere and we have seen that for an idealized ionized gas to be called as plasma the number of particles must be very large and the first plasma criteria was omega P tau c should be much greater than 1. So, put together these three conditions are known as plasma criteria and validity of these conditions is a must for calling an ionized gas as a plasma. Now, looking back if you remember one equation that we have derived we will write that equation and that equation is the basis of today's discussion which is d square x by d r square minus x by lambda d square is equal to 0 where x has r phi and phi is the device potential. So, this equation tells you the origin of the device length. So, we will take this equation and we know what we did before we reached this equation and how we can slightly modify the picture to draw more meaningful insights about the plasma potential is today's objective.

$$\frac{d^2\phi}{dx^2} = \frac{-e(n_i - n_e)}{\epsilon_0} \qquad e(n_e - n_i) = \rho$$

$$x = r\phi$$

$$n_i = n_\infty e^{-e\phi/k_BT_i} \qquad e^{-E/kT} \qquad r$$

$$n_e = n_\infty e^{e\phi/k_BT_e} \qquad \frac{x}{\phi}$$

$$\frac{d^2\phi}{dx^2} = -\frac{1}{\epsilon_0} e \left[n_\infty e^{-e\phi/k_BT_i} - n_\infty e^{e\phi/k_BT_e} \right]$$

$$T_e, T_i$$

$$n_i = n_\infty e^{-e\phi/k_BT_i}$$

$$= -\frac{n_\infty e}{\epsilon_0}\left[1 - \frac{e\phi}{k_BT_i} - \left(1 + \frac{e\phi}{k_BT_e}\right)\right]$$

$$= -\frac{n_\infty e}{\epsilon_0}\left[\cancel{1} - \frac{e\phi}{k_BT_i} - \cancel{1} - \frac{e\phi}{k_BT_e}\right]$$

$$\doteq -\frac{n_\infty e}{\epsilon_0}\left[-\frac{e\phi}{k_BT_i} - \frac{e\phi}{k_BT_e}\right]$$

So, let us say we start from the distribution of charges. So, in the earlier case we have d square phi by d x square. Now, I have made a shift from r to phi where x was earlier just a quantity which is just r phi and r is the radial distance away from the positive charge. Now, I am calling x as the position coordinate and phi as the potential itself. So, d square phi by d x square can be written as minus E times n i minus n e divided by epsilon naught.

You can look into the notes of last class or the class before that and you will find a similar equation. Where earlier we have taken the possibility that the charge multiplied by number of electrons minus number of ions is responsible for giving the charge density. So, d square phi by d x square is resulting from the Poisson equation and what the modification that I have done is that I have not considered a case where number of electrons are invariably larger in comparison to number of ions within the Debye sphere. I have considered a more general case in which what I am going to do is I will keep minus E by epsilon naught as it is and since I am allowing the possibility of number of ions as well as number of electrons to be distributed according to their own temperatures. So, by now you must have figured out generally in kinetic theory we refer to temperature as a single quantity which represents the entire measure of the energy or kinetic energy that is there within the gas system.

$$= \frac{n_\infty e}{\epsilon_0}\left[\frac{e\phi}{k_B T_i} + \frac{e\phi}{k_B T_e}\right]$$

$$\frac{d^2\phi}{dx^2} = \frac{n_\infty e^2}{k_B \epsilon_0}\left[\frac{1}{T_i} + \frac{1}{T_e}\right]\phi$$

$$\frac{d^2\phi}{dx^2} - \frac{n_\infty e^2}{k_B \epsilon_0}\left[\frac{1}{T_i} + \frac{1}{T_e}\right]\phi = 0$$

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n e^2}}$$

$$\lambda_D \propto T$$

$$\lambda_D \propto \frac{1}{n}$$

$$\frac{d^2 x}{dr^2} - \frac{x}{\lambda_D^2} = 0$$

$$\boxed{\frac{1}{\lambda_D^2} = \frac{n_\infty e^2}{k_B \epsilon_0}\left[\frac{1}{T_i} + \frac{1}{T_e}\right]}$$

(i) $T_e \gg T_i$

(ii) $T_i \gg T_e$

But in plasma we consider the electron temperature and ion temperature. The temperature is different these two temperatures represent the kinetic energy the average kinetic energy of electron species as well as ion species because of all the things that we have understood about plasma so far these two temperatures have to be different. So, electron the ion density n i can be written as n infinity e to the power of minus E phi by k B T i I will write it here. So, n i is n infinity n infinity is the equilibrium or let us say we consider the total number of electrons n i sorry ions n i e to the power of minus E phi by k B T i. This is nothing but e to the power of minus E by k T.

So, this is if you have studied any second or first year level BSc this may have been introduced while discussing lasers and n e similarly n e e is n infinity e to the power of E phi by k B T. You see the disappearance of the negative symbol in the exponential was already discussed in the last class because within that sphere the device sphere the number of electrons are extremely large. So, that is where it comes actually. If you substitute these two things we can write d square phi by d x square is equals to minus 1 by epsilon naught to e times n infinity e to the power of minus E phi by k B T i minus n infinity e to the power of E phi by k B T e. Doing some simplification we can write it as minus n infinity e divided by epsilon naught.

Taking only the relevant terms when you expand the exponential in the Taylor series we can write it as 1 minus E phi by k B T i minus 1 plus E phi by k B T e which will be minus n infinity e by epsilon naught times 1 minus E phi by k B T i minus 1 minus E phi by k B T e. So, this will get cancelled and what we have is this will be approximately equal to n infinity e by epsilon naught minus E phi by k B T i minus E phi by k B T e.

