Plasma Physics and Applications

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Lecture 37: Plasma as a fluid: Electric and Magnetic Properties

Hello dear students. Today we will try to understand the electric and magnetic properties of plasma. While we are discussing plasma as a fluid, we will try to see how plasma can be characterized based on its electrical and magnetic properties. So, this topic is electric and magnetic properties of plasma. This is still the discussion pertaining to plasma as a fluid. So, the equation of motion of a plasma particle of charge Q and mass M in a self consistent electric and magnetic fields can be written as M dV by dt is equals to Q times E plus V cross V.

Now when we take plasma as a fluid, we are talking about collection of many particles. Suppose all these particles in the plasma feel the same amount of force they move in the same way so that all of them have an average velocity V which is actually same as the individual particle velocity. So, you are collecting the plasma in which all the particles have the same velocity and the average velocity of the all the particles is also the same. Then we can multiply this equation with suppose the velocity of the particles is V and all of the particles are moving with the same velocity.

Then we can write this equation as number of particles N mass of particle dV by dt is equals to N Q E plus V cross V. So, this is for the entire volume of the fluid. We can replace the derivative with the total derivative understanding that we are trying to account changes with respect to time while following the motion. So, where we have the total derivative d by dt or is written as dou by dou t plus U or V dot del. So, we can write N M dou V by dou t plus V dot del times V is equals to Q N times E plus V cross V.

Electric & Magnetic properties of Plasma

$$m\frac{d\vec{v}}{dt} = q(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{v}$$

$$nm\frac{d\vec{v}}{dt} = nq(\vec{E}+\vec{v}\times\vec{B})$$

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\vec{\nabla}$$

$$\boxed{nm\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = qn\left[\vec{E}+\vec{v}\times\vec{B}\right]}$$

$$nm\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] = qn\left[\vec{E}+\vec{v}\times\vec{B}\right] - \nabla p$$

Pressure force.

So, this is the generalized equation of motion for plasma which is considered as a fluid. Now, this represents only the electromagnetic fields and their effect on the movement of plasma. But in addition to that since we have considered plasma as a fluid there is also a possibility that the fluid can also be experiencing additional forces such as pressure force. We can also include that effects into this equation by writing simply we can write this as N M total derivative of dou V over dou t plus V dot del dot times V is equals to Q N E plus V cross V minus del P. So, what is this? This is called as the pressure force.

So, this is the basic framework. Now, let us try to understand what is the role of electric and magnetic fields on the plasma which is now being considered as a fluid. Generally when you want to understand the electromagnetic effects Maxwell equations are the ones that we think of first. So, Maxwell equations when we consider them in vacuum they appear something like this. They are written something like this.

So, we have del dot E the divergence of electric field is let us say sigma by epsilon naught the charge density divided by epsilon naught and del dot P is 0 and del cross E is minus dou B by dou t or del cross B is mu naught times J plus epsilon naught dou E by dou t. So, these equations are valid in vacuum. How do we characterize vacuum? Vacuum can be the condition in which there are no bound charges. So, effects only due to free charges are considered in this set of equations. What it means is that the effects such as magnetization, polarization all these things are not included in these equations.

Maxwell equations

$$\vec{\nabla}\cdot\vec{E} = \frac{\sigma}{\epsilon_0}$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\left(\vec{J}+\epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

Vacuum

bound charges

only free charges.

In a medium [free charges & bound charges]

σ_b J_b $\vec{D}, \vec{H}$ ϵ, μ

$$\vec{\nabla}\cdot\vec{D} = \sigma$$
$$\vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$
$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}+\mu_0\epsilon_0\dot{\vec{E}}$$
$$\vec{\nabla}\times\vec{H} = \vec{J}+\dot{\vec{D}}$$

$D = \epsilon E$

$H = \frac{B}{\mu}$

What are free charge? Free charges are just free electrons that are free and if you consider a material subjected to an electric field then you have to think of polarization and the displacement electric field which comes into existence all that. If you do not consider all that you just take vacuum where only free charges are present and their effects are felt you consider these four Maxwell equations. But when you consider in a medium you have to account both the free charges as well as bound charges. Bound charges let us say electron is bound to the nucleus or the inside the atom then they can be considered as the bound charges free charges are which are basically free. In a medium when you are talking about a medium will have both free charges and bound charges.

