Plasma Physics and Applications

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Lecture 53: Ambipolar Diffusion

Hello dear students. In today's lecture we will discuss more important aspects of diffusion and collisions in plasma. So far we have understood what is the diffusion coefficient and we have derived some very important relations to understand how diffusion changes with respect to the concentration and with respect to time. So, one very important equation that we have derived is this where we write dN by dt and diffusion coefficient as d square N by dH square plus. Now you see this in this expression we have realized the difficulty of estimating the variation of number density or concentration with respect to time as well as with respect to the space simultaneously. Then we made an assumption we will consider at a particular height or at a particular distance and then with respect to that we will try to evaluate the time evolution.

The most important consequence of this relation is we should remember the diffusion coefficient as kT by m nu. You see the diffusion coefficient will be higher if you have lower collisions and for lower masses the diffusion coefficient will be larger or for larger temperatures the diffusion will be larger. So, basically what we are trying to understand is if you have a major species in a fluid if you have a major species how the minor species will diffuse through this by the means of collisions or by avoiding collisions is the main topic of discussion. Now diffusion is a natural consequence of pressure gradient.

Pressure gradient force will be opposite to the differences in pressure or the gradient and diffusion is a process which is actually happening to nullify this pressure gradient as such. So, it is natural for us to expect lighter mass particle will diffuse faster. So, one conclusion is particles with lighter mass will diffuse faster and lower collisions collision frequency leads to larger diffusion naturally. So that is why when you talk about the diffusion coefficient we have realized the d can be written as d naught exponential plus H by H d. So, this increases as we go up as we travel across a distance whereas the

number density decreases.

$$\frac{dn}{dt} = D\left\{\frac{d^2n}{dh^2} + \left(\frac{1}{H_N} + \frac{1}{H_D}\right)\frac{dn}{dh} + \frac{n}{H_D H_N}\right\}$$

$$\uparrow D = \frac{kT\uparrow}{\downarrow m\nu\downarrow}$$

1) lighter mass will diffuse faster

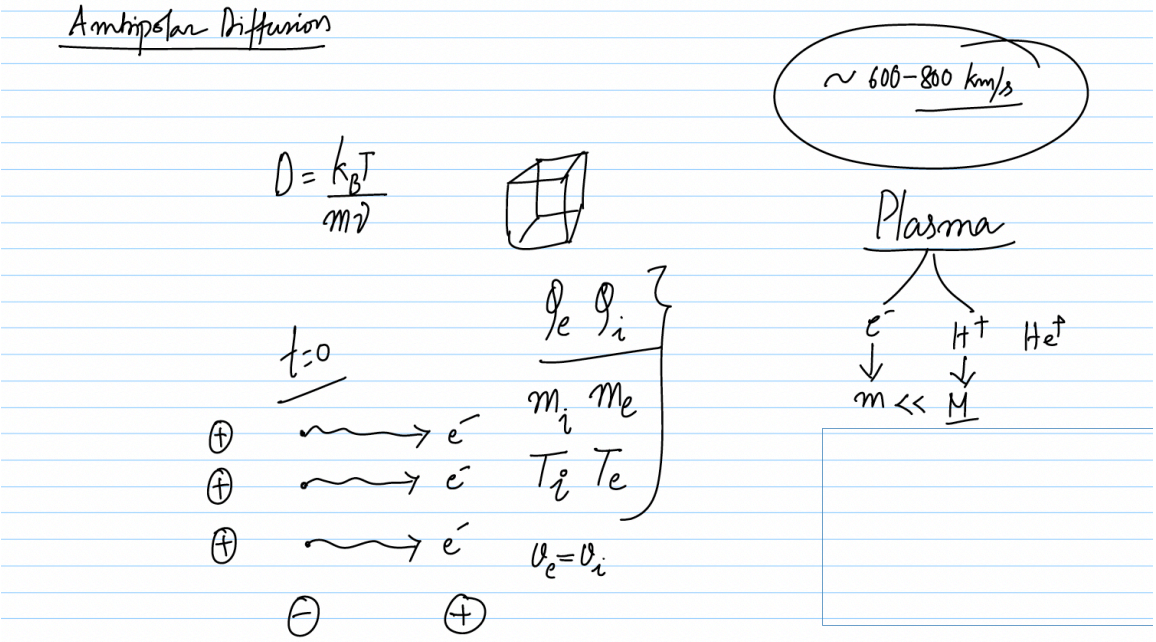
2) Lower collision frequency $\Rightarrow$ larger diffusion

