Plasma Physics and Applications

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Week :11

Lecture 52: Collisions and Diffusion in Plasma -II

Hello dear students, we are discussing diffusion processes in plasma. So far we have considered two species which are a major species J and minor species N. The major species is uniform at equilibrium and minor species has a concentration gradient along x and when this minor species goes through major species it will encounter a lot of collisions and because of these collisions there will be a force and there will be additional force due to the concentration gradient. These two forces should be equal so that we have the drift velocity constant. So, now in the last lecture we have seen that we can write a relation between the pressure gradient and collisional force. So, dP by dx is Nm W nu.

So, P the partial pressure of the minor species can be written as P is equals to N kT. Using this we can write minus kT dN by dx is equals to Nm W or minus kT by m nu is times dN by dx is equals to N W. So, this kT by m nu the factor kT by m nu is written as capital D dN by dx is N W. So, where D is called as the diffusion coefficient.

D is the diffusion coefficients. So, so far so this seems to be valid. So, let us say we write here D times dN by dx is equals to N W. This is a valid relation as long as we do not have any gravity field and the concentration gradient is only along the x direction. So, let us say we include the gravity field in the y direction or in the vertical direction.

Now we are looking in the vertical direction and we will also include the gravitational force which acts like an additional force in addition to the collisional and the pressure gradient force. We still have the concentration gradient along x. So, if there are N number of particles per unit volume and m represents the average mass of each of these particles and g is the gravitational force. Then we can assume the force N mg to be acting on the particles per unit volume. So, the modified equation will look like minus dP by dH minus N mg is equals to N m nu W.

So, using P is equals to N k T just like we did last time we can write minus k T by m nu dN by dH minus N mg by m nu is equals to N W or minus k T by m nu times dN by dH plus N mg by k T is equals to N W. We are shifted from x direction to y to the vertical direction. We call it as H. So, representing this factor as d the diffusion coefficient minus d times dN by dH plus N mg by k T will become equal to N W. k T by mg is the scale height we can write it as d times dN by dH plus N by H is equals to N W.

$$-\frac{dP}{dx} = nmW\nu$$

$$P = nkT$$

$$-kT\frac{dn}{dx} = nmW\nu$$

$$-\frac{kT}{m\nu}\frac{dn}{dx} = nW$$

$$-D\frac{dn}{dx} = nW$$

$$\downarrow$$

Diffusion Coefficient

$$\boxed{-D\frac{dn}{dx} = nW}$$

This is how the force balance works like. What is H? H is nothing but k T by m g which is the scale height. This scale height is different at different temperatures and also different for different species depending on the mass. But the scale height is not the distribution height that you should remember. Now at this point it is very important to know how the concentration of minor gas changes as a function of time due to diffusion.

See so far the minor gas species concentration if you do not include collisions all this how does it change? Since there is a gravity field you can write N to be the minor gas concentration per unit volume with reference to a standard value it is something like this.

Simple, but what have we done in addition to this how far have we come from this simple relation. This simple relation is of course valid, but when the particles are moving they will encounter an additional force because of collisions. So, this diffusion process of minor gas through the major gas gives out an additional force which is the collisional force. When you bring that into the picture this is how the modified equation looks like.

$$nmg$$

$$-\frac{dP}{dh} - nmg = nm\nu w$$

$$-\frac{kT}{m\nu}\frac{dn}{dh} - \frac{nmg}{m\nu} = nw$$

$$-\frac{kT}{m\nu}\left[\frac{dn}{dh} + \frac{nmg}{kT}\right] = nw$$

$$-D\left[\frac{dn}{dh} + \frac{nmg}{kT}\right] = nw$$

$$\boxed{-D\left[\frac{dn}{dh} + \frac{n}{H}\right] = nw}$$

$$H = \frac{kT}{mg}.$$

$$\boxed{n = n_0 \exp(-h/H)}$$

$$n$$

Now is there any chance that we can invert this relation and get a modified expression for the number of particles per unit volume including the d that is what we are actually trying to do. So, what we are doing is we are trying to know how the concentration of minor gas changes as a function of time or distance due to diffusion. Now when you want to find out how the minor gas species concentration changes with respect to time. So, far we have been doing dN by dH, but we want to convert that into time change with respect to time the rate of change of minor gas species. Then it is nothing, but how is the rate can be understood as how many number of particles are entering into a volume and how many particles are leaving the volume.

