Plasma Physics and Applications

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Week – 08

Lecture 39: Plasma as a fluid: Fluid Drift

Hello dear students. In today's lecture we will try to understand more important aspects about plasma as a fluid. We have discussed various aspects related to treatment of plasma as a fluid, some basic equations and we also understood how it is the description of fluid is different from the particle description. In today's class we will try to see what are the fluid drifts. So, let us recall our discussion on understanding particle trajectories in different types of fields. We realized we come across something called as drift velocity.

So, in the very familiar setting we have an electric field and we have a magnetic field. We derived what is called as the E cross B drift velocity which looks something like this E cross B by B square. So, this drift velocity expression is valid as long as the electric field and the magnetic field are constants with respect to time or this velocity itself is actually a constant. This does not vary with respect to time as long as electric and magnetic fields are constants with respect to time.

You can deduce what is x of t from this that is a different thing, but we can consider that this velocity is a result of particle being acted upon by the electric and magnetic fields at the same time or perpendicular to each other. So, if you look at the basic governing equation for plasma as a fluid we have a set of governing equations we just do not have one equation we have like the momentum equation we have the continuity equation and so on. So, the basic equation looks something like this m n dou u by dou t plus u dot del times u is equals to q n into E plus u cross B minus del P. So, this equation is valid for a fluid where m is the mass of the particle n is the number of particles per unit volume we can effectively write the density as m times n mass per unit volume. But what is different is that you see this the term that appears here carries a multiple small n which is the number of particles per unit volume.

Fluid drifts

$$mn\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right] = qn\left[\vec{E}+\vec{u}\times\vec{B}\right] - \vec{\nabla}p$$

$\vec{E}$ $\vec{B}$

$$V_D = \frac{\vec{E}\times\vec{B}}{B^2}$$

$\rho = mn$

Pressure force term

P.G.F

Drift

$$\rho\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u}\right] = qn\left(\vec{E}+\vec{u}\times\vec{B}\right) - \vec{\nabla}p \quad \text{— (1)}$$

$$\left.\begin{aligned} &\frac{d\rho}{dt} + \nabla\cdot(\rho u) = 0 \\ &\frac{\partial n}{\partial t} + \nabla\cdot(nu) = 0 \end{aligned}\right\} \quad (2)$$

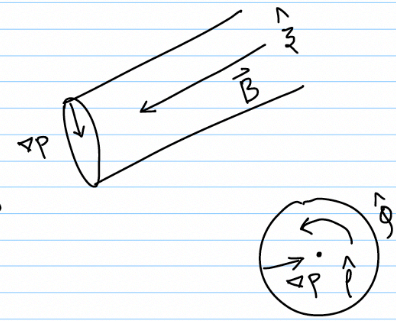
So, we are not writing for a single particle rather we are writing for a collection of particles small n. So, we know what will be the effect of this term we will get this E cross B drift velocity. But how about this additional term this is the generally referred to as the pressure term or pressure force pressure force in a very familiar way we can remember it to be the pressure gradient force. The origin of this force is that whenever you have a density gradient in the fluid it can build up a force making the fluid to move from high density to low density or high pressure to low pressure threat. That means the rate of change of velocity of the fluid will also be affected by the presence of pressure gradient.

Now we never accounted or we never came across this pressure force and we do not know what will be the effect of this force on the particles drift. So, if we now include the effects of pressure term or pressure force and get a derived get a modified drift velocity expression then we can consider that we are consistent our understanding is consistent with the particles description. Because when you consider plasma to be a particle you have accounted for everything. For example, you have accounted for its mass and getting influenced by the gravity we have an expression for the drift velocity under gravitational pull. We have an expression for the particles motion when it is experiencing different sort of electric and magnetic fields.

Fluid drift ($\perp^{r}$ to Magnetic field)

$$v = v_\perp$$

$$\frac{dv_\perp}{dt} = 0$$



$$\rho\left[\cancelto{0}{\frac{\partial v_\perp}{dt}} + \cancelto{0}{(v_\perp \cdot \nabla) v_\perp}\right] = nq\,(E + v_\perp \times B) - \nabla p$$

$$nq\,(\vec{E} + \vec{v}_\perp \times \vec{B}) - \nabla p = 0$$

let us take $\times \vec{B}$ from right

$$nq\left[(\vec{E}\times\vec{B}) + \underbrace{(\vec{v}_\perp \times \vec{B})\times\vec{B}}\right] - \vec{\nabla} p \times \vec{B} = 0$$

$$(\vec{V}_\perp \times \vec{B})\times\vec{B} = \cancelto{0}{(\vec{V}_\perp \cdot \vec{B})}\vec{B} - \vec{v}_\perp B^2$$

But the moment you take it to be a fluid you have to accommodate one additional force which is called as the pressure force. So, in this discussion we are going to see how we can evaluate an expression or a drift velocity for the plasma fluid. We can rewrite this expression this basic expression actually this is basic equation rho times dou u by dou t plus u dot del times u is q n times e plus u cross B minus del P. So, this is one equation that you need this is the momentum equation let us say we call it as equation number 1. Then we will need the continuity equation which is d rho by dt plus del dot rho u is equals to 0 or dou n by dou t plus del dot n u is equals to 0.