$$D = D_0 \exp\left\{+h/H_D\right\}$$

So, these two things are exactly compensating each other and in total we have a good agreement between the three key variables of this diffusion picture. Now when you talk about diffusion it is very important to understand a specific phenomena in plasma physics which is called as the ambipolar diffusion. What is ambipolar diffusion? We will try to understand the importance of ambipolar diffusion or present a simple thought to understand where does the ambipolar diffusion exist. If you consider solar wind for example, we know that solar wind is a stream of particles which are emanating from the sun and they are reaching towards the edges of the solar system. One very important characteristic is there are this is basically plasma highly energetic plasma which is made up of electrons, hydrogen ions and things like that.

Now this plasma as it travels its velocity increases and reaches supersonic velocity at a point which is below or which is within sun earth distance. These are all characteristic features of solar wind. But what I am actually trying to tell you is the mechanism by which the solar wind is escaping the sun's gravitational pull and following an accelerated motion towards the edges of the solar system cannot be differential for electrons and ions. That means, it has to be the same say it coronal heating or any mechanism leading from the magnetic disruptions on the surface of the sun whatever it is or if you just consider the surface temperature of the sun or coronal temperatures to be the main cause for driving away solar wind at very large velocities. This mechanism must have been the same for electrons and ions because ultimately the plasma that comes out seems to be

made up of electrons and ions nothing else.



Now these electrons and ions for example, when they reach the vicinity of the earth are approximately travelling at let us say 600 to 800 kilometres per second. This is a very large velocity depending on whether this fast solar wind or slow solar wind depending on the conditions and many other things this velocity can sometimes be typically of this order. Now the question is this is the velocity of the plasma that means, this is the velocity of the entire plasma which is a combination of electrons and ions like this. Now there was no differentiation because let us say this is a lighter mass particle and this is a very large mass. Now when these particles were thrown out into the space the mechanism must not have differentiated between these masses.

So, both of them have been through the same process by which they got the energy and left the gravitational pull of the sun. And more importantly if you give an amount of energy to a particle the kinetic energy of the particle or the velocity with which the particle will move taking that energy will obviously depend on the mass because the kinetic energy is half mv square. So, lighter mass particles will have a larger velocity and heavier mass particles will have a very small velocity. That means m being very small in comparison to capital M the electrons velocity out of that source of energy should be tremendously larger in comparison to the velocity of the ion. But what you see at the end is that when you take plasma both the electrons and ions are travelling at the same speed there is no difference.

$$-\frac{dP}{dh} - mng = nmvw \quad \text{——} (1)$$

$$-\frac{dP_i}{dh} - nm_i g + E_{em} = nwm_i v_i \quad \text{for ions} \quad \text{——} (2)$$

$$-\frac{dP_e}{dh} - nm_e g - E_{em} = nwm_e v_e \quad \text{for } e^- \quad \text{——} (3)$$

Add (2) & (3)

$P_i = nkT_i$, $P_e = nkT_e$

$m_i \gg m_e$; $m_i v_i \gg m_e v_e$

$m_i v_i \gg m_e v_e$

So, the solar wind plasma if you take a unit volume of solar wind plasma you will realize that all the constituents inside the solar wind plasma are travelling at the same speed. That means the flux of electrons and the flux of ions is the same. So, how is it possible? How is it possible that the electrons being lighter are travelling at the same speed in comparison to the ions or put it other way the ions being so much heavier are still travelling at the same velocity as that of electrons. So, this can be understood by bringing the electro dynamical aspects of plasma physics and this discussion actually leads to a unique process which is called as the ambipolar diffusion. Ambipolar diffusion is the main cause for particles of different masses to travel with equal velocities inside plasma.

