Plasma Physics and Applications

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Week – 02

Lecture 07: Debye Length and Plasma Criteria

Hello dear students. In this lecture, we will try to understand the meaning of device length. We have already derived an expression for device potential. We will try to see some consequences of this device length and we will discuss a problem based on device length. So far in our discussions, we have seen that the potential phi of r can be written as Q by 4 pi epsilon naught into 1 by r into e to the power of minus r by lambda d. This is the form of potential.

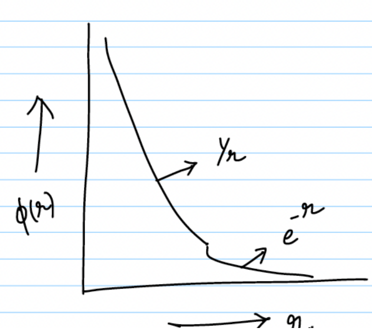
Now, this part is coulombic potential and this is an exponential part. In essence, what we have realized is if there is a positive charge which is kept inside the plasma, this charge will only be felt or its presence can only be realized as long as you are within the device length. Outside it, the plasma potential, the plasma will not make this charge to be visible. So, plasma is effectively shielding the positive charge or we can say that plasma by the means of its collective behavior has an ability to screen charges or has an ability to shield itself from external electric fields.

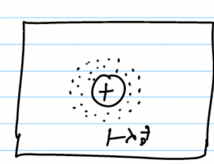
$$\phi(r) = \underbrace{\frac{Q}{4\pi\epsilon_0}\cdot\frac{1}{r}}_{C.\ Part}\cdot\underbrace{e^{-r/\lambda_D}}_{Exp.\ Part}$$

$r \cong \lambda_D$

$r \to \lambda_D \Rightarrow C$

$\sim \lambda_D \Rightarrow \frac{1}{e}\phi(r)$





So, this understanding can be similar to what we see in the case of skin depth. Now, more importantly, if you take r the distance away from the positive charge to be approximately equal to lambda d until you approach when as long as when r is tending to lambda d from 0, the potential is coulombic in nature. It is just 1 by 4 pi epsilon naught Q by r. The moment you reach when in the vicinity of lambda d, the potential will fall to 1 by e of the potential inside. And if you go further away, the potential will drastically decrease very fast.

So, in a sense, we have drawn this curve which explains how the potential will decrease. So, it will be 1 by r for some time and then it will be very fast to decay like this. So, this is supposed to be decay. So, it will be very fast and it will decay like this. So, this is 1 by r dependence and this is e to the power of minus r dependence.

λ_D = Debye length

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T_e}{n e^2}}$$

ϵ_0 : Permittivity of free space

k_B : Boltzmann Constant

T : Temperature

n : Particles/m^3

e : 1.6×10^{-19} C

$\lambda_D \propto \frac{1}{\sqrt{n}}$ $\lambda_D \propto \sqrt{T_e}$

$n \uparrow \quad \lambda_D \downarrow$ $T \uparrow \quad \lambda_D \uparrow$

This is as you going away from r and this tells you the magnitude of the potential. So, this is a very important characteristic of plasma. So, the quantity lambda d which was defined to be lambda d is called as the Debye's length. So, this Debye's length, if you want to define it, Debye's length is the distance up to which an external electric field can penetrate into the plasma as simple as that. So, lambda d can be written as square root of epsilon naught k B T divided by n e square.

Now, this Debye's length, what are the parameters? Epsilon naught is the permittivity of free space, k B is the Boltzmann's constant, epsilon naught is the permittivity of free space, k B Boltzmann's constant, T is the temperature, n is particles, number of particles per unit volume and e is 1.6 10 to the power of minus 19 coulombs. So, from this expression, we can realize that lambda d is proportional to square root of T. What does it mean? It means that as the temperature is increased, the lambda d will increase. What does it mean? It means that if the plasma is maintained at larger temperatures, then the Debye's length would be larger.

Sometimes to larger distances, the visibility of the positive charge can be felt, can be increased. How do you explain that? You can explain that because we know that electrons are trying to move around this positive charge just because of their inertia, they cannot neutralize the positive charge completely. So, if they have more velocity, they will occupy because of the larger thermal velocity that they have at the larger temperature, they will occupy more space and as a result, effectively the screening distance will increase. And one more consequence is the Debye's length is proportional to lambda d is proportional to 1 by square root of n. Number of electrons or number of charged particles in the plasma is inversely proportional to So, if you increase charge density, the plasma density, the Debye's length will decrease.

