Plasma Physics and Applications

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Lecture 41: Magnetic Pressure

Hello dear students. In continuation to our understanding of idealized MHD equations, in today's class, we will try to see what is magnetic pressure. How does the magnetic field exert pressure? We are familiar with the kinetic pressure, which is basically a resultant of molecular motions or the exchange of momentum between the molecules. So, let us say we consider plasma which is neutral in nature and there are no electric fields. So, we consider a plasma in which there are no electric fields. And in addition, we consider the idealized magneto hydrodynamic equations in which we have clearly demonstrated that plasma is no longer be treated like a two fluid entity rather it is a single fluid and the equations pertaining to the electron and ion were mixed and the entire framework of MHD equations was obtained.

So, we consider idealized MHD equations and then we say that the plasma is in a steady state. What does it mean? Steady state means the velocity is a constant. The plasma motion is such that the velocity is a constant or we can also say that it is in equilibrium. So, we can use the idealized MHD equation, Ne Me plus Ni Mi dou Um by dou T is equals to minus del P plus j cross P.

So, this is the momentum equation which says the effects of electromagnetic behavior are in addition to the effects of pressure force are totally responsible for the rate of change of velocity with respect to time. Now, since we have assumed steady state, we are going to make this derivative as 0 which implies we can write del P is equals to j cross P. What does it mean? It means that the total effect of pressure force is equal to the effects of electricity and magnetism. So, in addition in order to go further we will use the Maxwell equations. The Maxwell equations are del cross B is equals to mu naught j and del dot B is equals to 0.

## Magnetic Pressure

1) $E = 0$

2) Idealised MHD equations

3) Steady state $\Rightarrow$ Velocity is constant

$$(n_e m_e + n_i m_i)\frac{\partial \bar{U}_m}{\partial t} = -\bar{\nabla} P + \bar{J}\times\bar{B}$$

$$\frac{\partial \bar{U}_m}{\partial t} = 0 \Rightarrow \bar{\nabla} p = \bar{J}\times\bar{B} \quad — (1)$$

Maxwell equations

$$\bar{\nabla}\times\bar{B} = \mu_0 \bar{J} \quad — (2)$$

$$\bar{\nabla}\cdot\bar{B} = 0 \quad — (3)$$

Pressure force = Lorentz force

$\nabla p \perp^{r}$ $\bar{J}$ & $\bar{B}$

Let us say we call this equation as equation number 1, 2 and 3. What do we have? We are trying to understand what is magnetic pressure and to use the idealized MHD equation we made an assumption that the electric field is 0, which plasma is completely neutral and we also assumed a steady state which means the flow is steady. So, there are no changes in the velocity with respect to time. So, then we use the idealized MHD equation and using this approximation we realize that the pressure force is exactly balanced by the electromagnetic force. So, in addition we are using the Maxwell equations.

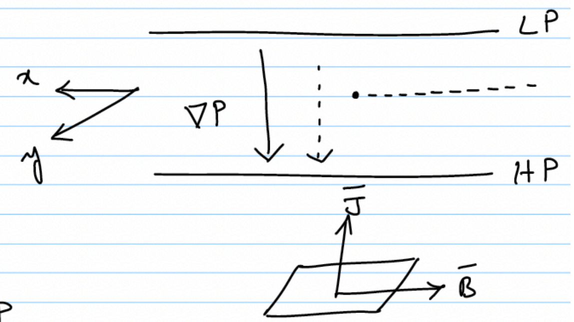
From equation number 1 we say that the pressure force is equals to the Lorentz force. But what we see is that del P the pressure gradient is perpendicular to both j and B. So, we have to imagine del P in such a way that it is perpendicular to both j as well as B. So, how do we do it? So, let us say we imagine we have a surface which is at a lower pressure and a surface at a higher pressure. The gradient of pressure is in this direction del P.

And let us put the reference for coordinates let us say this is x and this is y. So, the surface j and B have to be in such a way that they have to be parallel to delta P. So, if this is the direction along which the pressure is changing, if this is the direction along which the pressure is increasing or the gradient of pressure is defined, it is obvious for us to understand that both j and B will be perpendicular to delta P or they have to exist at a surface where the pressure is constant. By no means they can be aligned in any direction at an angle to delta P rather than only at right angle. So, we can imagine j and B vectors are at a constant pressure surface.

So, within this constant plane if you define this constant surface then you can expect the j vector to be like this and B vector to be like this. And with an understanding that the pressure gradient is perpendicular to both of them, but here it appears that j is vertically upwards rather what I actually wanted to draw is j is in a plane or along the plane. So, the clear message that is so delta P is perpendicular to j and B or we can expect j and B to be in a direction or j and B to be at a constant pressure surface. So, let us say we go back equation number 1 and 2 from equation number 1 and 2 if we eliminate j from equation number 1 and 2 what will we get we can write j as 1 by mu naught del cross B from the Maxwell equation and if we use the definition 1 by mu naught times del cross B cross B as per the equation number 1 from 1 we can write this and from 2 we can write this is equals to del P. So, del P if you resolve this cross product we can write del P as 1 by mu naught B dot del times B minus half del B square or we can write del P plus half del B square.

