Plasma Physics and Applications

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Week – 12

Lecture 58: Instabilities in Plasma

Hello dear students. In today's lecture we will try to understand the idea of stability in plasma and what are the processes by which plasma can become unstable. We will try to briefly see how we can understand the instabilities of plasma, what are the different approaches to treat it and what are the applications which are going to be affected when plasma becomes unstable. Now, all of you at this point of time understand what is plasma and how plasma is fundamentally different from any other state of matter, what are the unique characteristics of plasma and how plasma can be generated, what kind of wave propagations or wave phenomena can be seen in plasma and lastly we have seen what is the role of collisions in plasma and how these collisional processes will help in the diffusion of plasma in the presence of magnetic field and the same collisional process will hinder the diffusion of plasma in the absence of magnetic field etcetera. Now, please keep in mind that plasma is the most abundant state of matter in the visible universe 99.9 percent of the state of matter is actually in plasma state.

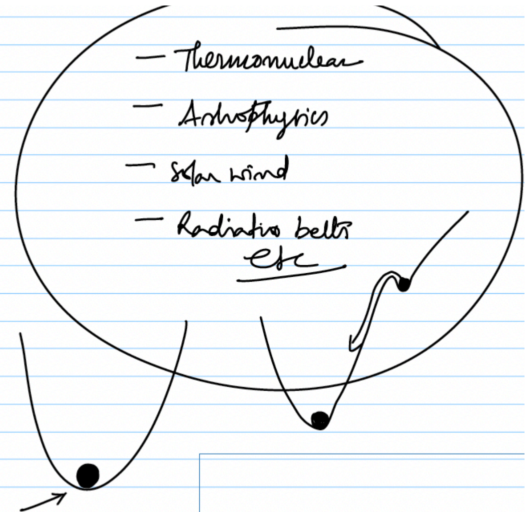
On this note it is very important that many of the technological developments take an exploit of the unique characteristics of plasma because it is a gas, but which has ability to be affected by the electromagnetic fields. It can accommodate electromagnetic phenomena in its propagation things like that. So if you want to understand anything that is surrounding the planet or if you want to devise some applications based on plasma we have to see we have to understand when the plasma can be stable or when it can be unstable. So, the fundamental idea of stability or instability is basically different from an equilibrium.

1) Streaming instabilities

2) Rayleigh-Taylor Instability

3) Universal instabilities

4) Kinetic Instabilities



An equilibrium is a state in which if you slightly change the physical characteristics of the system it may try to come back to its original state, but stability is slightly different the idea of stability is different. Now, the reason that we want to understand plasma stability is that in applications such as thermonuclear fusion or extracting or devising applications for energy using plasma it becomes very important that you make conditions such that plasma is stable. If the governing conditions are not favorable there is a possibility that plasma can become unstable. So, in addition to these controlled thermonuclear processes stability also plays a very important role in understanding like things like solar wind, sunspots, vanillin radiation belts, pulses, astrophysics, astrophysical processes all of this. So, understanding stability is important for thermonuclear processes fusion, astrophysics, stellar evolution, solar wind, radiation belts there are many etcetera.

So, a plasma is said to be in an equilibrium state if all the forces that are acting on it are balanced so that you can extract a time dependent solution. What is time dependent solution? If so like I must have said many number of times if you are able to get the velocity as a function of time or if you are able to tell how the position will change with respect to time then like 90 percent of the task is completed. So, you have a time dependent solution. Time dependent solution can only be obtained if all the forces that are acting on the plasma are accounted for and known. So, the macroscopic fluid equations for a collisionless plasma have steady state solution and it might be expected to persist indefinitely.

1) Intuitive approach

2) Energy approach.

3) Normal mode analysis

What it means is that what I have just said is that even though if you know the nature of forces, their directions, their magnitudes everything you may think that since I know all of these things the time dependent solution will be valid indefinitely but whereas it may not be the case. So, the solution may not be valid solution always. So, there is a chance that it may not be valid all the times. Because a particular equilibrium configuration may be stable or unstable. If a small let us say if a small perturbation is given to the system and the system behaves as if this small perturbation is growing with respect to time.

