Plasma Physics and Applications

Prof. MV Sunil Krishna

Department of Physics

Indian Institute of Technology Roorkee

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Lecture 38: Plasma as a fluid: Electric and Magnetic Properties - II

Hello dear students. In this lecture we will try to understand more important aspects about the magnetic and electric properties of plasma. So, in the last class we have discussed what is capital B, what is capital H, what is capital M. So, there are other alternate names for B and H. B is also known as the magnetic field or we can also call it as magnetic flux density. Sometimes it is also referred to as magnetic induction.

And capital H on the other hand is referred to as magnetic field intensity or magnetic field strength or magnetizing field or simply magnetic field. And one relation which connects all of this B is equals to mu naught H plus M. Now we have to get a simple relation between B H. We are trying to see what is the magnetic nature of plasma? Is it a paramagnetic, diamagnetic or is it a magnetic material to begin with or not? All of these questions can be answered depending on how plasma will get affected in the presence of an external magnetic field.

So, at this point of time one thing that is very clear to us that capital H is basically due to the free currents and capital M is because of the bound currents. In general in a magnetic material we can write the magnetization to be proportional to the applied field strength. So, if H is the applied field strength that you have control on the magnetization the amount of magnetic moment that will be induced into the substance per unit volume would be proportional to the field strength that you apply from the outside. We can break this proportionality by introducing a constant which is chi M times H. What is chi M? It is a magnetic susceptibility of the material.

B	H
1) Magnetic field	1) Magnetic field Intensity
2) Magnetic flux density	2) Magnetic field strength
3) Magnetic induction	3) Magnetizing field
	4) Magnetic field

$\vec{B}$ $\vec{H}$ $\vec{M}$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

$\vec{H}$

$$\vec{M} \propto \vec{H}$$

$$\vec{M} = \chi_m \vec{H}$$

$$B = \mu H = \frac{M}{\chi_m}$$

Since we know that B is equal to mu H not in free space we can write H to be equal to M by chi M. So, the amount of magnetization is proportional to the magnetic flux density which is being applied from the outside. So, capital M will increase when you apply very strong field and vice versa and this is basically valid for any magnetic material. Now for the case of plasma the idea of magnetic moment is simple because when you have particles they are gyrating you have an area that is enclosed and you have a current the magnetic moment is current by area. We have a measure of this RL which is M V perpendicular by qB and the frequency is qB by M.

So, this frequency has a measure of time and we have charge q these two things will give you what is the current that is produced and this RL will give you a measure of the area of the loop. So, in effect all these things put together will give you the magnetic moment that is generated per single particle. When you consider the collection of several thousands or millions of particles in a plasma the net magnetic moment per unit volume will be the magnetization. Now if you see magnetic moment inside the plasma was written to be equal to M V perpendicular square by 2B. This is a very important formula that we have to remember and we very well know how to get here how to derive this formula.

$$\boxed{\mu = \frac{m v_\perp^2}{2B}}$$

$$\downarrow$$

$$\frac{\mu}{V} = \frac{m v_\perp^2}{2B}$$

$$\boxed{\mu \propto \frac{1}{B}}$$

$$\boxed{M \propto \frac{1}{B}}$$

$$\mu \left\{ \begin{array}{l} A \; i \\ \mu = \frac{i}{A} \\ r_L = \frac{m v_\perp}{qB} \rightarrow A \\ \omega_c = \frac{qB}{m} \\ \downarrow \\ \text{time} \leftarrow q \quad i \end{array} \right.$$

But more importantly one crucial aspect that we have to keep in mind if the magnetic moment is defined like this the magnetic moment per unit volume seems to be M V perpendicular square by 2B whatever. But in a sense we have to grab this proportionality which is the magnetic moment is proportional to 1 by B. If you have already divided with the volume then it is more appropriate to call it as the magnetization is proportional to 1 by B. So, this is something that we are getting from the particle gyration inside the plasma. Is it different or not? So we know that B is equal to mu H and we have realized the magnetization is basically proportional to the applied field strength in the earlier slide this is what we have.

More the field that you apply the magnetization will be proportional to that. If you apply strong field more dipoles will align themselves in the direction of external magnetic field. But what you see here is that the magnetization so you can write B is equal to mu H you can realize that M is proportional to 1 by H. That means that this proportionality is no longer valid or in plasma the magnetization is not proportional to the applied field strength. Magnetization is not directly proportional to the applied field strength.