Now in order to make a difference between the heavier mass and lighter mass we let us also bring in all other variables Me, Mi the masses of electron and ion. Since we are used to using the notation of temperature being different from for electrons and ions the temperature difference makes more sense in a diffused plasma or in a weaker plasma. Now these things are different but what you see the velocity of electrons and velocity of ions is the same. If you bring something that you know already then the diffusion coefficient for plasma k B T by m mu. So you would expect that lighter mass particles will diffuse faster that means the electrons will diffuse way faster in comparison to the ions.

$$-\left(\frac{dP_i}{dh}+\frac{dP_e}{dh}\right)-nm_ig-nm_eg+F_{en}-F_{en}=nWm_iv_i+nWm_ev_e$$

$$-k(T_i+T_e)\frac{dn}{dh}-nm_ig=nWm_iv_i$$

$$-k_B(T_i+T_e)\frac{dn}{dh}-nm_ig=nWm_iv_i$$

$$\frac{-k_B(T_e+T_i)}{m_iv_i}\frac{dn}{dh}-\frac{nm_ig}{m_iv_i}=nW$$

$$\frac{-k_B(T_e+T_i)}{m_iv_i}\left\{\frac{dn}{dh}+\frac{nm_ig}{k_B(T_e+T_i}\right\}=nW$$

So what happens is so we have to have a starting point let us say at T is equals to 0 let us assume the lighter mass particles which are electrons are diffusing faster into the second medium and as a result what happens is these particles will move further or will move away. So when they move away we have to recollect some discussion that we had at the early stages of this course which is when they move away they set up space charge effects. They set up space charge the difference of charges so polarity is being introduced. So they set up space charge and electric field will be set up which will be opposite to the direction of electrons movement that is how it is. Electrons go against the electric field and ions go along the electric field.

So when they move out when being the lighter ones they move out and they will set up a space charge and an electric field. So this electric field will obviously try to hinder the faster rate of diffusion of electrons at the same time the same electric field is going to be conducive for the movement of ions. So the ions seeing the when this electric field they will be faster. So the electric field which slows down the electrons will act exactly in the opposite direction and will accelerate the ions. So electrons are moving ahead ions are lagging here let us say this positive sign indicates the ions.

$$D_{amb} = \frac{k_B (T_e + T_i)}{m_i \nu_i}$$

$$-D_{amb} \left\{ \frac{dn}{dh} + \frac{n m_i g}{k_B (T_e + T_i)} \right\} = nw$$

$$H = \frac{k_B T}{mg}$$

$$-D_{amb} \left\{ \frac{dn}{dh} + \frac{n}{H_p} \right\} = nw$$

$$H_p = \frac{k_B (T_e + T_i)}{m_i g}$$

So electrons are moving away the electric field is set up and so this is now going to be positive and this is going to be negative. So this is the electrons the ions will be accelerated by the electric field. So ultimately we can consider a steady state will be reached what will happen after so much of time after let us say certain amount of time electrons although they are very mobile their velocity itself creates an electric field and it will try to decelerate the electrons motion because electrons are now asked to go against the direction of electric field. So after certain amount of time a steady state is reached where the electrons and ions are moving with the same velocity which is decided by the electric field. So once the electric field is trying to slow down the electrons and the same electric field is trying to make the ions move even faster.

Now the forces that are acting so this resulting velocity can be a drift velocity which is as a result of all the collisions and electric field modifications etc. So the forces acting on the particles are the partial pressures of course when there is a density gradient the moment you talk about diffusion there is a density gradient or there is a concentration gradient and the partial pressures difference in partial pressure is what drives the plasma from one point to another point. And if you imagine a picture in which the plasma is under the effect of gravitational pull so each particle will experience a gravitational force

obviously and in addition to all this every charged particle electron and ion will experience an electric field. So this electric field can add to the PGF or can get subtracted from the PGF or the resultant of gravity plus PGF something like that. So we can without involving the electric field we can write a simple equation indicating the force balance as let us say dP by dH minus dP by dH minus mNg is equals to Nm omega W.