<u>Two stream Instability</u>

$\vec{u}_0(x)$

1) $\vec{u} = u_0 \hat{i}$

2) $v_i = 0$

3) $T_e = T_i = 0$

If the perturbation ceases to exist or if it decays exponentially or in any way and the system regains this original equilibrium situation then you call that particular situation as stable and if it so happens that the small perturbation that you have given grows with time, then you are actually talking about unstable situation or instability in plasma. So, what we will see in this particular lecture is that how the fluid equations that we have studied or written many number of times can be used to understand the instabilities in plasma. So, in a nutshell equilibrium is of course what you expect to be there. So, at equilibrium you are able to write down a time dependent solution because you know the nature of forces, the magnitudes of forces, the directions of forces, all the information that is required about the forces. So, in every possible angle you have accounted for all the possible forces which can affect the motion or which can affect the behaviour of the system that is only up to equilibrium.

Linearized governing equations $\qquad n_0 e^{i(kx - \omega t)}$

$$\frac{\partial n_{i1}}{\partial t} + n_0 \vec{\nabla} \cdot \vec{u}_{i1} = 0 \quad \text{---} \quad (1)$$

$$\frac{\partial n_{e1}}{\partial t} + n_0 \vec{\nabla} \cdot \vec{u}_{e1} + u_0 \vec{\nabla} n_{e1} = 0 \quad \text{---} \quad (2)$$

$$m_i n_0 \frac{\partial \vec{u}_{i1}}{\partial t} = e n_0 \vec{E}_1 \quad \text{---} \quad (3)$$

$$m_e n_0 \left[\frac{\partial u_{e1}}{\partial t} + (\vec{u}_0 \cdot \vec{\nabla}) \vec{u}_{e1} \right] = -e n_0 \vec{E}_1 \quad \text{---} \quad (4)$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E}_1 = e(n_{i1} - n_{e1}) \quad \text{---} \quad (5)$$

Now if you are able to write the equilibrium solution fine, but that does not guarantee that the system is stable or not. How do you test it? You put a small perturbation to the system and if it so happens that the perturbation is growing with respect to time, then you call that as unstable. So, system coming back to equilibrium is actually stable, system not coming back to equilibrium despite the perturbation or in the presence of perturbation is unstable. So, plasma can have or there are many different types of instabilities in plasma or in plasma physics. So, for plasma it is possible to have equilibrium states in which all the forces are in balance yet, they are not in perfect thermodynamical equilibrium.

So, there is a chance that some amount of free energy is available which can cause the equilibrium to be unstable one. So, it is these non equilibrium or meta equilibrium states that are to be examined if you want to understand the idea of stability. So, an instability is always associated with the decreasing in the free energy and leads the plasma to be in a new equilibrium state which is closer to the perfect thermodynamical equilibrium. So, you cannot ignore the fact that a perfect thermodynamical equilibrium is required to establish a stable equilibrium. You know what is stable equilibrium, what is unstable equilibrium you know all that.

$$\vec{u}_0 = u_0 \hat{i}$$

$$-i\omega n_{i1} + ik n_0 u_{i1} = 0 \quad \text{——} \quad (6)$$

$$-i\omega n_{e1} + ik n_0 u_{e1} + ik u_0 n_{e1} = 0 \quad \text{——} \quad (7)$$

$$-i\omega m_i n_0 u_{i1} = e n_0 E_1 \quad \text{——} \quad (8)$$

$$m_e n_0 \left[-i\omega n_{e1} + ik u_0 u_{e1}\right] = -e n_0 E_1 \quad \text{——} \quad (9)$$

$$ik \epsilon_0 E_1 = e(n_{i1} - n_{e1}) \quad \text{——} \quad (10)$$

So, depending on the type of free energy that is available, so this free energy is a very important thing. You can rewatch what I said about free energy. Free energy depending on the type of free energy that is available, the instabilities may broadly be categorized into four different types. Then the fundamental nature of instabilities that you can find in plasma can be broadly categorized in four different types. Number 1, the first type of instabilities are streaming instabilities.

