Plasma Physics and Applications

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Lecture 51: Collisions and Diffusion in Plasma

Hello dear students. In today's lecture, we will try to understand what is the importance of collisions and diffusion processes in plasma. So far in our discussions, we have understood what is plasma, how is plasma different from other types of matter and how plasma can be treated in different ways, what is the propagation of waves look like in plasma, obtained various dispersion relations etcetera. Now, when the plasma is moving from one point to another point or we all know that plasma is made up of charged particles like electrons and ions, what is the role of diffusion in plasma physics or how collisions will affect the behaviour of plasma is going to be the topic of discussion for today. So, this is basically understanding collisions and diffusion in plasma. It is very important to consider how the movement of charged particles happens inside the plasma.

Let us consider for instance, a set of particles or a group of identical particles denoted by one. One denotes a certain type of charged particles and what happens is this type of charged particles will come in contact with another type of charged particles or neutrals which are denoted by two. So, another type of particles. What we are expecting is we want to understand how the motion of these first type of particles will get affected when they interact or when they collide with another type of particles which is two.

So, for instance, we can consider the type one to be electrons and type two to be ions or you can consider type one to be ions and type two to be neutrals or electrons and neutrals. So, how the motion of particles represented by type one will get affected by colliding with another type of particles is the topic of discussion for today. So, let us say we make some approximations. Let us say number one, we will consider the particles move very slowly and such that there is no ionization that means the thermal velocities associated with the particles are very small and ionization or excitations are not present inside the plasma. And we assume that the kinetic energy of the particles let us say type one particles kinetic energy is conserved and we will also assume that the collisions that they will do with type two are going to be elastic in nature.

Collisions & Diffusion in Plasma

I : type of charged particles ← e^- , ions , e^-

II : Another type of particles ⟸ ions , Neut , Neut

1) Ionizations, excitations are not present

2) KE is Conserved & Elastic

x-direction

3) V_1 I Bulk velocity

4) V_2 II

Maxwellian

And the third assumption is type one charged particles are made up of several particles of equal mass. Now, all these particles will have random velocities, their velocities will be different and at the same time the direction of these velocities would also be different. But what we will do is we will assume a group velocity or a bulk velocity v1 associated with type one particles which is actually an average velocity of all the particles that are present within this type. So, it is implicit that some particles may have a smaller velocity than this or some particles may have a larger velocity than this. This v1 is called as the bulk velocity of type one particles.

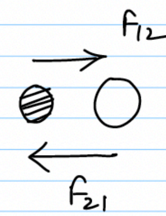
And similarly, we will assume that v2 is the bulk velocity of type two particles and we will assume that both of these are Maxwellian in nature. So, both these particles velocity are according to the Maxwell distribution that means there are large number of particles whose velocity is average, there are small number of particles on either sides of this average velocity. So, what happens when these particles collide, the transfer of bulk momentum occurs. So, when the particles collide, there is momentum transfer depending on the masses of these two particles, the masses of these two particles in addition to the direction of velocities of these two particles will decide the direction of momentum transfer. So, in order to simplify this, we will assume that these two velocities are of course Maxwellian and we will also assume that they are in the x direction.

$$\frac{2m_1m_2}{(m_1+m_2)}(V_2-V_1)$$

ν_{12}

Momentum transfer/time $\Rightarrow \nu_{12}\underbrace{\left(\frac{2m_1m_2}{m_1+m_2}\right)(V_2-V_1)}$

Collision frequency.



$$F_{12} = n_1\nu_{12}\left(\frac{2m_1m_2}{m_1+m_2}\right)(V_2-V_1) \quad \text{— (1)}$$

(II)

$$F_{21} = n_2\nu_{21}\left(\frac{2m_1m_2}{m_1+m_2}\right)(V_1-V_2) \quad (2)$$

So, we are considering a picture which is oversimplified which is just in the x direction. So, when the particles collide, the transfer of bulk momentum happens in the x direction and when this bulk momentum transfer is happening, this will be in such a way that the individual particle velocities does not really matter. So, effectively what is the amount of momentum that is being transferred from one species to another species will only depend on the bulk velocity of the first species and the bulk velocity of the second species. So, individual particles velocity do not really matter. Now, for example, when we have a head on collision of type 1 particles with type 2.

So, what would happen? The momentum that is that would be transferred or gained would be something like this would be 2 m1 m2 by m1 plus m2 times V2 minus V1. So, this is the for head on collisions the particles in group 1 will gain this much of momentum by the collisions. And if you now assume that there are the collisions are happening. So, the collisions are happening at a frequency let us say at a frequency mu. There are new number of collisions per unit time.