So, effectively we can take this minus outside and all of the right hand side will become positive. Then we can write n infinity e by epsilon naught times E phi by k B T i plus E phi by k B T e or n infinity e square divided by epsilon naught times 1 by T i plus 1 by T e times phi. What do we have on the left hand side? We have d square phi by d x square.

If we write all of the terms on the left hand side we will get d square phi by d x square minus n infinity e square divided by epsilon naught times 1 by T i plus 1 by T e times phi is equal to 0. Let us compare this equation with what we had when we considered the electrons only to be distributed according to their temperature and the ions to be constituting a constant background. What was that equation? This one d square x by d r square minus x by lambda d square is equal to 0 which is d square phi by d r square minus x by if I am writing everything in terms of x then I should write x here x by lambda d square is equal to 0. Comparing these two equations what I can write is 1 by lambda d square is n infinity e square n infinity e square divided by epsilon naught into 1 by T i plus 1 by T e. So, lambda d square now has some terms relevant to the temperature of ion as well.

Earlier lambda d is square root of I think we missed one K B in between. So, K B is here the Boltzmann constant. Now, K B should appear here. Lambda d is equals to earlier epsilon naught K B T divided by n e square. So, in this we thought lambda d is proportional to temperature and lambda d is inversely proportional to the number density which means at larger temperatures when you have more energy available to the electrons they will move more fast and they will not be able to be restricted within a small value of lambda d because of the larger velocity you have to accommodate a provision in which the larger lambda d takes place to nullify the positive test charge.

$$\frac{1}{\lambda_D^2} = \frac{n_\infty e^2}{\epsilon_0 k_B}\left[\frac{1}{T_i} + \frac{1}{T_e}\right] \qquad \frac{T_i + T_e}{T_e T_i} = \frac{\cancel{T_e}}{\cancel{T_e} T_i}$$

(i) $T_e \gg T_i$

$$\frac{1}{\lambda_D^2} = \frac{n_\infty e^2}{\epsilon_0 k_B}\left[\frac{1}{T_i}\right]$$

$$\lambda_D^2 = \frac{\epsilon_0 k_B T_i}{n_\infty e^2} \Rightarrow \lambda_D = \sqrt{\frac{\epsilon_0 k_B T_i}{n_\infty e^2}}$$

$\therefore$ Debye's length will depend on temperature of colder speci

Debye's length will depend on the temperature of colder species

(ii) $T_i \gg T_e$

$$\frac{1}{\lambda_D^2} = \frac{n_\infty e^2}{\epsilon_0 k_B}\left[\frac{1}{T_e}\right]$$

$$\boxed{\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n_\infty e^2}}}$$

So, the proportionality can easily be explained in terms of physical perception and it is proportional to 1 by n that means more is the number of particles per unit volume smaller or will be the device length. That means if there are more number of electrons within a small volume then the concentration of electrons that you would need to nullify the positive charge are available within a smaller distance that means lambda d will be small. So, both of these things are very well acceptable in terms of easier applying simple common sense. How the expression has changed is this. So, let us say this is an important result we will highlight this result so that it will be easy for you when you look at this lecture notes.

Now comes the most important conclusion of this discussion. What is it? We can have a situation in which the electron temperature is very large. We can have a situation say one is the electron temperature very large in comparison to the ion temperature. We can have another situation in which the exactly opposite may be the possibility. The ion temperature is very large in comparison to electron temperature.

So, what will happen in that case? This is the most important conclusion that we are going to draw. So, let us take this expression as it is 1 by lambda d square is equals to 1 by lambda d square is equals to gen infinity e square by epsilon naught k B times 1 by T i plus 1 by T e. Let us take a situation in which the electron temperature is very large in comparison to ion temperature. So, without actually doing this summation we can straight forward say that if the electron temperature is very large this term becomes very small. So, we can neglect it altogether and we can write 1 by lambda d square is equals to n infinity e square divided by epsilon naught k B into 1 by T i or lambda d square is epsilon naught k B T i by n infinity e square.

We can write it as the device length is under root epsilon naught k B T i divided by n infinity e square. We can arrive at the same result saying that let us say this is the T e T i and T i plus T e. When electron temperature is very large we can write it as T e divided by T e T i this will get cancelled you will be left with T i that is what we have. That means when electron temperature is larger the device length is decided by the ion temperature. Is it what we are seeing now? So, that means the device length will depend on the temperature of colder species.

This is a remarkable result actually and exactly when you do it for the case of ion temperature becoming very large in comparison to the electron temperature the expression will reduce to something like this 1 by lambda d square is n infinity e square by epsilon naught k B times 1 by T e. I hope you can figure out the algebra by yourself it is not complicated after all. 1 by lambda d square becomes n infinity e square epsilon naught k B into 1 by T i which means now the temperature that appears in this expression is not the higher temperature rather it is the lower temperature. So, we can write lambda d is equals to epsilon naught k B T e divided by n infinity e square under root. Now you see these two expressions which are telling a simple fact that under a situation let us say if you do not assume that the number of ions or the distribution of ions is independent of the ion temperature then your expression for the device length is not the same.

It will be decided by the colder temperature or the temperature of the species which is colder. So, that is why we have these two expressions lambda d is equals to as a function of T i when electron temperature is large and lambda d as a function of T e when the ion temperature is large. So, I will write the conclusion here. So, the conclusion is device length will depend on the temperature of colder species. Now we can discuss a numerical based on what we have understood and then probably this will even help us to visualize the nulling or shielding of the unit positive test charge.

Now having seen this the effect of temperature on the distribution of ions and electrons and how it modifies the definition of lambda d. We will take up some numerical examples or problems based on the derivations itself to clarify these concepts and to make a more concrete understanding of the device shielding.