So this equation is familiar we have already written the same equation in the last class we have written dN by dH. So P is the partial pressure so as the plasma travels along the length dimension which is indicated by small h to nullify the pressure gradients it will come under the influence of gravitational pull and a net drift velocity which is attributed out of the collisions. Now here as long as you do not talk about different charges or you do not talk about ambipolar diffusion we need to this equation looks fine but the moment you talk about that you have to add an additional term which is basically an electric field. Now the direction of electric field will be exactly opposite to both the charges so we can write two equations now for electron and ion. So I will write minus dP i by dH minus Nm i g plus e e N is equals to N W m i gamma i so this is for ions this is what is this let us say we call this equation as equation number 1 and this is equation number 2.

You see this dP by dH the force is exactly opposite to the pressure gradient so this minus sign is justified. dP i is the pressure gradient in the partial pressure of ions N is the number of particles per unit volume each of this particles has an average mass of m i. So the total gravitational force that the plasma will experience is per unit volume is Nm i g and this is e e N so you see this force the e F is equals to e Q the force per unit volume due to the electric field is e N and on the right hand side you have the collisional force. So similarly we can also write an equation for the electron dP e by dH minus N m e g minus e e N is equals to N W m e mu e this is for an electron so we call this as equation number 3. So what are we trying to do? We have discussed some basic idea behind particles of plasma travelling at the same velocity that is a question that we are trying to answer.

How do you expect the particles to have the same velocity despite having so much disparity in terms of their masses? Having posed that question we resorted to an arrangement in which there is an electric field which is making the heavier particles to move with larger velocity at the same time slowing down the lighter particles to move with slower velocity effectively bringing down a balance where all of them are travelling in unison with the same velocity. So that is the basic thought. In order to establish this that thought we have taken this mathematical equations. First is we have the simple equation of force balance and since we are now adding an electric field we have written two variations of that equations. Now let us say now what we do is we are going to add 2

and 3 adding equations 2 and 3 because we want to write down the equation for the entire plasma let us see.

So the partial pressure Pi is Nk Ti the partial pressure Pe is Nk Te and with the understanding that mi will be much greater than me and mi vi will be much greater than me ve and mi nu i will be much greater than me nu e. With all these things we will write an equation which is just a combination of equation number 2 and 3 it will look something like this minus d Pi by dh plus d Pe by dh minus Nmig plus Eem minus Een is equals to Nw mi nu i plus Nw me. So this will get cancelled and this will be very small because of the mass of electron being very small can be neglected and this one also can be neglected. Coming back removing all the terms which have been neglected or cancelled we can write minus K Ti plus Te times dN by dh I have used P is equals to NkT and got this minus Nmig is equals to Nw mi nu i slightly rearranging the equation we have minus K times it is better to write Kb the Boltzmann constant Kb times Ti plus Te dN by dh minus Nmig is equals to Nw mi nu i or we will rearrange it minus Kb Te plus Ti times mi nu i dN by dh minus Nmig or minus Kb times Te plus Ti divided by mi nu i is dN by dh plus Nmig divided by dN by dh plus Kb Te plus Ti is equals to Nw. So in this factor that appears Kb times Te plus Ti divided by mi nu i is in the dimensions of diffusion coefficient and we are going to call this as d suffix a and b which is Kb times Te plus Ti divided by mi nu i.

So, with this we can write the equation as minus the diffusion coefficient of amepolar diffusion times dN by dh plus Nmig divided by Kb Te plus Ti is equals to Nw. So, this is a very important relation that we have to remember. If you look inside the bracket there are few terms which are already familiar to us because we know the plasma scale height as Kb T by mg generally. So, we can write it as minus d amepolar diffusion coefficient times dN by dh plus N by hp the plasma scale height is Nw. So, this is the modified equation for plasma which is including the amepolar diffusion coefficient.

So, despite the differences in mass and velocity both the electrons and ions diffuse at the same rate which is characterized by the amepolar diffusion coefficient. So, this type of diffusion process is known as the amepolar diffusion and this one so is hp the plasma scale height is Kb times Te plus Ti divided by mg. So, what is this? This is called as the plasma scale height. So, we will try to understand few more interesting aspects of the amepolar diffusion in the next lecture. Thank you.