So, it is basically the idea of continuity equation if you bring that you can write dN by dT is equal to minus d by dH of N w it is same as that. So, d by dH of N w from the earlier relation we know that d by dH of d times dN by dH plus N by H N w if you go here you see this is I have just taken d by dH of N w will give you this. Now that minus gets cancelled. So, now dN by dT can be written as d by dH of d times dN by dH plus N by capital H. Now d is the diffusion coefficient is kT by m nu which implies d is proportional to 1 by nu at a constant temperature.

$$\frac{dn}{dt} = -\frac{d}{dh}(nw)$$

$$= \frac{d}{dh}\left\{D\left[\frac{dn}{dh}+\frac{n}{H}\right]\right\}$$

$$\frac{dn}{dt} = \frac{d}{dh}\left\{D\left[\frac{dn}{dh}+\frac{n}{H}\right]\right\}$$

$$D = \frac{kT}{m\nu} \Rightarrow D \propto \frac{1}{\nu} \Rightarrow D \propto \frac{1}{n_J}$$

$$\frac{D_1}{D_2} = \frac{n_{J_2}}{n_{J_1}}$$

$$\frac{D}{D_0} = \frac{n_{J_0}}{n_J}$$

So, collisional frequencies obviously it will depend on the number of particles. So, it will be N j the number of major species with which the particles are colliding. If you put a reference so d you have to write d1 by d2 is equals to N j2 by N j1. So, this relation can be written like this. So, if you assume your reference to be d0 so d by d0 can be written as N j0 divided by N j so with respect to a reference or diffusion coefficient d can be written as d0 times N j0 by N j.

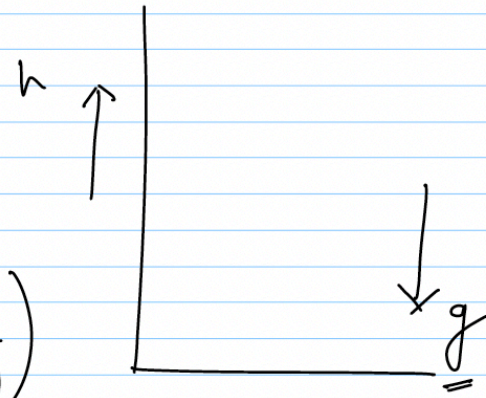
So, when the major species N j j species is in equilibrium with respect to time we can write including the gravity field we can write N j0 times exponential minus H by H d using this into this relation we can write d as d0 N j0 divided by N j0 times exponential minus H by H d. So, d is equals to d0 exponential small h by H d. So, the diffusion coefficient as a function of distance seems to be increasing exponentially with respect to height. We have this picture in which gravity is acting vertically downwards the height is taken in this way if the major species is distributed as per this scale height the diffusion constant which is k T by m nu seems to be increasing with height this is an interesting finding. This is exactly opposite to what we have seen with respect to the number densities diffusion coefficient increasing with height or with distance.

$$D = D_0\left(\frac{n_{J_0}}{n_J}\right)$$

$$n_J = n_{J_0}\left(\exp\left(-h/H_D\right)\right)$$

$$D = D_0\left(\frac{\cancel{n_{J_0}}}{\cancel{n_{J_0}}\exp\left(-h/H_D\right)}\right)$$

$$\boxed{D = D_0 \exp\left(h/H_D\right)}$$



$$\frac{kT}{m\nu}$$

So, bringing back that here so we can write dN by dT is d by dh of d times dN by dh plus N by hN one thing. So, earlier we had d as constant but d is not a constant now. So, we can write dN by dT as dD by dh times you see what I have done what I have done is just took the derivative inside and d is also changing with respect to height exponentially positive variation. So, we have used the chain rule to apply the derivative on d once and apply the derivative on everything that is there inside this bracket. So, we can rearrange the terms such that dN by dT is equals to d is d naught exponential h by hD hD is the scale height for the diffusion coefficient.

So, dD by dH is d naught exponential h by hD times 1 by hD. So, all of this is nothing but d itself dD by dH is d by hD. We can use it here d by hD times dN by dH plus N by hN plus d times the rate at which the concentration of minor species changes with respect to time due to collisions and diffusion dN by dT. Rearranging all these terms we can write d times d square N by dH square plus 1 by hD plus 1 by hN times dN by dH plus N by hD hN. You see this is a very important relation it has the scale heights of small n as well as capital D where d is kT by m nu and varies as d naught exponential h by hD and the concentration of minor species with respect to distance N naught exponential minus h by hN.

$$\frac{dn}{dt} = \frac{d}{dh}\left\{ D\left(\frac{dn}{dh} + \frac{n}{H_n}\right)\right\}$$

$$D = D_0 \exp(h/H_D)$$

$$\frac{dD}{dh} = \underbrace{D_0 \exp(h/H_D)}\cdot\frac{1}{H_D}$$

$$\frac{dn}{dt} = \frac{dD}{dh}\left(\frac{dn}{dh} + \frac{n}{H_n}\right) + D\left(\frac{d^2n}{dh^2} + \frac{1}{H_n}\frac{dn}{dh}\right)$$

$$\frac{dD}{dh} = \frac{D}{H_D}$$

$$\frac{dn}{dt} = \frac{D}{H_D}\left(\frac{dn}{dh} + \frac{n}{H_N}\right) + D\left\{\frac{d^2n}{dh^2} + \frac{1}{H_n}\frac{dn}{dh}\right\}$$