Plasma is not a magnetic medium

Polarization

$$\vec{P} = \sum_i p_i$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\sigma_b = -\vec{\nabla} \cdot \vec{P}$$

$$\epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \sigma_f + \sigma_b$$

$$\epsilon_0 (\vec{\nabla} \cdot \vec{E}) = \sigma_f - \vec{\nabla} \cdot \vec{P}$$

$$\epsilon_0 \left(\vec{\nabla} \cdot \left(\vec{E} + \frac{\vec{P}}{\epsilon_0} \right) \right) = \sigma_f$$

$$\vec{\nabla} \cdot (\epsilon_0 \vec{E} + \vec{P}) = \sigma_f$$

As a result something very interesting comes out is that the plasma should not be considered as a magnetic material at all. So, you can call the plasma as a diamagnetic material or simply put we can not consider plasma as a magnetic medium at all. Plasma is not a magnetic medium. This is the basic idea. What have we learned? We have established the magnetic nature of plasma.

Now let us talk about the electrical properties or the nature of how the electrical behavior of the plasma will depend on the externally applied electric field. So, just like magnetization you talk about magnetization in when you have externally applied magnetic field you expect all the individual magnetic dipoles to align themselves in the external fields direction. If you have an electric field you expect something similar and you call this process as polarization. So, if there is polarization you define this vector polarization or you are trying to establish the direct nature of plasma. You define this vector P as the sum of individual dipole moments.

$$\vec{\nabla} \cdot \vec{D} = \sigma_f$$

$$\vec{D} = \epsilon \vec{E}$$

$$\boxed{\begin{aligned} &\vec{P} \propto \vec{E} \\ &P = \epsilon_0 \chi_e E \\ &\epsilon = (1+\chi_e)\epsilon_0 \end{aligned}}$$

$$\frac{\partial \sigma_p}{\partial t} + \vec{\nabla} \cdot \vec{J}_p = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(J_f + J_p + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

What is an electric dipole? Plus or minus separated by distance constitute a simple electric dipole. So, when you have externally applied electric field these small electric dipoles will be created and this will lead to a charge density which is called as bond charge density. If you have an atom, if you have electrons revolving around this nucleus and if you have a very strong electric field it may be possible the electron cloud may shift or may be attracted towards a particular direction. And as a result the nucleus being positively charged and the electron cloud being negatively charged can constitute a small electric dipole. Since these electrons are not coming out this dipole can be considered as bound and it will constitute a bond charge density which is sigma b.

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(J_f + \epsilon \frac{\partial E}{\partial t} \right)$$

$$\epsilon = \epsilon_0 + \frac{J_p}{\dot{E}}$$

$$J_p = ne(v_{ip} - v_{ep}) = \frac{\rho}{B^2} \dot{E}$$

$$\epsilon = \epsilon_0 + \frac{\rho}{B^2}$$

$$\epsilon_r = \frac{\epsilon}{\epsilon_0} = \frac{\epsilon_0 + \frac{\rho}{B^2}}{\epsilon_0}$$

$$\epsilon_r = 1 + \frac{\rho}{\epsilon_0 B^2}$$

$$\epsilon_r = 1 + \frac{\mu_0 \rho c^2}{B^2}$$

Charge density I am calling it as charge density which is charge per unit area let us say for now is minus del dot P. So, in vacuum we must include both free and bound charges like in the last lecture we have learned that although plasma is a bound system we will continue working with Maxwell equations for free space or vacuum. And we will include the effect of all these bound charges into the definitions of epsilon and mu that is what the basic understanding is so far. Now if you want that to be done we have to write epsilon naught times del dot E is sigma f plus sigma b. So, if you want to write it in a simple form what we can do is we can write using this epsilon b we can write it as epsilon naught times del dot E is equals to sigma f minus del dot P or epsilon naught times del dot E plus P by epsilon naught is equals to sigma f or we can write it as del dot epsilon naught E plus P is sigma f.

Now we know what this quantity is d is equals to epsilon naught E plus P. So, we can write del dot D is equals to sigma f or D is equals to epsilon E. So, what have we done? We just look back we wrote the Maxwell equation including the bound charge densities and this bound charge density which is expressed in terms of the polarization electric field simply absorbs all of this and gives you a simple equation which is del dot D is equals to sigma f. So, the displacement electric field or the divergence is equal to the free charge densities that means the polarization is proportional to the electric field. Now we can remove this by writing P is equals to epsilon naught chi e E where we can write epsilon as 1 by epsilon E epsilon naught 1 plus epsilon naught epsilon E is equals to epsilon R.

$$\boxed{\epsilon_r = 1 + \frac{\mu_0 \rho c^2}{B^2}}$$

Low frequency Plasma dielectric constant

So, this is what the understanding is so far. Now when you have an alternating electric field it will lead to polarization current density and we can think of a polarizing charge density to be representative or to be responsible for it. So, we can write if you remember the continuity equation sorts dou sigma P by dou t plus del dot J P is equals to 0. So, the polarizing current density will not come into picture until and unless the electric field is changing with respect to time. So, you have to remember the fourth Maxwell equation which connects del cross B is equals to something plus dou E by dou t.