Magnetic nature of Plasma

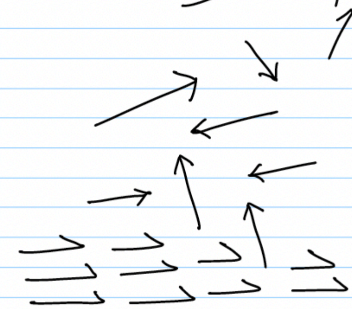
$\odot$ — Diamagnetic

$$M = \frac{\sum_i \mu_i}{V}$$

henry

μ_m

$B = \mu_0 H + M$



$$\vec{J}_b = \vec{\nabla} \times \vec{M}$$

$$\vec{J} = \vec{J}_b + \vec{J}_f$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} = J_b + J_f + \epsilon_0 \dot{\vec{E}}$$

$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} = J_f + (\vec{\nabla} \times \vec{M}) + \epsilon_0 \dot{\vec{E}}$$

$$\vec{\nabla} \times \left[\frac{B}{\mu_0} - M \right] = J_f + \epsilon_0 \dot{E}$$

$$\boxed{\vec{\nabla} \times \vec{H} = J_f + \epsilon_0 \frac{\partial \vec{E}}{\partial t}}$$

So, how do we write the Maxwell equations? In a medium we write them as del dot T is equal to sigma del dot B is 0 del cross E is minus dou B by dou T and del cross B is mu naught J plus mu naught epsilon naught E dot or we can write it as J del cross H is J plus d dot where d is epsilon E and H is B by mu. So, we know all these vectors what are the standard definitions. So, now in these set of equations how are these two different? These two are different because we are no longer using the permeability and permittivity of free space in writing these equations. So, they represent the conditions which will exist inside a medium. So, the bound charge and the current densities that means bound charge density the bound densities that arise from polarization and magnetization are included into these equations in terms of capital D and capital H and epsilon and mu.

Vectors
- B: Magnetic flux density
- H: magnetic field strength
- E: Electric field
- M Magnetization

$\underline{B}$ (Tesla)

$$\vec{H} = \frac{\vec{B}}{\mu_m} = \frac{\vec{B}}{\mu} = \frac{B}{\mu_r \mu_0}$$

$$= \frac{B}{\mu_0 (1+\chi)}$$

$$\vec{B} \quad \vec{H}$$

$$\uparrow \quad \uparrow$$

So, capital D and capital H have the information of these bound charges. Bound charge density is sigma B and bound current densities is J B and the relevant permittivity is epsilon and the permeability is mu. Now in a plasma when you consider the ions and electrons comprising the plasma which will actually constitute the plasma are actually equivalent to the bound charges. So, they cannot move freely. So, they have to the moment has to be consistent with the movement of the bulk plasma.

So, generally the ions and electrons which comprise the plasma are equivalent to bound and current charge densities. Since these ions and electrons move in a very complicated way we cannot consider their effects and include them in epsilon and mu. This is the basic point you are saying they are equivalent to bound charges and currents but then since the motion of these particles is very complex we cannot actually calculate or we cannot measure these effects and include them into the definitions of epsilon and mu. What we do is hence we generally work with Maxwell equations in free space in which we only replace sigma and J and assume that these will include all charges free as well as bound charges and all the currents free as well as bound current densities. So, we do not modify the equations and work with the set of Maxwell equations in a medium rather we use the Maxwell equations in the free space and we change the definition of capital D and sigma.

Now let us first consider the magnetic nature of plasma. The magnetic nature of plasma what do you mean by that? So, the magnetic nature can be broadly speaking it can be a diamagnetic material, it can be a paramagnetic material or it can be a ferromagnetic material. The definition of these is basically dependent on the behaviour of this material when subjected to an external magnetic field. If it gets strongly magnetized you call it as a ferromagnetic material and so on. So, now let us try to understand what kind of magnetic material is plasma.

So, is it a ferromagnetic material or a diamagnetic material or a paramagnetic material? So, the sign and the magnitude of the susceptibility actually gives away the information about the magnetic nature of the any material as such. So, now we know that the plasma when it is subjected to a magnetic field the particles will start gyrating they mean they will move around. So, they will around a fixed point they will circle. So, this circular motion or the helical motion can be along the field lines all that we have had enough discussion on that already. But the point is since when the particle is gyrating it is constituting a small current and current in a loop gives you the idea of magnetic moment.