This is the frequency of collision of 1 with 2. Then if there are new 1 2 number of collisions per unit time, the gain of momentum would happen at a rate of mu 1 2 times 2 m1 m2 divided by m1 plus m2 times V2 minus V1. What is this? The momentum transfer per unit time is given by this. And what is this? Mu 1 2 is the collision frequency or collision frequency for the momentum transfer. If there is a momentum transfer, we can estimate what is the force that is being exerted on the species 1 due to species 2.

That means the force that is experienced by the species 1 when they collide with the species 2. Let us call that force as F 1 2. The force is N 1 mu 1 2 times 2 m1 m2 by m1 plus you can derive this relations by simply using the law of conservation of momentum. So, this is the force. What is N 1? N 1 is the number of particles in the class 1 per unit volume times this is the average amount of momentum transfer and when you multiply with number of particles, you would get the force that is exerted on the species 1 when they collide with species 2 and this is the force per unit volume.

$$F_{12} = -F_{21}$$

$$n_1 \nu_{12} \cancel{\left(\frac{2m_1 m_2}{m_1+m_2}\right)} \cancel{(V_2-V_1)} = + n_2 \nu_{21} \cancel{\left(\frac{2m_1 m_2}{m_1+m_2}\right)} \cancel{(V_1-V_2)}$$

$$\boxed{\frac{\nu_{12}}{\nu_{21}} = \frac{n_2}{n_1}}$$

$$t \;\&\; t+dt$$

$$\propto \nu \exp(-\nu t)$$

$$\frac{1}{\nu}$$

Now this is one side of the coin. Now the species 2 when you say collision with species 2, the species 1 colliding with species 2 has resulted this. You can also look at it in another way saying that species 2 is colliding with species 1. So, there should be a force which is resulting from the momentum transfer from species 2 to species 1 which we are going to call as F 2 1 is going to be N 2 the number of particles within the class 2 per unit volume and multiplied by the collisional frequency which is nu 2 1, species 2 colliding with species 1 and this factor would of course remain same 2 m 1 m 2 divided by m 1 plus m 2 times V 1 minus V 2. You would obviously expect that these two velocities are different.

If these two velocities are same, there cannot be any momentum transfer. Now ideally these two forces which is let us say we assume this to be species 1 and this to be species 2. So, species 1 is the shaded one. So this is F 1 2 is this and F 2 1 is this. Now obviously we would expect that these two forces are exactly same to each other and but in the opposite direction.

So we would expect that F 1 2 is equal to minus F 2 1. They are exactly opposite to each other. So, we can bring these two relations and this two let us say we call this as 1 and we call this as 2. We will bring these two things together and write N 1 nu 1 2 times 2 m 1 m 2 divided by m 1 plus m 2 times V 2 minus V 1 is equals to N 2 nu 2 1 2 m 1 m 2 divided by m 1 plus m 2 times V 1 minus V 2 0. So what you can write is of course this can also be cancelled which will give you a minus sign here.

So, nu 1 2 divided by nu 2 1 is nothing but N 2 by N 1. Of course there is a minus here already. So this will give you plus. So what does it mean? The collisional frequency of species 1 colliding with 2 is inversely proportional to the number density of species 1. This is one very important relation.

The probability of collisions that terminate within a duration of T and T plus dT would be proportional to the collisional frequency times exponential. The average time between the collisions if nu number of collisions are happening per unit time then obviously the time that takes for one collision to happen would be just 1 by nu. Now let us say we will bring some information which is not actually derived at this point of time rather this is based on prior understanding. So let us say particles in class 1 are electrons and particles in class 2 now are neutrals. So like we say plasma not just contains electrons and ions it also contains a large number of neutral species.

So in this case we can denote the frequency nu E n. What is E n? Collisional frequency of electrons with neutrals small n represents the neutrals. Collisional frequency we can write that it will be equal to the number of neutral species per unit volume multiplied by the velocity multiplied by sigma. What is sigma? Sigma is called as a cross section of neutral atom. So sigma tells you what is the probability or sigma quantifies the probability of a collision to happen.

So if you have an electron which is going to be very small sigma which is the cross section of the atom which is in the unit of area will only quantify what is the probability that this collision may happen. Larger the sigma, larger is the probability for the collision. Coming back the collisional frequency of electrons with the neutral species depends on the number of neutrals that you have per unit volume multiplied by the velocity multiplied by the cross section. Now what is this velocity? This velocity is obviously the velocity of electron but not the neutral species. So electron velocity V can be written as square root of 3 k B T by m e.