$\epsilon_0 : 8.85 \times 10^{-12}$

$e : 1.6 \times 10^{-19} C$

$k_B : 1.38 \times 10^{-23} J/k^{-1}$

$1 \, eV = 11{,}600 \, k$

$$\lambda_D = \sqrt{\frac{\epsilon_0 k_B T}{n e^2}}$$

$$\lambda_D = 69 \sqrt{\frac{T_e}{n}}$$ T is in kelvin

$$\lambda_D = 7431 \sqrt{\frac{T_e}{n}}$$

T is in eV

How do we understand that? If there are more number of electrons per unit volume, the positive charge will be nullified because there are more number of electrons, it will take more number of electrons will be surrounded in the vicinity of the positive charge. That means, the effectively the distance up to which you want the electrons to be available to shield the positive charge will decrease. So, that is why lambda d can be assumed to be decreasing. So, here the only parameters which will carry information about a particular plasma are these. One is temperature, another is number of charged particles per unit volume.

So, the remaining parameters or remaining k B T, k B epsilon naught and e square are constants there, we know the value. So, for example, epsilon naught is 8.85 10 to the power of minus 12 value of constant. The charge of electron is 1.6 10 to the power of minus 19 coulombs and what else is there? The Boltzmann's constant is 1.

Debye Sphere

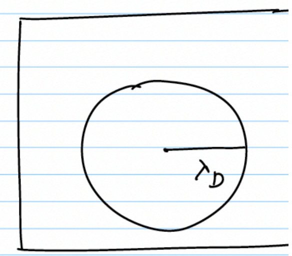
λ_D

$V = \frac{4}{3}\pi\lambda_D^3$

$N = n_0 \times \frac{4}{3}\pi\lambda_D^3$

$\downarrow$

$N_D = \frac{\text{no of particles}}{m^3} \times m^3 = \text{no of particles}$



$T\uparrow \ \lambda_D\uparrow$

$n\uparrow \ \lambda_D\downarrow$

$N = n_0 \times \frac{4}{3}\pi\left[\sqrt{\frac{\epsilon_0 k_B T}{ne^2}}\right]^3$

$$\boxed{N = n_0 \times \frac{4}{3}\pi\left(\frac{\epsilon_0 k_B T_e}{ne^2}\right)^{3/2}}$$

38 10 to the power of minus 23 joule per Kelvin. So, by substituting all of these in this expression lambda d is square root of epsilon naught k B T by n e square. By substituting all of this, we can write lambda d is 69 times square root of T e by n as long as the temperature is n the units of Kelvin. The same expression can be written if you are using the plasma temperature in the units of electron volts, you can write it as 7 4 3 1 square root of T e by n, where the temperature is in electron volt. So, you can remember that 1 electron volt is equals to 11600 Kelvin.

So, this is about device length. So, for whatever we have discussed pertains to the understanding of device length. So, what and all we have learnt in this module is that plasma has a unique characteristic of exhibiting collective behavior and because of this collective behavior, it is able to screen out positive external electric fields or external charges. So, before all this happens plasma is completely electrically neutral that is the condition is referred to as quasi neutrality in which in small packets there can be an excess of positive or negative charges. But if you look at the entire system, there are equal number of positive and negative charges thereby plasma is assumed to be electrically neutral.

If you try to disturb this electric neutrality by introducing foreign charge or if you try to polarize the plasma medium by applying electric field, plasma rearranges itself such that this external electric fields are stopped. They cannot protrude into the plasma beyond a certain distance. So, this certain distance is what is called as device length. So, we constructed a simple mathematical picture in which we assumed the potential to be columbic near the vicinity of the charge and we have to account how this potential will change within the sphere electron cloud. And one more condition that we have imposed if the plasma has to shield out the external positive charge, it has to happen in such a way that this positive charges effect or influence cannot be felt of outside lambda.

$$\omega_p = \sqrt{\frac{ne^2}{m\epsilon_0}} \Rightarrow f_p = \frac{e}{2\pi}\sqrt{\frac{n}{m\epsilon_0}}$$

$$\boxed{f = 9\sqrt{n_0}\ \text{MHz}}$$

$$v_{th} = \sqrt{\frac{k_B T_e}{m_e}}$$

$$\tau_p$$

$$\underline{v_{th}\tau_p} = \frac{v_{th}}{\omega_p}$$

$$\underline{\text{Distance}} = \sqrt{\frac{k_B T_e}{m_e}} \times \sqrt{\frac{\epsilon_0 m_e}{ne^2}}$$

Then mathematically when we combined all these things we realized the potential is of course columbic in nature, but because of this collective behavior there is an exponential part which comes into picture and this exponential part will make sure the potential decreases very fastly after you cross this characteristic distance. Now, there is no point if you cannot explain why the positive charge is invisible beyond the electron cloud with mathematics there is no point actually. But we have a self consistent picture in which the mathematics is reinforcing the understanding that we did and now we have an expression for the device length. So, device length is very important in characterizing plasma. What kind of plasma is it? Is it a very strong plasma or what whether we can call a particular ionized gas as a plasma or we can call it as a gas.