Number 2 are Rayleigh Taylor instability or instability. Number 3, universal instabilities. Number 4, kinetic instabilities. So, what are these streaming instabilities, Rayleigh Taylor instabilities, universal instabilities and kinetic instabilities? Let us try to understand each one of them briefly. These topics are so advanced and so elaborate that we cannot possibly cover these topics individually and this becomes clearly out of context or out of scope for this particular course.

$$⑥ \Rightarrow \quad u_{i1} = \frac{i\omega n_{i1}}{ik n_0} \quad — (11)$$

$$⑧ \Rightarrow \quad u_{i1} = \frac{e n_0 E_1}{-i\omega m_i n_0} = \frac{ie}{m_i \omega} E_1 \quad — (12)$$

(11) & (12)

$$n_{i1} = \frac{ik e n_0}{m_i \omega^2} E_1 \quad — (13)$$

$$u_{e1} = \frac{-ieE_1}{m_e(\omega - k u_0)} \quad — (14)$$

$$n_{e1} = \frac{-iekn_0 E_1}{m_e(\omega - k u_0)^2} \quad — (15)$$

(10) (12) (15)

So, we will try to understand these instabilities briefly. So, the streaming instabilities they basically arise from the interaction of two groups of plasma which are drifting with respect to each other. Here the instabilities gain energy from the kinetic energy associated with the relative drift velocity. So, these two fluids of plasma are drifting with respect to each other. So, that means they are not in cohesion, they are not moving together, their velocities are non-zero with respect to each other.

So, the resulting difference in the kinetic energy drives these instability. So, such instabilities may be generated when a beam of energetic particles travel through plasma or a current is driven through plasma. So, just some examples. Then comes what is called as the Rayleigh Taylor instability. These instabilities arise due to the spatial confinement of a plasma.

$$ik\epsilon_0 E_1 = ie^2 k n_0 \left[\frac{1}{m_i \omega^2} + \frac{1}{m_e(\omega - k u_0)^2}\right] E_1 \quad \text{— (a)}$$

$$\boxed{1 = \omega_{pe}^2 \left[\frac{\frac{m_e}{m_i}}{\omega^2} + \frac{1}{(\omega - k u_0)^2}\right]}$$

$$\omega = \omega_r + i\omega_i$$

$$\frac{\omega}{\omega_{pe}} = x \qquad \frac{k u_0}{\omega_{pe}} = y.$$

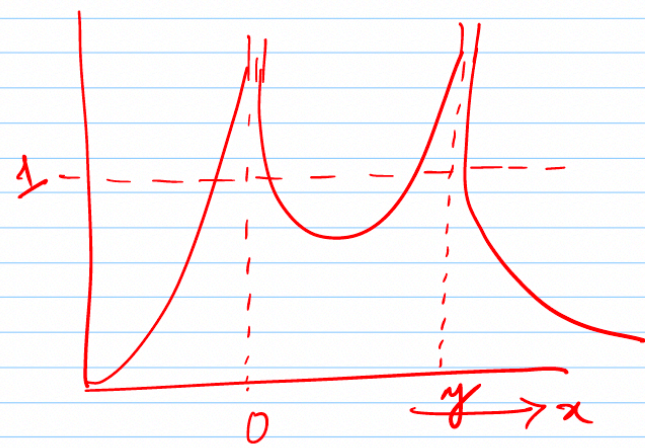
For example, if a uniform plasma is supported against gravity by a magnetic field, the equilibrium becomes unstable. If a plasma is held against gravitational pull, but with the support of a magnetic field, then you can have a situation which appears to be in equilibrium, but it is not an equilibrium, it is an unstable equilibrium. So, if you slightly disturb this, the system will tend towards a higher state of equilibrium. Let us say you have this potential well. You see this particle which is existing here, you put a small perturbation to this particle.

