

# Plasma Physics and Applications

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Week – 01

## Lecture 04: Debye Shielding

Hello students, in today's lecture we will try to understand the mechanism which we call as the Debye shielding. So far in our understanding of basic plasma physics we have seen how plasma tries to maintain charge neutrality by the means of oscillations. So, we have derived the characteristic frequency which we call as the plasma frequency and the basic purpose of these oscillations is to maintain charge neutrality. So, although charged particles constitute plasma electrons and ions, but plasma on the whole is electrically neutral. So, we have seen the charged particles or the presence of charged particles is the reason why plasmas behavior is drastically different from a neutral gas. But at the same time if you look at the overall charge concentration it is electrically neutral.

Now in addition to this, so the frequency of oscillations  $\omega_p$  is square root of  $n q^2 / m \epsilon_0$ . So, where  $n$  is the number of electrons,  $m$  is the mass of electrons and  $q$  is the charge of electrons. So, this is the plasma frequency. So, this is the plasma frequency we call as the plasma frequency.

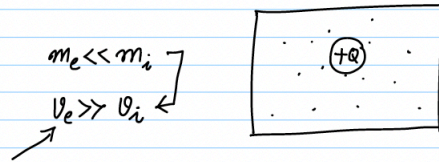
The purpose of this frequency is to maintain charge neutrality. So, of course the plasma frequency depends on the concentration of charges, it also depends on the constituents or the mass of this constituents. Now, in extension we try to discuss the Debye shielding which is a more important concept and also helps us in getting the plasma criteria. What are plasma criteria? Just relations which will make sure that it is not an ideal gas it is actually a plasma. So, based on plasma frequency we have one plasma criteria which is  $\omega_p \tau \gg 1$ .

# Debye's Shielding

- Collective behavior

$$N_e = N_i$$

$$\omega_p = \sqrt{\frac{n_e q^2}{m_e \epsilon_0}}$$



Plasma frequency

$$\omega_p \tau_m > 1$$

$$e(N_e) = e(N_i)$$

$$\phi(r) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q}{r^2}$$

$$f \propto \frac{q_1 q_2}{r^2}$$

Omega p is the plasma frequency which quantifies the oscillations and tau is the mean free time between the collisions then this product should be 1. Now, device shielding establishes the nature of plasma which is the collective behavior of plasma. What is collective behavior? So, when an electric field is subjected this electric fields effect will not be seen everywhere inside the plasma. So, plasma will realign itself such that this external electric field is shielded. So, for most of the plasma this electric field will be invisible.

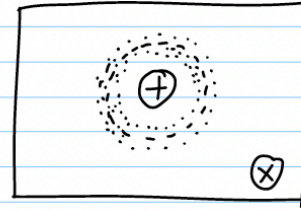
$$n_e = n_i, v_e \gg v_i, m_e \ll m_i$$

$$n_e > n_i \Rightarrow n_e - n_i = 0$$

charge separation

$$\rho = (n_e - n_i)q \neq 0$$

$$\rho = \frac{\text{charge}}{\text{volume}} = \frac{\text{Difference of charges}}{V} \times \text{charge}$$



$\phi$  (Potential)

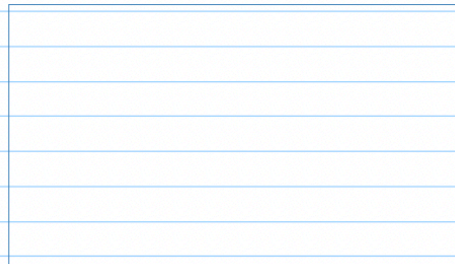
$\lambda_D$  (Debye's length)

$$\rho = q(n_e - n_i) \quad \text{--- (1)}$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{E} = -\vec{\nabla} \phi$$

$$\Rightarrow \nabla^2 \phi = \frac{-\rho}{\epsilon_0}$$



So, this is the basic idea where plasma behaves collectively as a single entity and shields out the effects of electric field. So, we can also discuss how far this electric field will be seen, how far the effect of this electric field can be felt inside this charged plasma. So, that is the basic discussion that we are going to have when we discuss the device shielding. So, the basic idea is the macroscopic charge neutrality of the plasma is preserved by the means of plasma oscillations macroscopic. Overall if you look at the entire system as it is this charge neutrality is preserved by the means of plasma oscillations.

$$\nabla^2 \phi = \frac{-\rho}{\epsilon_0} = \frac{-q(n_e - n_i)}{\epsilon_0}$$

$$q = -e$$

$$\nabla^2 \phi = \frac{e(n_e - n_i)}{\epsilon_0}$$

And since there is there are no charge separation. So, we have realized if even if we try to separate charges large electric fields will be set up and the role is to bring back the

electrons which were being taken away. And in the process we get to see the oscillations. So, as a result what we can conclude is that it is not possible to separate the charges in the plasma. Let us say we take out the electrons and move it away it is not possible.