Let us say we consider this to be equation number 2. So, we can now consider the plasma as a system of two interpenetrating fluids. What are these two interpenetrating fluids? One is an electron fluid another an ion fluid. So, in this situation we will discuss drifts in two different ways. One perpendicular to the magnetic field another parallel to the magnetic field.

$$qn\left[(\vec{E}\times\vec{B}) - \vec{v}_\perp B^2\right] - \vec{\nabla}p\times\vec{B} = 0$$

$$\vec{E}\times\vec{B} - \vec{v}_\perp B^2 - \frac{\vec{\nabla}p\times\vec{B}}{qn} = 0$$

$$\vec{v}_\perp B^2 = \vec{E}\times\vec{B} - \frac{\vec{\nabla}p\times\vec{B}}{qn}$$

$$\boxed{\vec{v}_\perp = \underbrace{\frac{\vec{E}\times\vec{B}}{B^2}}_{\substack{\vec{v}_E \\ \vec{E}\times\vec{B}}} - \underbrace{\frac{\vec{\nabla}p\times\vec{B}}{qnB^2}}_{\substack{\vec{v}_D \\ \text{Diamagnetic drift velocity}}}}$$

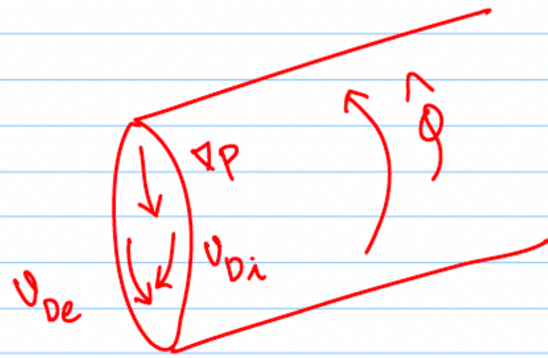
$$v_D = \frac{-\vec{\nabla}p\times\vec{B}}{qnB^2} \qquad q: \pm$$

$$v_{De},\ v_{Di}$$

$$\vec{v}_\perp = \vec{v}_E + \vec{v}_D$$

So, we will start with perpendicular drift where we will consider a geometry a cylindrical geometry in which we have a cylinder for example and this is the axis of symmetry of the cylinder and this direction is chosen to be z cap and the magnetic field is along this direction and you have to account for the pressure gradient. So, it is radially inwards. So, del P that means if you look at the cross section this is the center the pressure gradient is building up in this way. Gradient always points towards the increasing direction of the physical quantity that means as you go radially inwards the pressure is increasing. So, this direction is represented by phi cap and you have the radius which is from the center.

$$\nabla p = -\hat{\rho} \times \hat{z} = +\hat{\varphi}$$



$$\boxed{v_D = \frac{-\nabla p \times B}{q n B^2}}$$

$$p = C\rho^{\gamma}$$

$$p = nkT$$

$$\frac{\nabla p}{p} = \frac{\cancel{kT}\,\nabla n}{n\cancel{kT}} \Rightarrow \frac{\nabla p}{p} = \frac{\nabla n}{n}$$

So, in this picture the magnetic field is pointing along the axis of symmetry which is called as the z axis and the pressure gradient is towards the center of the cylinder and this direction is rho cap. Now, let us try to evaluate the drift velocity perpendicular to the magnetic field. Why is it perpendicular to the magnetic field? The magnetic field is along the axis of symmetry in this direction and the gradient that you see the pressure gradient is perpendicular to the magnetic field that is why we refer to as this is the fluid drift actually perpendicular to magnetic field. So, we will say that the velocity V is V perpendicular. Now, we have to consider a limiting condition for the velocity.

Let us say the velocity V perpendicular is small such that it is in comparison to the cyclotron motion of the particle. This is one V perpendicular we are limiting the value of V perpendicular to be something that is below the cyclotron frequency or cyclotron motion. So, when V perpendicular is very small compared to the cyclotron motion of the particle we can write dV perpendicular by dt to be equal to 0. So, the convective term in the total derivative if you go back and see this term becomes very small in comparison to the other term. So, for numerical calculations we can start writing the equation as rho times dou V perpendicular by dt plus V perpendicular dot del times V perpendicular is n cube times e plus V perpendicular cross B minus del P.

$$V_D = \frac{-kT\,\vec{\nabla} n \times \vec{B}}{q n B^2} = \pm \frac{kT\,\nabla n B}{q n B^2}$$

$$\boxed{V_D = \pm \frac{kT\,\nabla n}{q n B}}$$

$n(r)$ is along $\hat{r}$

$n(r) = n'\hat{r}$

$B = B\hat{z}$

$V = \pm\hat{\theta}$

$$\boxed{V_D = \pm \frac{kT\, n\hat{r} \times B\hat{z}}{q n B^2}}$$

So, this is already 0 because V perpendicular is very small. Since it is already small this second order term may also be neglected. So, effectively what we have is n cube times e plus V perpendicular cross B minus del P is equal to 0. All of these forces effectively contribute to nothing. These forces will sum up and give you 0.