I) e^-

II) Neutrals

$$\nu_{en} = n_n v \sigma$$

$$v = \sqrt{\frac{3k_BT}{m_e}}$$

$$\nu_{en} = n_n \sqrt{\frac{3k_BT}{m_e}} \times \sigma$$

$$\sigma = \pi r^2 \cong \pi \left(10^{-10}\right)^2 m^2$$

$$\nu_{en} = 2\times 10^{-16} \times n_n \times T^{1/2} s^{-1}$$

$$\boxed{\nu_{en} \cong 6\times 10^{-15} \times n_n \; s^{-1}}$$

So we can write the collisional frequency of electrons with the neutrals as N N multiplied by square root of 3 k B T by m e multiplied by sigma. Sigma is the cross section of the neutral atom. So if you take for instance an atom sigma is written as pi R square which will be approximately of the order of pi times 10 to the power of minus 10 square meter square. Now nu e N 2 10 to the power of minus 16 times N N times T to the power of 1 by 2 second inverse which is actually calculated using the known densities and for plasma temperatures of approximately 600 Kelvin this will be approximately equal to 6 10 to the power of minus 15 times N N second inverse. So this is just one number one relation which tells you how many collisions may happen at a particular temperature between electrons and neutrals.

Basically what happens when a charged particle approaches an atom? An atom is nothing but a positive charge at the center which is called as a nucleus and the electrons which are negatively charged revolving around the center. Without any external field this is perfectly neutral the charge cloud of electrons compensates the positive charge as a result the atom is electrically neutral. But when a charged particle approaches the neutral atom the dipole moment is induced that means this charged cloud will be separated away from its nucleus. So that means the effective cross section which actually determines the probability of a collision will increase because the atom is now being stretched apart to become or to look like a electric dipole. So basically the cross section will now depend on the force.

What has been found is the sigma the cross section is proportional to sigma is proportional to T to the power of half which means we can take this new E n as N N times some constant which is just giving you that everything other than the temperature is T to the power of half multiplied by sigma. Sigma is again a constant times T to the power of half. So the collisional cross section of electrons with the neutral species is going to be proportional to the temperature or the collisional cross section is proportional to the energy of the electron. So, this is the most important point or when you try to do the same thing for a collision between ion and a neutral species then what has been found is that if it is the ion the sigma the collisional cross section generally is proportional to T to the power of minus half. So, which means new I n collisional frequency of an ion with respect to or colliding with neutral species will be of the order of 2.

$$\sigma \propto T^{1/2}$$

$$\nu_{en} = n_n \times k \times T^{1/2} \times C \times T^{1/2}$$

$$\boxed{\nu_{en} \propto T}$$

$$\sigma \propto T^{-1/2}$$

$$\nu_{in} = 2.6 \times 10^{-15} \times n_n \times M^{-1/2} \; s^{-1}$$

6 10 to the power of minus 15 times number of neutral species per unit volume times m to the power of minus half per second. You can combine these two things and write a relation for sigma E i and sigma E n as 2 10 to the power of 11 T to the power of minus 2. Now, this is how we can understand the collisional processes in plasma which seems to be dependent on the temperature and some other number densities. So, moving ahead it is very important to understand diffusion processes in plasma diffusion of plasma or

diffusion in plasma. What is the role of diffusion in plasma? That means, how a species will diffuse from a higher density or higher concentration towards a lower concentration and how this process will lead sometimes will lead to neutralizing of plasma.

So, this is what we have to understand in diffusion of plasma. Generally, when several gases rest in equilibrium under the effect of gravity each of the gas will be distributed as if it alone is occupying the entire volume. So, when temperature is uniform for all the gases the number density of any gas can be written as n is equals to N naught exponential minus h by h. What is n? n is the number of particles per unit volume N naught is some reference with respect to that reference. If you go across when you bring in gravity of course, you have to have gravity.

So, this is the length dimension that I am talking about and at a given temperature what you can see is the number density seems to be varying like this because gravitational pull is in this direction the gravity field is primarily responsible for this distribution. So, which is basically an exponential decay. Now, you see here capital H what is capital H? Capital H is the scale height. It tells you by what increment or by what decrement actually the number density changes over some distance. How much distance does it take for the number density to decay to 1 by e th of its original value is what is being given by the scale height.

What we will do in this sub topic which is the diffusion of plasma we will try to understand how a gas or how a particular species will move inside or will move through another species how it will diffuse how it will go from one point to another point in the presence of another species. So, for that to happen we will consider a major gas. What is major gas? We will consider one species let us say which is larger in concentration and we will consider another species which is minor. Minor species or minor gas because we are considering plasma as a fluid it is convenient to call them as gas. We will consider a major gas which is in equilibrium and equilibrium and a minor gas which is not in equilibrium.