So, when the plasma is subjected to an electric field its free charges redistribute such that the major part of the plasma is shielded from the external electric field. What are we talking about it will redistribute the charged particles will redistribute so, as to nullify the effect of external electric field. Now, let us consider a plasma which is having number of electrons equal to the number of ions. That means, macroscopically it is electrically neutral total positive charge is equal to the total negative charge. Now, for instance what we do is let us say we keep we take this plasma this chamber or this is the plasma system and in this there are all electrons and ions which are inside.

Now, what we do is let us say we take a test charge and keep it inside. What is the charge plus  $Q$ ? The polarity is positive and the  $Q$  is the magnitude of the charge. Now, obviously when you keep it you can expect some polarization all the negative charges getting attracted to it. Now, here it is more important for us to consider this situation where mass of electron is much less than mass of ion and at a given thermal state or at a given temperature it is natural for us to expect that the velocity of electron will be much larger than the velocity of ion. Why? Because any given temperature will only be able to give some energy to the particle and this energy will obviously depend on the mass of the particle lighter the mass larger will be the energy or larger will be the velocity.

So, as it is this is the consequence let us say this forces this condition to be valid and this is the initial condition in which both the charges are equivalent numbers. And we also say that just to avoid complexity we can say that of course number of electrons is equal to the number of ions, but ions are singly ionized that means there is only one positive charge per atom or per molecule. It is not multiply ionized, doubly ionized, triply ionized we rule out all the possibility. So, effectively even if I have to force this the charge  $1.6 \times 10^{-19}$  multiplied by the number of electrons is equal to the same multiplied by the number of ions.

So, plasma is of course existing at a particular temperature which gives the constituents thermal velocity. Now if you have this positive test charge this will have a potential the amount of potential that you can observe as a result of this positive test charge is simply  $\frac{1}{4\pi\epsilon_0} \frac{q}{r}$ ,  $q$  is the magnitude of charge and  $r$  is the distance away from the charge. Now the Coulombic attraction will start attracting electrons towards it. This is something that is inevitable whenever there is a positive charge the negative electrons will obviously be attracted towards it and according to the force that the Coulombic force we can say that  $F = \frac{q_1 q_2}{r^2}$ . So, the force of attraction will

start pulling the electrons towards it.

But electron as it has a large velocity  $V_e$  due to its thermal velocity will try to neutralize the positive charge. But it will always overshoot the position of the positive charge because electron by the virtue of its light mass it has a high velocity that means due to its inertia it may not be able to just go and stick to the positive charge and just neutralize it. So, it will always in the process of going and colliding with the positive charge it will always overshoot the position and as a result it will not the positive charge will not be completely compensated within a matter of time. What will happen? So, the ions at the same time we have not talked about the ions. The ions will also be influenced but considering the fact that the ions are massive their motion can be neglected.

So, there is no disturbance in the ion concentration or ion distribution inside the plasma. So, the ions are there where they are actually. So, they are there. So, they are constituting a constant positive background in the plasma but whereas the negative charges are disturbed or are influenced by the presence of this positive charge and they are trying to move towards the positive charge. What is the purpose that they are moving? They are trying to move so that they can compensate the positive charge and result in net neutrality.

So, over the period of time after some time what will happen is so we have this system in which we have an enclosure in which plasma is confined and inside we have kept a test charge which is positive polarity. So, as the electrons most of the electrons which come under the influence of this positive test charge they will constitute a cloud of electrons around the positive charge. So, this is just so all these electrons which have. So, in a three dimensional picture what you are trying to see is we have a lot of electrons just surrounding the positive test charge. Now what is the purpose of this all the negative charge it is just to compensate the positive charge that you have kept inside the system.

So, in the vicinity of the positive charges the negative charge cloud is present is it not? All the negative charge cloud is present in the vicinity of the positive charge. So, after sufficient amount of time elapses it may so happen that you are only able to see this negative charge negative cloud of electrons surrounding completely surrounding the positive charge. So, at this point of time it is probably ideal for us to expect that the remaining parts of the plasma or outside this cloud of electrons we cannot see the presence of the positive charge. So, this that means that this let us at this point we see the plasma cannot sense the presence of this external electric field the electric field that you have actually implanted. So, this is what is called as the shielding.

So, plasmas collective behaviour makes sure that any external electric field which tries

to disrupt the charge neutrality or which tries to polarize the plasma because plasma is a charged medium it has negative and positive charges. So, I obviously when you say polarization this external charge or the electric field for that matter is just trying to separate all the electrons and ions. So, any such influence is shielded by the collective behaviour of plasma this process or this mechanism is referred to as device shielding. Now, what is important for us to understand is of course this is the device shielding, but the discussion is not complete just by defining what is device shielding or just by saying this is a mechanism in which is active in plasma. But rather we should see how the potential that is created by the positive test charge does not protrude beyond this electron cloud.

That means one thing very clear the potential can only be seen as long as you are there inside the charged electron cloud and it suddenly vanishes because it is assumed that because we know it for the fact that for regions outside this electron cloud there is no potential. So, we have to get this form of potential  $\phi$  which is valid or which can be non-zero within this electron cloud and immediately when you cross this electron cloud it is 0. So, we have to find the mathematical or analytical form of this potential. So, this potential is called as the device potential and the length up to which you can sense the presence of external electric field is called as the device length. So, within device length only you can see the effect outside device length you cannot see.