That means you can understand the idea of perturbation just by making the particle slightly move. What will happen? The small perturbation will decay or what do you call I mean in a familiar sense you can say that this perturbation gets dampened and the particle will come to its original position. So, you have a situation in which you have something like this and you put this particle here rather than here. Now it appears as if the particle is now in a stable situation or it is an equilibrium situation, but this equilibrium is not a stable equilibrium. That means if you slightly put a perturbation, the particle may be pushed by the virtue of the availability of free energy.

The particle may be pushed into an equilibrium state which is actually a stable equilibrium. So, this particle if it is displaced, this particle while the perturbation is getting dampened it may so happen that this will go here and settle here. Now this equilibrium state is more stable or it is called as the stable equilibrium. Now coming back to the Rayleigh Taylor instability, so if a fluid of higher density is sitting on the top of a fluid which is of lower density, then what happens is you have an interface and if you slightly give a perturbation the equilibrium situation may get disturbed. So, any force such as a non electromagnetic gravitational force can drive this instability.

So the basic idea is very simple. If a uniform plasma is supported against gravity by a magnetic field, you can think of it as you can think of a perfect condition for the Rayleigh Taylor instability to occur or a non electromagnetic gravitational force drives this instability. An analogy to this instability can be found in hydrodynamics. When a heavy fluid is supported by light fluid, any ripple on the interface tends to grow at the expense of the gravitational potential energy. You have an interface, heavy fluid is sitting on the top of the lighter fluid for some reason that can be anything.

$$1 = \frac{\frac{m_e}{m_i}}{x^2} + \frac{1}{(x-y)^2} = F(x,y)$$



The moment you disturb this, any ripple that goes along the interface can grow at the expense of the gravitational potential energy. Then you have this universal instability. A plasma is never actually in a true thermodynamically equilibrium. What happens is all laboratory or natural plasmas in the universe have density gradients. We have seen how important is the density.

You can never expect the plasma to be uniformly distributed in space. Although we say that plasma is uniform, the plasma obeys what is called as quasi neutrality. We have understood what exactly it means to say quasi neutrality but not neutrality. So recall all the discussion, you may appreciate that plasma is never in a true thermodynamical equilibrium. The non thermal character of plasma equilibrium can be understood to be the reason for the universal instability.

The pressure gradients that build up because of the density gradients tend to make the plasma expand and this expansion energy will drive the instability. So this instability is called as the universal instability. Then you have the kinetic instabilities. So these instabilities are associated with the departure of the velocity from the maxwellian distribution. So, you know maxwellian distribution that means you have maximum number of particles which have an average velocity and minimum number of particles which has the least velocity and so does the particles with highest velocity.

So if there is a clear departure from this distribution, so because of the anisotropy in the velocity distribution, an instability can be driven and those type of instabilities are called as the kinetic instabilities. So you can study these instabilities by using the plasma kinetic equations. Unfortunately we could not cover the kinetic theory of plasma but still the discussion that we had so far should still be sufficient to appreciate the instabilities. So there are various methods to actually analyze or understand plasmas instability. These methods are let us say number one is intuitive approach or method.

So what is this intuitive approach? So in this method the plasma in equilibrium which is supposedly in equilibrium is given a small perturbation so as to slightly alter the forces which are keeping the plasma in equilibrium. So if these slightly modified forces tend to increase the initial perturbation then you can assume the original equilibrium or the initial equilibrium to be unstable. So this is how you probe whether a particular plasma is stable or whether a particular plasma is existing in a stable equilibrium or an unstable equilibrium. The basic idea is you know the forces which are there, you slightly modify these forces and if it so happens that this modification tends to increase the initial perturbation itself then you can figure out whether you are not going to actually write equations for the instability. With this approach you can find out whether the particular plasma at a particular state is existing in a stable equilibrium or an unstable equilibrium.

Then the second method is the energy approach. What is this? So in this method the plasma at equilibrium is given a perturbation and the change in the potential energy of the plasma as a result of this perturbation is calculated. You are not changing the forces now rather what you are doing is you are trying to account for the change in the potential energy of the system as a result of the small perturbation that you have given. Now if the change in the potential energy is negative the equilibrium is unstable. What does it mean? If the change in the potential energy is negative the plasma the system is moving towards and stable situation.