$\bar{J}$ & $\bar{B}$ are at a constant pressure surface

$\nabla p$ is $\perp^r$ $\vec{J}$ & $\vec{B}$




Eliminate $\vec{J}$ from ① & ②

$$② \Rightarrow \vec{J} = \frac{1}{\mu_0}(\bar{\nabla} \times \bar{B})$$

$$\text{from } ① \Rightarrow \frac{1}{\mu_0}(\bar{\nabla} \times \bar{B}) \times \bar{B} = \bar{\nabla} p$$

$$\vec{\nabla} p = \frac{1}{\mu_0}\left[(\bar{B}\cdot\bar{\nabla})\bar{B} - \frac{1}{2}\nabla B^2\right]$$

$$\vec{\nabla} p + \frac{1}{2\mu_0}\nabla B^2 = \frac{1}{\mu_0}\left[(\bar{B}\cdot\bar{\nabla})\bar{B}\right]$$

del B square is equals to 1 by mu naught B dot del times B. We can simplify it further by writing grad of P plus P square by mu naught is equals to 1 by mu naught times B dot del times B. So, if we expect the B not to be changing spatially or when it is very small we can simply write the derivative on the right hand side to be 0 we can write gradient of this P plus B square by 2 mu naught is equals to 0 or if B is not changing spatially we can write P plus B square by 2 mu naught as the constant. Now this parameter which is being added to the pressure is referred to as the magnetic pressure. So, in the presence of an externally applied magnetic field the sum of kinetic pressure plus the magnetic pressure will be 0 or it will be a constant wherever the plasma goes.

$$\nabla\left(P+\frac{B^2}{2\mu_0}\right) = \frac{1}{\mu_0}\left[(\bar{B}\cdot\bar{\nabla})\bar{B}\right]$$

$$\Rightarrow \nabla\left(P+\frac{B^2}{2\mu_0}\right) = 0$$

$$\Rightarrow P+\underbrace{\frac{B^2}{2\mu_0}}_{\text{Magnetic pressure}} = \text{constant}$$

high $\beta$ : kinetic effects dominant

Low $\beta$ : Magnetic effects dominant

Solar wind — Low $\beta$ plasma

$\beta$ - Value of plasma

$$\boxed{\beta = \frac{P}{B^2/2\mu_0}}$$

So, the most important message is in the presence of an externally applied magnetic field the sum of kinetic pressure is by the virtue of molecular motions and B is the externally applied magnetic field these two things balance each other so that it will the sum of these two terms will remain a constant and if there is a pressure gradient there must be a gradient in the magnetic pressure exactly in the opposite direction. So, these two things compensate each other all the time so that the net quantity the summation will remain a constant. Now to characterize the physical effects of this magnetic field or the magnetic pressure we define a parameter which is called as the beta. Beta is defined as a ratio of pressure the kinetic pressure divided by the magnetic pressure. So, this is a very important parameter to characterize the nature of plasma.

So, beta is the ratio of kinetic pressure to the ratio to the magnetic pressure what is this called as this is called as beta value of plasma what does it infer. So, there may be situation when the kinetic effects are more that means properties exhibited by the plasma as a result of the motion particles like electrons and ions which is inside the plasma maybe more there may be some effects which are established by the magnetic pressure of plasma. So, in order to quantify or in order to differentiate these two effects we define this beta value it is generally seen that in load beta plasma when beta value is very low the magnetic effects will dominate and in high beta plasma the kinetic effects will dominate. So, in high beta it is the kinetic effects which are dominant and in low beta plasma it is the magnetic effects which will dominate. So, for reference we can say that the solar wind which is a plasma system the solar wind which is made up of electrons and ions is generally considered as a low beta plasma.