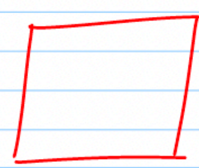
$$\text{Distance} = \sqrt{\frac{k_B T_e}{m_e}} \times \sqrt{\frac{\epsilon_0 m_e}{n e^2}}$$

$$\boxed{\lambda_D \ll L}$$

$$\lambda_D = \sqrt{\frac{k_B T_e \epsilon_0}{n e^2}} \quad \underline{m}$$

Distance travelled by plasma particle in One period (a measure of ω_p) is equal to Debye length.

$\underline{\omega_p, \lambda_D}$



L

$\underline{L > \lambda_D}$

In order to make this difference we need the measure of device length for a particular plasma. So, in extension to this concept we discuss what is called as Debye sphere. Debye what is Debye sphere? Construct a sphere with radius of lambda d. You have a sphere whose radius is lambda d. This sphere is called as Debye sphere.

So, what is the volume of this sphere with radius lambda d? The volume of such a sphere would be 4 by 3 pi lambda d cube. Now, if this is the volume how many number of particles can exist within this Debye sphere? The number of particles inside the Debye sphere would be N naught the charge density of plasma multiplied by 4 by 3 pi lambda d cube. See N naught is in the unit of number of particles per unit volume that means you have volume in the denominator and when you multiply this with volume what you will get is number of particles. We are actually looking to find out the number of particles only. We are trying to find out the number of particles per unit volume inside the Debye sphere.

So, if you construct a sphere of lambda d within the plasma for example, then there will be this many number of particles. We call this as N d. N d denotes the number of particles inside the Debye sphere. So, here we can substitute the value of lambda d which we have derived and then N is equals to N naught into 4 by 3 pi lambda d is square root of epsilon naught k B T by N e square all of this raise to the power of 3 or N is equals to N naught into 4 by 3 pi epsilon naught k B T by N e square raise to the power of 3 by 2. So, this will tell you how many number of particles are there inside the Debye sphere in a three dimensional construct.

Here everything is a constant actually apart from N and T. So, here our earlier understanding was if temperature increases, the Debye's length would increase. If the number density of the particles increases, the Debye's length would decrease. You see here if it is valid or not. It is valid in the case of temperature, it is still directly proportional.

And if you use the known values of constants, we can write N d as 1.38 10 to the power of 6 into T to the power of 3 by 2 into N naught raise to the power of minus half when T is in Kelvin. The same expression by using temperature in the unit of electron volt, you can write 10 to the power of 12 into T to the power of 3 by 2 into N naught raise to the power of minus half. So, temperature is in the unit of electron volt. Now, you see the number of particles inside this volume seems to be inversely proportional to the number of particles per unit volume.

So, if you have a very dense plasma, the number of particles would be small. But on the other side, if you have a very weak plasma where N naught is very small, the number of particles inside the Debye's length would be larger. That means you will require more number of particles to be present to neutralize the positive charge that is there inside. So, this is our discussion conclusion about Debye length, Debye shielding, Debye potential and Debye sphere. Now, we can write expression for the Debye's length using what we know about plasma frequency as well.

$\omega_p \tau_c > 1$ Plasma Criteria - 1

$L \gg \lambda_D$ Plasma Criteria - 2

$N_D \gg 1$ Plasma Criteria - 3

So, we know that the plasma frequency omega p is square root of N e square by m epsilon naught which implies this is the angle of frequency. So, frequency in hertz f p is e by 2 pi square root of N by m epsilon naught. So, substituting all the known values of constants for example, charge of electron, mass of electron, the permittivity of free space, all that we can write the frequency as 9 square root of N naught hertz. This is the frequency of the plasma. Let us say the particles of course, we have in order to construct the Debye's shielding, the particles should have some thermal velocity because otherwise if it is a cold plasma the particles cannot move and the shielding effect may not happen.