And when you have so many magnetic moments the net magnetic moment per unit volume gives you a measure of the effect of external magnetic field on the plasma. Now let us say we consider a plasma which is subjected to an external magnetic field we can naturally consider the plasma with a magnetic permeability let us say something like mu m. So, plasma by nature at the end of this we are just going to conclude that plasma is basically a diamagnetic material. We will see how it is diamagnetic material. Now according to the classical theory of magnetism individual dipoles or magnetic moments due to all domains or individual if it is a ferromagnetic material you consider small group of arranged dipoles as a domain and these domains align in the direction of external magnetic field constitute a heavy polarization or magnetization.

So, it is basically called as the ferromagnetic material. But the basic idea is you have all these dipoles which are aligned in random directions in the absence of any external magnetic field but depending on the nature of the material when you put it under the effect all these magnetic dipoles will align in a single direction and thus they constitute a magnetic material. The measure of the amount of magnetization or the measure of the effect of the external magnetic field on the material is M. What is M? M is the magnetization which is basically the net magnetic moment mu i per unit volume.

The units are Henry. So magnetic moment per unit volume total magnetic moment per unit volume is the magnetization. Now when you have a magnetic field it is obvious that we need a current density can be attributed to be the cause of this magnetic field. So in the absence of any free currents in the material like we said plasma can be considered as a bound system bound charges bound current densities. So in the absence of any free currents or free current densities we can assume the total current density to be let us say JB the bound current density is del cross f this is standard. So in vacuum but in vacuum we must include all the effects of free or any external applied currents such as JF.

So JB is the bound current density but we need to also have a provision for any external current density or free current density. So we can write the total current density J as JB plus JF. So we can rewrite this expression the Maxwell equation as 1 by mu 0 del cross B is JB plus JF plus epsilon 0 E dot. So now we have to include the effect of bound current density in B or H. If you have included the effects of both bound and free you call your magnetic field with H.

If you are talking about free space you refer to it as capital B. Now since we know JB is del cross M we can simply write this expression as 1 by mu 0 del cross B is JF plus del cross M plus epsilon 0 E dot or we can rearrange this equation as del cross B by mu 0 minus capital M is equal to JF plus epsilon 0 E dot. We know from earlier discussion this B is equal to mu 0 H plus M. So this can be using this we can write it as del cross H is equal to JF plus epsilon 0 dou E by dou T. So this is a very important expression that we have.

Now this expression has the effects of bound as well as free current densities. Now in this expression we see two vectors. We see capital H and capital E. Capital H is the magnetic field strength, E is the electric field and we also had B before this equation was written. So we have B, we have H and we have E the electric field and we also have M all these vectors all are vectors.

What are these? There are various names to these vector quantities. Capital B is magnetic flux density, capital H is the externally applied magnetic field or we can also call it as the magnetic field strength, capital E is the electric field and capital M is the magnetization. So the magnetic field that is generated by a current carrying wire is generally calculated by using the Ampere's law or the Biot-Savas law and we write it in the units of Tesla. But in vacuum the magnetic field generated by a current carrying wire is written to be B, B is equals to B0 I by 2 pi R or something which is basically we write it in the units of Tesla B, we write it as Tesla. So when the magnetic field passes through a material as such instead not vacuum it passes through a material.

Ambiguities may arise to distinguish which part comes from the vacuum or which part comes from the external currents and which part is from the material itself. So in order to differentiate these things so it is a general practice that we define another magnetic field quantity called as in addition to B we define what is called as capital H which is the what do we call it as? We call it as magnetic field strength which is B by mu M the permeability of the material that we take into account. So generally we write it as B by mu or we can write it as B by mu R times mu 0 which is equals to B by mu 0 into 1 plus chi. What is chi? Chi is the susceptibility of the material. So in addition to this there are various other names by which we refer to the magnetic field B or H.

So as long as it is in vacuum we use B and if it is a material we use H. What are the other names for these two vector quantities and what is the definition or how the understanding changes between these two vectors? We will discuss all that in the next lecture. Thank you very much.