So, this characteristic length is called as the device length or which is generally written like this. The symbol is  $\lambda_d$  which is called as the device length and this is the device potential. Now, in our discussion we are going to derive mathematical expressions for both device length as well as potential. At the same time we will also try to understand what is the physics behind each mathematical step.

So, we can start. So, our assumptions so far are that number of electrons is equal to number of ions, velocity of electron is much larger than the velocity of ion, the mass of electron is much smaller than the mass of ion. We have these three things in place macroscopically. Now, let us consider the situation within the vicinity of this positive charge or within the vicinity of the electron cloud. So, one thing very clear is that within this vicinity the number of electrons is much larger than the number of ions. So, we assume a homogeneous distribution of electrons and ions before we put the positive test charge then we can say that since there are a lot of electrons which are concentrated around the positive charge it is obvious that all the negative charges are 4 I mean in this sphere.

And within that within the vicinity the number of electrons is greater than the number of ions which means that  $N_e - N_i$  is 0. You see this is nothing but charge separation.

This condition refers to charge separation. How is charge separation going to play a role? If you want to account for the charge density let us say charge the amount of charge that is available per unit volume then charge density has to be  $N_e$  minus  $N_i$  we have a number then number of electrons per unit volume minus number of ions per unit volume that means if there are more electrons we take  $N_e$  minus  $N_i$ . So, effectively we have more electrons and you multiply with the charge then you get the charge density.

This is the charge per unit volume this is the charge density and this is the difference of charge carriers or charges per unit volume multiplied by charge. You see difference is just a number. So, we have the left hand side which is equal in dimension with the right hand side. Now if you are able to create a situation in which all the ions are surrounding a huge negative charge so that ions being very heavy still able to constitute this cloud then if you are accounting for the total charge in that sphere you will have to place  $N_i$  minus  $N_e$  because number of ions will be larger in comparison to the number of electrons. Earlier when all of this was not there when you did not place the positive test charge macroscopically the first statement that we made is that macroscopically the total number of electrons is equal to the total number of ions.

So, when you talk about quasi neutrality it may so happen that if you go to a very microscopic places maybe you will have some difference of charges but on the whole it is electrically neutral. So, there is no concept of charge separation so that is the basic idea. So, we have now deviated away from the basic charge separation idea. Now charge separation is actually established. Now we will write the charge density here so at least now it is not equal to 0.

So, charge density  $\rho$  is equal to  $q$  times  $N_e$  minus  $N_i$ . Let us say we call this equation as equation number 1. Equation number 1 is the direct consequence of all the assumptions that you have made or all this small experiment that you have done within the plasma by keeping a positive test charge. Now since there is a non-zero charge density it is natural for us to expect that there will be a creation of electric field. Why this electric field was not there earlier? In the beginning itself I made a statement there is no charge separation inside the plasma and there is no formation of electric fields.

Electric fields will be resulted only when you displace they will be compensated by the moment of electrons resulting in oscillations same story again and again. But the message is very simple and very clear. So, because of the non-zero charge density the electric field will result. So, we can write an expression for the electric field which is  $\nabla \cdot \mathbf{E}$  is  $\rho$  by  $\epsilon_0$ . Due to a non-zero charge density  $\rho$  the amount of electric field that will be generated is  $E$ .

$\nabla \cdot E$  is equal to  $\rho$  by  $\epsilon_0$ . Now what is electric field? Electric field is the negative potential gradient. So,  $E$  is equal to minus  $\nabla \phi$ . Electric field is the negative potential gradient.

This is basic. You have a potential difference the electric field a gradient of the potential will be pointing towards the higher potential from the lower potential. The idea is the electric field will exactly be in the opposite direction from the higher to the lower that is why we have a negative symbol here. So, we can combine these two expressions  $\nabla \cdot E$  is equal to  $\rho$  by  $\epsilon_0$  and  $E$  is equal to minus  $\nabla \phi$ . We can write what am I doing? I am just trying to put  $E$  is equal to minus  $\nabla \phi$  into this equation into the divergence equation then we can write  $\nabla^2 \phi$  is minus  $\rho$  by  $\epsilon_0$ . Since we know  $\rho$  the charge density equal to  $Q$  times  $n_e$  minus  $n_i$  divided by  $\epsilon_0$ .

So, there is a minus here. So, let us say we are talking about electrons the charge of electron  $Q$  we can take it to be minus  $e$ . So, we will write  $\nabla^2 \phi$  is  $E$  times  $n_e$  minus  $n_i$  divided by  $\epsilon_0$ . Now what is this  $\phi$ ? So,  $\phi$  is the original potential and electron experiences this potential. So, because of the charge separation there is a potential that is created. Of course, electron will try to nullify this potential because that is a basic idea when it is nullified there is no there is the charge neutrality is retained or is maintained.

Now we will try to derive the device length and device potential from this expression in the next lecture. Thank you.