Now, let us consider that a situation when there is no gravitational field and the major gas J let us say the major gas is represented by J and the minor gas is represented by N. Let us say the major gas J is at rest with respect to the distance. So, it is in equilibrium and it is not distributed as such it is in equilibrium and it is at rest. So, it is uniformly distributed that is what I wanted. The major gas is at rest and uniformly distributed within the volume or whatever that you take the major gas is uniformly distributed and there is no gravity field.

Let us say we confine ourselves to one direction which is x. Wherever you go along x you will find equal number of constituents of major gas per unit volume. But whereas the minor gas is not in equilibrium and minor gas is distributed as if there is a gradient in its concentration. So, dN by dx is positive along x direction. So, each particle will make new number of collisions as it moves along this particular path makes new collisions with the major gas.

So, when you take N to be the minor gas the minor gas species has a concentration gradient and these minor gas since they are not in equilibrium as they move along they will encounter collisions at a rate at a frequency of nu 1. So, there are new number of collisions per unit. Now as a result of these collisions the minor gas particles are given an average velocity average velocity which is also called as the average drift velocity which is omega. As a result of collisions if there are no collisions things are different, but collisions will not make all the minor gas particles to move in the same direction or move in the original direction. So, they will collide they will alter their direction move with a different velocity so on and so forth.

But a net drift velocity is attributed which is called as W. And let us say we have a partial pressure P for the minor gas. And since there is a gradient this gradient in the number densities transcends towards a gradient in the pressure. So, P is equals to n k T. Now each unit volume of the minor gas species will experience force.

What type of forces will it experience? Since there is a concentration gradient in the minor species there will be some pressure gradient force. Now the minor gas species are moving through the major gas species. So, there will be collisions and as a result some momentum transfer will be there and if you take that momentum transfer per unit time you will get some you will get a measure of the amount of force that is there. So, there are two forces number one is let us say we take minus n nu m W. I have given the minus because it is exactly opposite direction to what the major gas species will encounter because there is equal and opposite things.

Is due to collisions between J and n. J is the major species n is the minor species. Number two there is some force which is minus dP by dx. What is this? This is dP by dx is the pressure gradient force due to partial pressure variations as there is a concentration gradient along the positive x axis. So, we know what is PGF. So, this is PGF as there is positive dn by dx along the x direction.

$$\boxed{\frac{\sigma_{ei}}{\sigma_{en}} = 2 \times 10^{11} \times T^{-2}}$$

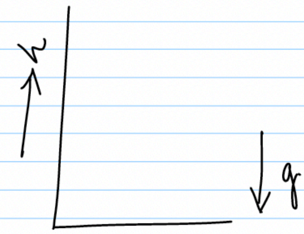
Now, if you want the drift velocity W to remain a constant you would obviously expect that the opposing force which is created which is causing the minor gas species to be retarded because of the collisions must be equally balanced with the pressure gradient force which is due to the concentration gradient. That means for a uniform W drift velocity you would expect these two forces to be equal to each other. That means these two forces when combined should give you 0 dP by dx minus nM W nu is equal to 0. We can write it as minus dP by dx is equal to nM W nu. This is just one relation that we have to understand or remember thoroughly.

Just a quick recap what are we trying to do? We are trying to understand the diffusion of one species through another species of gas. Now, since plasma is an entity which can have three different species minimum three different species there can be different types of ions, different types of neutral species and one species of electron densities. You can also have some negative ions. So, we are trying to see how one particular species can move through another species and what is the role of collisions when such a thing happens that is the basic idea. So, in order to do that we have assumed one species to be at rest and uniformly distributed no gradients and we have taken another species moving through that major species and we also attributed some concentration gradient to the minor species.

Diffusion of Plasma

T

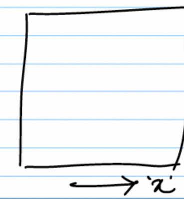
$n = n_0 \exp(-h/H)$



$\underline{H}$

Major gas } equilibrium 'J'

Minor gas } Not in equilibrium 'N'



$\frac{dn}{dx}$ = +ve along 'x'

ν W

$nkT = P$ $\frac{dP}{dx}$

And we have worked out the force balance when this diffusive process goes on and it appears that we have a simple relation which is dP by dx is equal to minus nM W nu. N is the number of particles the minor species number of particles per unit volume, M is the mass, W is the drift velocity, nu is the collisional frequency. We will continue this discussion in the next video. Thank you

1) $-n\nu mW$ due to collisions b/w J & N

2) $-\frac{dP}{dx}$ PGF as there is $\frac{dn}{dx}$ = +ve along 'x'

$\underline{W}$

$$-\frac{dP}{dx} - nmW\nu = 0$$

$$\boxed{-\frac{dP}{dx} = nmW\nu}$$

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