Normal mode analysis in this method so the third method for probing the stability of plasma is called as the normal mode analysis. You see in the energy approach if the change is negative that means initial potential energy is smaller than the final potential energy. So you are expecting the situation to be moving in a unstable equilibrium. So in the normal mode analysis the plasma equilibrium is given a perturbation and the linearized plasma equations are solved for time development of perturbation by applying appropriate boundary conditions and seeking exponential solution for the perturbed variables. This is similar to the perturbation theory that we have studied in the earlier lectures.

Now these are the methods these are the instabilities and this is the relevance of studying plasma instabilities. Now let us take one type of instability for reference we will try to solve that mathematics and try to see what are the inferences of the result. So one instability that we are particularly interested is called as a two stream instability. What is the two stream instability? We will just consider two fluids as an example we will consider streaming instability in which uniform unmagnetized plasma in which electrons and ions are basically the two fluids they are drifting with respect to each other. Now we are not talking about a situation in which electrons and ions are moving with the same velocity.

We are talking about a situation in which they are drifting with respect to each other. So for simplicity what we will do is the electrons being lighter will be given a velocity u let us say u naught in the x direction to be simple and we will assume that the ions being lighter are at rest they are at rest. So this is just an example what is the system? The system is a two fluid plasma in which both the fluids are drifting with respect to each other. They are not in cohesion they are not moving together with the same velocity rather there is a relative motion involved between these two fluids that is why there may be chance that some instability may occur and grow with time. So electron fluid is moving with a uniform velocity u naught along x so you write that u naught u is u naught i cap.

Number 1, number 2 velocity of ions is equal to 0 that means they are at rest. So where do we start? We start with the fluid equations and for simplicity we will also assume that both the fluids T e and T i is equal to 0 it is a classic case of cold plasma. So we will start with the linearized governing equations and we will try to couple these equations to get a solution. So the linearized equations these are something that I have done multiple times. So take the liberty and I will directly write the linearized equations not you must remember the process.

The process is simple you take the equations you figure out what are the variables which may get perturbed you split those variables into an equilibrium part and a perturbed part. The equilibrium part will remain constant with respect to time and the perturbed part is the one which will actually give you physical insight of the perturbation. Here after you substitute this into the governing equations you neglect all the higher ordered terms which are appearing as a product of perturbed variables and since you have written the each variable as a constant plus perturbed part all the derivatives acting on the constant parts or the equilibrium parts will become 0. So doing all that so probably this is possibly the best exercise for you to get this linearized equations starting from the governing equations. So the linearized these are linearized governing equations linearized governing equations by considering plasma as a fluid two fluid which are dou n i 1 divided by dou t plus n naught del dot u i 1 is equals to 0.

What is this? This is the continuity equation. This equation is written for ions and it so happens that the perturbed variables are written with a subscript of 1. Then I will keep writing the other equations dou n e 1 divided by dou t plus n naught del dot u e 1 plus u dot del n e 1 is equals to 0 m i n naught dou u i 1 by dou t is equals to e n naught e 1 m e n naught plus u naught del u. This is equation number 1, 2, 3, 4. Now when the electrons and ions if they are moving with different speeds it will be obvious that there is a polarity of charge concentration spatially that means there will be an electric field. This information is appropriately embedded in the Poisson equation which is epsilon naught times del dot e 1 is e times n i 1 minus n e 1.

So you are assuming that the electrons are moving ahead and the ions are actually at rest. So we call this equation as equation number 5. Now these equations before you get these equations there are at least 10 different steps 10 other equations after which you get these equations. Now after this the process is simple we know that the velocity u naught is basically u naught i cap. You will assume sinusoidal solution for all the perturbed variables you see all the perturbed variables like u i 1 n e 1 every variable that appears with a subscript 1 is a perturbed variable.