## Plasma Waves

— Any periodic motion

$$n = \bar{n} \exp[i(k \cdot r - \omega t)]$$

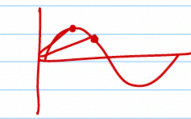
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\vec{K} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$$

$$k \cdot r = k_x x + k_y y + k_z z$$

$\bar{n}$ : Amplitude of wave

$K$ : Propagation constant



Propagating in $x$-direction

$$n = \bar{n} e^{i(kx - \omega t)}$$

$$Re(n) = \bar{n}\cos(kx - \omega t)$$

$$\frac{d}{dt}(kx - \omega t) = 0$$

$$\frac{dx}{dt} = \frac{\omega}{k} = \underline{v_p}$$

What does it mean whenever you are referring to plasma as a gas. So, the kinetic properties are not so important because it will not exert much pressure because of its molecular or ion or electron composition it is the magnetic effects which need much better understanding. So, in addition to this so this is the basic understanding of magnetic pressure if you just go back what we have started with is an idealized MHD equation and we have used these two Maxwell equations in combination with this and we imposed a condition that there are no electric fields and the plasma is moving in a steady state. If the plasma is moving in a steady state we made this velocity term or the change of velocity with respect to time to be 0 then we realized that the pressure force is balanced by the Lorentz force completely balanced by the Lorentz force and if such a balance exists it has to impose a condition that the current density and the magnetic field are perpendicular to the pressure force and if this balance has to be established both of these should be at a constant pressure surface. So, that means it has to be dynamically stable between these three parameters.

$v_p$ : Phase velocity

if $\omega/k = \underline{+ve}$ $\Rightarrow$ wave is moving in +ve '$x$' direction

$= -ve$ $\Rightarrow$ −ve '$x$' direction

So, doing little algebra we eliminated the current density from equation 1 which is idealized MHD equation and the Maxwell equation and then what we realized is that we wrote del P the gradient in the pressure to be equal to 1 by mu naught of this vector product vector triple product. Doing some algebra we have now realized that the change of the gradient of the sum of pressure plus b square by 2 mu naught is equals to 0 or we can say that since this is a gradient if it has to be 0 on the right hand side we have b dot del then P plus b square by 2 mu naught should be a constant. We started calling this quantity b square by 2 mu naught as the magnetic pressure and since plasma demonstrates kinetic effects as well as magnetic effects in order to characterize the plasma we can define ratio of kinetic pressure to the magnetic pressure and call it as a beta value of plasma and a low beta value of plasma always indicates that its magnetic effects are going to be more dominant in comparison to the kinetic effects and high beta plasma tells us that it kinetic effects are going to be more prominent in comparison to the magnetic effects. And for reference solar wind a plasma flowing away from the Sun constantly is referred to as a low beta plasma. Whenever you are studying solar wind its magnetic properties are worth attention rather than the kinetic properties.

In addition to this plasma allows or plasma facilitates the movement of or different types of waves. What is a wave? Wave is a transportation of energy from one point to another point. Plasma is very rich in waves. Plasma allows several different types of waves to permeate through it or to pass through it. What we will do is we will try to understand the wave motion in plasma how different waves are transported through plasma and things like that.

In order to understand we have to see what is the fundamental definition of a wave. Any periodic motion can be referred to as a wave. Wave is any periodic motion or transfer of information from one point to another point. So, any periodic motion can be decomposed into a Fourier series which is several small sinusoids with different amplitudes and different frequencies adding up to represent the full wave. So, any sample wave can be decomposed into Fourier series.

So, let us say we take how do we write an expression for the moment of a wave. So, we write n is equals to n bar exponential i k dot r minus omega t. What is k dot r? k dot r can be written as k x x plus k y y plus k z z with an understanding that the r x i cap plus y j cap plus z k cap and the wave vector is k x i cap plus k y j cap plus k z k cap. Now what is n bar is the amplitude of wave and what is k? k is called as the propagation constant. So, for a wave which is propagating in a three-dimensional phase, this is valid.

Let us consider a wave which is propagating in x direction. Wave propagating in x direction can be written as n is equals to n bar e to the power of i k x minus omega t. So, by notation what we can say is that the exponential means only the real part is to be taken as a measurable quantity. So, that means the real part of n the wave function is n bar cos of k x minus omega t. So, let us consider a point on this wave.

The point is moving or vibrating so that the information is transmitted from one point to the other point. So, this movement of this point in a phase let us say is actually referred to the wave. So, this movement of a point let us consider a point of constant phase within this wave in the wave will always move such that the d by dt of k x minus omega t is always 0. You consider a wave like this. Let us say you have this point, these two points.

Let us consider these two points. These two points are not at the same phase, they are in a different phase. So, you consider one point at a constant phase and this point or will move such that the rate of change of k x minus omega t is always 0. So, if you say dx by dt is equals to omega by k which is called as vp dx by dt is omega by k from this which is called as vp. What is vp? vp is referred to as the phase velocity.

What is the importance of this phase velocity? Phase velocity tells you how individual wave is moving in space. So, if omega by k is positive, what does it mean? It means that the wave is moving in the positive x direction or if it is negative, it is moving in the negative x direction. So, this is some basic information about the magnetic pressure and some fundamental understanding of the wave phenomena. So, we will try to see how plasma can facilitate different types of waves and if they are propagating through plasma, how can we obtain different characteristic features of these waves? That will be in the next class. Thank you.