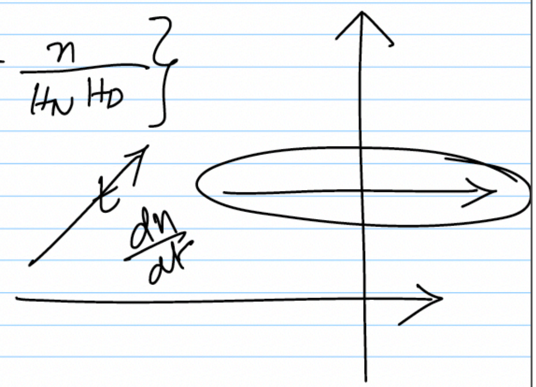
$$\boxed{\frac{dn}{dt} = D\left\{\frac{d^2n}{dh^2} + \left(\frac{1}{H_D} + \frac{1}{H_n}\right)\frac{dn}{dh} + \frac{n}{H_D H_N}\right\}}$$

$$n = n_0 \exp(-h/H_n) \qquad D = \frac{kT}{m\nu}, \quad D_0 \exp(h/H_D)$$

So, this is with respect to space this is with respect to time. So, we have this relation let us take this relation once again dN by dT the rate at which concentration of minor species changes with respect to time seems to be dependent on how it changes with respect to space and how the diffusion coefficient changes with respect to space is d times d square N by dH square plus 1 by hN plus 1 by hD times dN by dH plus N by hN hD. So, if you see this I have probably skipped one or two steps of algebra between these two relations you can work it out you will eventually get the same form that I have written here. So, what it implies is that when the major gas is distributed as per the gravitational field it decays exponentially you will find that the diffusion coefficient is increasing with respect to the it is exactly opposite. So, in this picture we can realize that the diffusion increases exponentially with respect to height.

So, till this point we do not know what will be the distribution of N with respect to height. So, what we have understood so far is collisions will happen more the rate the number of collisions will be more which will be a function of the number of major species. Major species densities are exponentially decreasing with respect to increasing height and due to this the number density decreases exponentially with increasing height. So, as a result of all this it is a natural consequence that the diffusion coefficient will increase with respect to height. So, from this discussion that we had we can realize one simple thing that we can only obtain one distribution either with respect to time or with respect to space at a given instance.

$$\frac{dn}{dt} = D\left\{\frac{d^2n}{dh^2} + \left(\frac{1}{H_N} + \frac{1}{H_D}\right)\frac{dn}{dh} + \frac{n}{H_N H_D}\right\}$$



$$n = n_0 \exp\left(-h/\delta\right)$$

$$\frac{dn}{dh} = n_0 \exp\left(-h/\delta\right)\left(-\frac{1}{\delta}\right)$$

$$\frac{dn}{dh} = \frac{-n}{\delta} \Longrightarrow \frac{d^2n}{dh^2} = \frac{n}{\delta^2}$$

So, let us see the concentration of minor species how it will change with respect to time. So, let us say we assume at a particular height. So, at a particular height so, we cannot take both the things together change things changing with respect to time and with respect to height it is not possible. So, what you can do is you can fix at a particular height and here you can vary the time and find out how it will change dN by dt. So, let us say at a particular height the minor species are varying there only as N is equals to N naught exponential minus H by delta.

So, which is dN by dH is equals to N naught exponential minus H by delta times 1 by minus 1 by delta. So, we can write dN by dH as minus N by delta or d square N by dH square as N by delta square. We will use this here. So, dN by dt is equals to the diffusion coefficient times N by delta square plus slightly rearranging the terms we can pull out N and write it as 1 by delta square minus 1 by delta times 1 by H d plus 1 by H N.

So, this is dN by dt. So, dN by dt seems to be proportional to d or seems to be proportional to N when you assume everything else to be a constant at a particular height of course. So, we can write dN by dt is some constant gamma times N with gamma is equals to d times 1 by delta minus 1 by H d times 1 by delta minus 1 by H N. If you are interested you can check out the algebra. So, as long as the minor species concentration changes with respect to height which is following with respect to a distribution height of delta its concentration increases at a rate gamma with respect to time.

$$\frac{dn}{dt} = D\left\{\frac{n}{\delta^2} + \left(\frac{1}{H_D} + \frac{1}{H_N}\right)\left(\frac{-n}{\delta}\right) + \frac{n}{H_N H_D}\right\}$$

$$\frac{dn}{dt} = Dn\left\{\frac{1}{\delta^2} - \frac{1}{\delta}\left(\frac{1}{H_D} + \frac{1}{H_N}\right) + \frac{1}{H_D H_N}\right\}$$

n δ

$\frac{dn}{dt} = +ve$

γ

$$\frac{dn}{dt} = \gamma n$$

$$\boxed{\gamma = D\left(\frac{1}{\delta} - \frac{1}{H_D}\right)\left(\frac{1}{\delta} - \frac{1}{H_N}\right)}$$

This is the conclusion. So, as long as N is changing at a rate of delta dN by dt will stay positive and at a rate gamma. Now, this is the conclusion of this discussion. What we have been able to achieve is we have been able to get a relation of how the concentration of a minor species changes with respect to time when it is diffusing through a major species through another species for example. What is the additional information that you have here when it is when you say that it is diffusing through the major species it is encountering collisions and the rate at which collisions are happening is embedded into this coefficient which is the diffusion coefficient. So, with these ideas we will try to understand what is called as the amepolar diffusion in the next lecture. Thank you.