So you take a sinusoidal solution let us say you are taking n naught e to the power of i k x minus omega t substitute that back into this equation then you will get the linearized equations which are minus i omega n i 1 plus i k n naught u i 1 is equals to 0. We will call this as equation number 6 minus i omega n e 1 plus i k n naught and this is minus i omega. There is a one to one correspondence between equation number 1 to 5 to these equations that I am writing m i n naught u i 1 is equals to e n naught e 1 8 m e n naught minus i omega s i k. So we have equation 1 2 3 4 5 which are basically the governing equations written in the perturbed variables after substituting the sinusoidal solutions we get the linearized equations. Now we can remove some variables or eliminate some variables let us say from equation number 6 we can write u i 1 is i omega n i 1 divided by i k n naught we call this as equation number 11.

So, using from equation number 8 u i 1 the perturbed ion velocity can be written as e n naught e 1 divided by minus i omega m i n naught after self cancelling we have i e divided by m i omega. So, comparing 11 and 12 we can write n i 1 the perturbed number density of ions can be written as i k e n naught divided by m i omega square e 1. So, we call this as equation number 13. So, this we have eliminated and we have just found out what is n i 1. So, in a similar way we can also find so we have u i 1 which is written in terms of n i 1.

So, u i 1 can be written by substituting back into this we already have it anyway. So, u e 1 the perturbed electron velocity because of that because of the instability or because of the drift of electron fluid with respect to the ion fluid. So, that means that is the basic thing that is what we assume to be the cause of instability. So, if that is the case this is how the velocities and the number densities are going to be perturbed and is equals to i e e 1 divided by m e omega minus k u naught let us say we call this as e 1 and n e 1 is minus i e k n naught divided by m e times omega minus k u naught square that we call as 15. And let me highlight the important variables 1, 2, 3, 4 by using the linearized equations we have been able to get all the terms.

So, let us say from equation number 15, 10, 12 and 15 you need the knowledge of n i 1 and n e 1 we have already found out. So, if you substitute this into this equation number the Poisson equation basically the perturbed Poisson equation what you can write is this we have i k epsilon naught u 1 is i e square k n naught times 1 by m i omega square plus 1 by e 1. So, this is the Poisson equation that you have e 1 will of course get cancelled. Now this is for this equation to have a non trivial solution for e 1 it is necessary that the following dispersion relation between omega and k is obeyed or is valid omega p square times. So, this equation this dispersion relation must be valid for let us say we call this as a now for a to have a non trivial solution for e 1.

So, the dispersion relation can be solved for any frequency as a function of the wave number because it is between omega and k. So, if the wave number k is assumed to be real then the frequency omega obtained from this equation may be complex which means omega can be written as omega real plus i times omega imaginary. Now if it so happens that if omega i is greater than 0 the oscillation mode will be growing in time that is we are going towards the instability. So, let us say we define omega by omega pe omega by omega pe to be x and k u naught by omega pe within this equation as y then the dispersion relation can be written as 1 is equals to m e by m i divided by x square plus 1 by x minus y whole square which can be written as a function of x and y. Now if you write if you plot this x and y with x on the x axis y on the not y rather F you will get a curve something like this.

So, this one this point because you have to see is 1. So, the stability is actually understood. So, this shows a plot of F x y as a function of x for a given value of y. So, intersection of this curve with F of x y is equals to 1 gives the real values of x satisfying this dispersion relation and the function has two different singularities at x is equals to 0 and x is equals to y. So, this value is a singularity because the slope because the figure is going towards infinity. So, this is expressing the instability of the system of or the plasma in which we have considered two fluids.

So, this is some brief discussion about instabilities, how instabilities can be understood and how a simple two stream instability can be understood mathematically. So, you can read more about this in any of the reference books. So, this concludes our discussions on plasma physics and applications. So, we have in principle we have covered many important or let me put it this way many fundamental aspects of plasma physics. This course is not intended to be an advanced plasma physics course rather it is a very basic course on plasma physics where if you compare the complexity of mathematics that you can find in advanced plasma physics course whatever that is being taught is very basic and very fundamental.

I hope you will understand what has been covered and you may have to solve all the mathematics the steps in between to really appreciate the physics that comes out of these mathematics. So, in the subsequent lectures we will try to address some applications of plasma, how systems based on plasma can be used or in research or in industries etcetera. Thank you so much. Thank you.