So, let us say there is some thermal velocity of the particle. So, which we write as V th the thermal velocity of particle is k B T e by m e where k B is the Boltzmann's constant electron temperature and m is the mass. What are we doing? We have so far just substituted and got an expression for the plasma frequency, the thermal velocity is this. Now, there are oscillations of electrons which is just a mechanism that plasma does to establish charge neutrality. So, in this process there are oscillations and the particles may collide and there can be a provision for a time period which is associated with these oscillations.

It is a frequency that there is a time period. Now, the distance travelled by the electron by the particle in one period because the velocity the distance would be V th times tau p. This is something. So, this is how much the electron will travel within that tau p which can be written as V th by omega p or if we do that, if we use the expression for V th that we have here and omega p the plasma frequency expression that we have here. We can write this ratio as square root of k B T e by m e which is actually the thermal velocity multiplied by 1 by omega p which is epsilon naught m e by n e square.

What is this? This is a measure of distance actually. What distance is this? This is the distance that the particle will travel, electron will travel within one tau p. What is tau p? time that is associated with the plasma frequency. So, there is frequency, there is a time period that is associated with that and within the time period given that the velocity of the electron is V th, how much distance would it travel? This is the distance that it would travel. Now, surprisingly if you simplify this, let us say we write it again.

So, this distance is ratio which is k B T e by m e thermal velocity into 1 by omega p is epsilon naught m e by n e square. So, if you multiply this, what you will realize is it will be k B T e by epsilon naught by n e square. Does this expression on the right hand side look familiar? Does it? You go back and see. Here it is or even here.

It is nothing but the device length. So, this distance can now be comfortably named as the device length. So, this device length is a natural consequence. So, if you have constructed the plasma frequency and associated length scales and time scales, then this device length is pretty much there already. It is not something that.

So, the units are meters. So, the distance travelled by the plasma particle in one period is device length. The distance travelled by the plasma particle in one period which is actually a measure of plasma frequency is equal to d by length. That means this omega p and lambda d are self related. They are related to each other. We are just looking at another side of it when we discuss the plasma frequency.

This concludes most of the discussion like what is plasma frequency? How is plasma frequency relevant for establishing the plasma criteria? What are plasma oscillations? What is the collective behaviour of plasma? And how is device shielding a natural consequence of plasmas ability to behave collectively? And what is device length? What is device shielding? What is device potential which facilitates all these mechanisms? And then what is device sphere? And all these things. Now at this point of time, we can naturally understand one more plasma criteria. What is it? If you have a plasma system or any place where plasma is there or you say that there is an ionized gas that is present in an enclosure or any system, then you would expect that the length scale of this system must always be greater than lambda d. Lambda d is at least the distance up to which this device sphere would be there. And you do not want to cut the device sphere in half for an existing temperature and number density and then say what is plasma that is ridiculous.

You would naturally expect that plasma has to be at least or more than the length scale of device shielding or generally we can expect that lambda d is much smaller than capital L. What is capital L? Capital L is the length scale of plasma. What is the earlier plasma criteria that we have discussed? Observations spanning the length scales shorter than lambda d cannot detect plasma. If you are trying to detect plasma and if you probe within lambda d cannot detect plasma. If the plasma state is true, its oscillations can be realized only over length scales which are greater than lambda d.

This is another way of saying it because if plasma is there, it will exhibit oscillations and these oscillations will have a characteristic time scale and with this characteristic time scale you will have a characteristic length which is nothing but the device length. So, you cannot have plasma oscillations until unless this characteristic length scale is established. So, your length scale of plasma should always be greater than the device length. This can be treated as another plasma criteria and we also discussed one plasma criteria in terms of plasma frequency omega p tau c should be much greater than 1.

This is one plasma criteria 1. Then the second condition is the length scale of plasma system should be much greater than the device length. This is plasma criteria number 2. And the third condition or criteria is going to be in terms of the number of particles inside the device sphere. So, if Nd is the number of particles in the device sphere for you to call an ionized gas as plasma, you would expect the number of particles should at least be 1, but for reasonable plasma behaviour you would expect that the number of particles is very large in comparison to 1.

So, this is the plasma criteria number 3. So, if this is not valid plasma shielding will not be effective. So effectively we have now established all the plasma criteria 1, 2 and 3 using the principles of plasma frequency and device length and device shielding etc. How these criteria are useful? Any plasma existing in the universe can be characterized based on these parameters. Now we are at a position where we can differentiate what is an ionized gas and what is a plasma. Like I said before every plasma is actually an ionized gas, but not every ionized gas is plasma.

For it to be called as plasma, it should obey these three conditions. These are called as the plasma criteria. So, we will continue this discussion in the next lecture. Thank you.