So, the point is the polarizing current density will only be valid when the electric field is changing with respect to time. So, if we now bring that equation del cross B is mu naught J F plus J P epsilon naught dou E by dou t or we can write it as del cross B is mu naught J F plus epsilon dou E by dou t using the fact that epsilon is epsilon naught J P by. So, this polarizing current density is N e V i p minus V e p J is equals to N e V d using this we can write it as rho by B square E dot. So, equating the two expressions we can realize that epsilon is epsilon naught plus rho by B square. If we talk about the permittivity epsilon r is you know that epsilon r is a ratio of permittivity to the permittivity of free space we can write it as epsilon naught plus rho by B square divided by epsilon naught or the relative permittivity is 1 plus rho by epsilon naught B square which is 1 plus mu naught rho C square divided by B square or epsilon r is equals to 1 plus mu naught rho C square by B square.

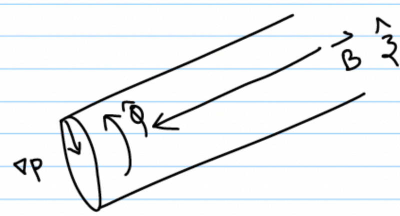
Fluid drifts : Diamagnetic drifts ($\perp^r$ to B)

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = N_i q_i + N_e q_e$$

$$mn \left[\frac{\partial u}{\partial t} + (u \cdot \nabla) u \right] = qn \left(\vec{E} + \vec{u} \times \vec{B} \right) - \nabla p$$

$$nm = \rho$$

$$\frac{d\rho}{dt} + \nabla \cdot (\rho u) = 0 \implies \frac{\partial n}{\partial t} + \nabla \cdot (nu) = 0$$



$\hat{p}$: Pressure gradient

So, this constant or epsilon r is called as the low frequency plasma dielectric constant is called as the low frequency plasma dielectric constant. So, plasma can be treated as a dielectric with this particular constant. So, this is some discussion about the electric and magnetic properties of plasma. We will continue this discussion with understanding what are called as fluid drifts. When we take plasma as a fluid it is important to understand how the electric and magnetic fields will affect the plasma.

So, this topic is called as the fluid drifts. So, we will consider in two different directions we will consider the fluid drifts one parallel to the magnetic field and the second one is perpendicular to the magnetic field. So, these are more or less similar to single particle drifts that we have studied earlier. But let us see what are the aspects can be understood when the same approach is done by considering plasma as a fluid. So, to begin with we will take up the drift perpendicular to the magnetic field which is called as the diamagnetic drift.

Fluid drifts which is sub topic is the diamagnetic drift or which is perpendicular to the perpendicular drift. So, where do we start? We take the essential equations let us say we take the Maxwell equation epsilon naught times del dot e is Ni Qi plus Ne Qe and for unit volume number of particles per unit volume and charge per particle. So, we can start with this equation and the momentum equations. We consider a collisionless plasma and the momentum equation can be written as M N dou u by dou t plus u dot del times u is Q N e plus u cross B minus del p. So, this can be rho N is the number of particles per unit volume and this is mass of particles.

So, this can be density. Now this is let us say this is one equation for electron and we can write another equation for ion. So, in effect we have two equations. We have to consider the plasma as a mixture of two interpenetrating fluids one electron fluid another is the ion fluid. So, in addition we will also need the continuity equation which is d rho by dt plus del dot rho u is equal to 0. This is for the electrons or we can write it in terms of a partial derivative as dou N by dou t plus del dot N u is equal to 0.

So, we are considering the plasma as a mixture of two interpenetrating fluids one electron fluid another ion fluid. So, in this case when we are considering diamagnetic drift or perpendicular drift we take the magnetic field to be pointing along the axis of symmetry. So, let us consider a simple geometry and then try to see how the field is aligned. So, we consider the magnetic field along the axis this is the direction of magnetic field and this is z cap. We take the pressure gradient to be in this direction that pressure is increasing inwards into this tube into this geometry.

The other coordinate is phi cap. So, the magnetic field is pointing along the axis of symmetry. So, you consider the center of this tube is the axis of symmetry along which you can make it into equal parts. So, along the z axis and that axis is the z axis. So, phi is varying like this and r is varying like this. The radius is one coordinate which is from edge to edge along the cut of this pipe z is along the pipe and phi is around the pipe.

So, the pressure gradient is towards the center of this cylinder pipe and the direction is rho cap. Rho cap is inwards rho cap is the direction of the pressure gradient. Now let us try to calculate the fluid drifts which will arise perpendicular to the magnetic field. Now that we have set up the basic geometry to discuss the fluid drifts we will take this discussion ahead in the next lecture. Thank you.