Let us take cross product with B from right then we have n cube times e cross B plus V perpendicular cross B cross B minus del P cross B is equal to 0. So, this is in the form of A cross B cross C which can be expanded by following a simple vector identity is equals to V perpendicular dot B. So, now if you look at the right hand side in the very beginning we made a assumption that V perpendicular is at right angles with the magnetic field. So, this term since it is a dot product and the angle being 90 this term becomes 0. So, what we are left with is q n times e cross B minus V perpendicular B square minus del P cross B is equals to 0.

We can write this as e cross B minus V perpendicular B square minus del P cross B by q n is equals to 0 or V perpendicular B square is equals to e cross B minus del P cross B by q n or V perpendicular is equals to e cross B by V square minus del P cross B by q n B square. What is this expression? This is for V perpendicular. Now, we have to understand the V perpendicular velocity represents the motion perpendicular to the direction of magnetic field, but at the same time V perpendicular has the characteristic feature of telling you the information about the pressure gradient. Because we have taken the pressure gradient to be perpendicular to the direction of magnetic field. Now, looking at this expression what we can understand is the first term which appears on the right hand side is nothing but V e which is the e cross B drift velocity and this other term that appears is called as V d let us say we call it as V d.

We may have referred to V d itself as the e cross B drift velocity, but we are slightly changing the notation here. V e is the e cross B drift velocity, V d from here onwards is going to be called as the diamagnetic drift velocity. V d is called as the diamagnetic drift velocity. Which means that the V perpendicular is equal to V e plus V d. The e cross B term that we are familiar with resembles the single particle motion in a combined electric and magnetic fields.

The second term is actually an additional term which is minus del P cross B by q n B square. Now, this term this diamagnetic drift velocity term we have not seen earlier. This is something completely new for us. But one thing is we started the discussion with this del P. We started with this discussion of del P, but what I have done the mathematics the simple algebra that I have done could have been avoided by simply using the generalized drift velocity expression.

But that would not give you the value of n appearing in the denominator. The variable n appears only you take when you take the fluid equation for n number of particles and that is when you get it. But the message is this diamagnetic drift velocity tells you what will be the additional drift the plasma will experience when you have a pressure gradient inside it. Since we are now dealing with two fluid theory this expression should be like written for an electron as well as ion. That means this Vd is minus del P cross B by q n B square.

Q is the charge. Q can be positive or negative that means you can have an expression for Vde and Vdi. If you talk about the direction let us see. So, del P is minus rho cap cross z cap which is plus phi cap. So, if you take this geometry that we have taken earlier this is your phi cap and this is your del P. Then this should be the Vde for electrons and exactly opposite you should have Vdi the diamagnetic drift velocity for electrons and for ions.

We will rewrite this expression in other ways. So, Vd is written as minus del P cross B by q n B square this is the main expression that we have to remember. So, Vd is minus del P cross B by q n B square. So, we know that the pressure P can be written as C times P rho to the power of gamma. We also know P is equals to n k T.

So, we can write del P over P is k T del n by n k T. What have I done? I have taken a derivative of P and k being a constant T is also assumed to be a constant. Then if there has to be a change in the pressure obviously there should be a change in the number of particles per unit volume or vice versa both of them tell you the same thing. So, you can cancel this and you can simply write del P by P is equals to del n by n. So, we can use this in the diamagnetic drift velocity expression Vd is minus k T del n cross B by q n B square.

So, this is the concentration gradient is perpendicular to the magnetic field. So, we can write del n B by q n B square which is equals to plus minus k T del n by q n B. This is the diamagnetic drift velocity written in terms of the number of particles or concentration gradient. So, we have identified that n of r is along the r cap direction. So, n of r can be written as n cap sorry n bar r cap.

So, and the B the magnetic field B is along z cap. So, the velocity since it is a cross product of these two should be plus minus along phi cap. So, using these directions in the diamagnetic drift velocity we can write Vd is plus minus k T n r cap cross B z cap divided by q n B square. So, this is another way of writing the diamagnetic drift velocity. So, one thing is very clear because it is charge dependent this will be in opposite directions for electrons and ions.

In a cylindrical plasma so, when the electrons and ions move in opposite directions it will lead to the formation of currents and this current may produce a secondary magnetic field which should obviously try to reduce or will be in opposite direction to the original magnetic field. So, at constant temperature if you take the temperature to be constant we can write the ideal gas equation as P is equals to let us say n k T. So, we will continue this discussion in the next lecture where we will conclude the discussion on the diamagnetic drift and also try to understand what is the parallel drift or if the pressure gradient is parallel to the magnetic field. Thank you.