Plasma Physics and Applications

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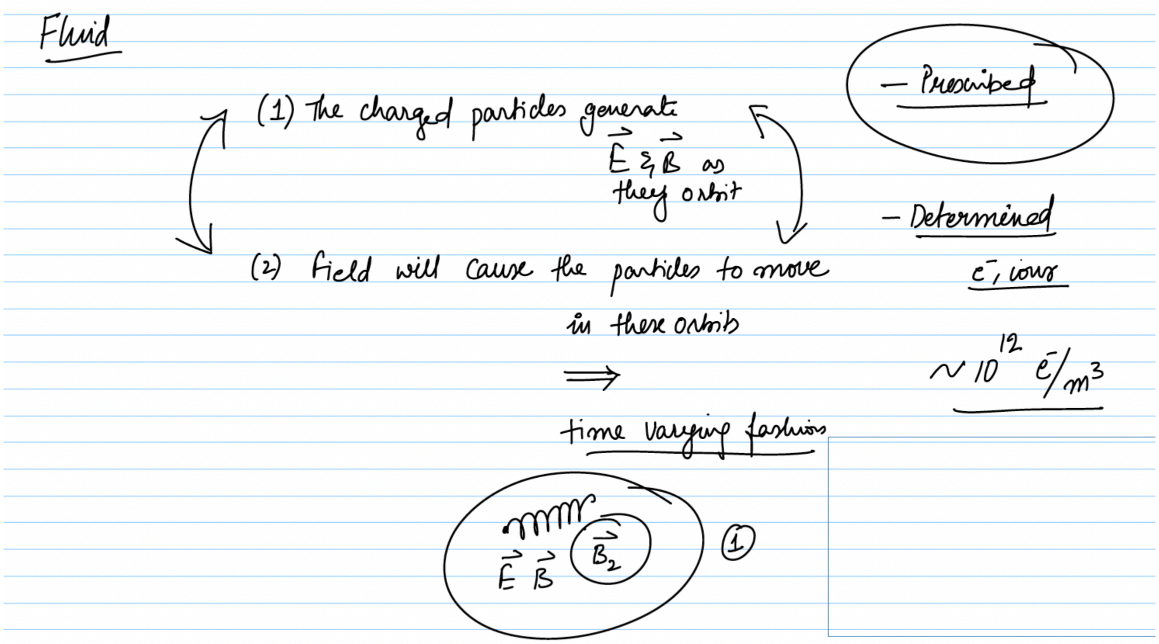
Week – 07

Lecture 31: Plasma as a Fluid: Equation of Continuity

Hello students, in today's lecture we will try to understand plasma as a force. We will try to see how plasma can be treated as a fluid and how this treatment is different from treating the plasma as a collection of particles and trying to understand how these particles will behave in different types of fields, electromagnetic fields. So, far in our discussions it has been very simple as we have considered individual particles and we have imposed or we have considered various electric and magnetic fields let us say time dependent or time independent electric fields and magnetic fields. So, most of this discussion was under the assumption that we prescribe certain type of electric field or magnetic field and then we study what happens to plasma. Now the entire plasma is summed up into a single particle that is what the single particle theory is all about. So, we were only focused about how a single particle let it be an electron or ion will behave in the presence of these fields.

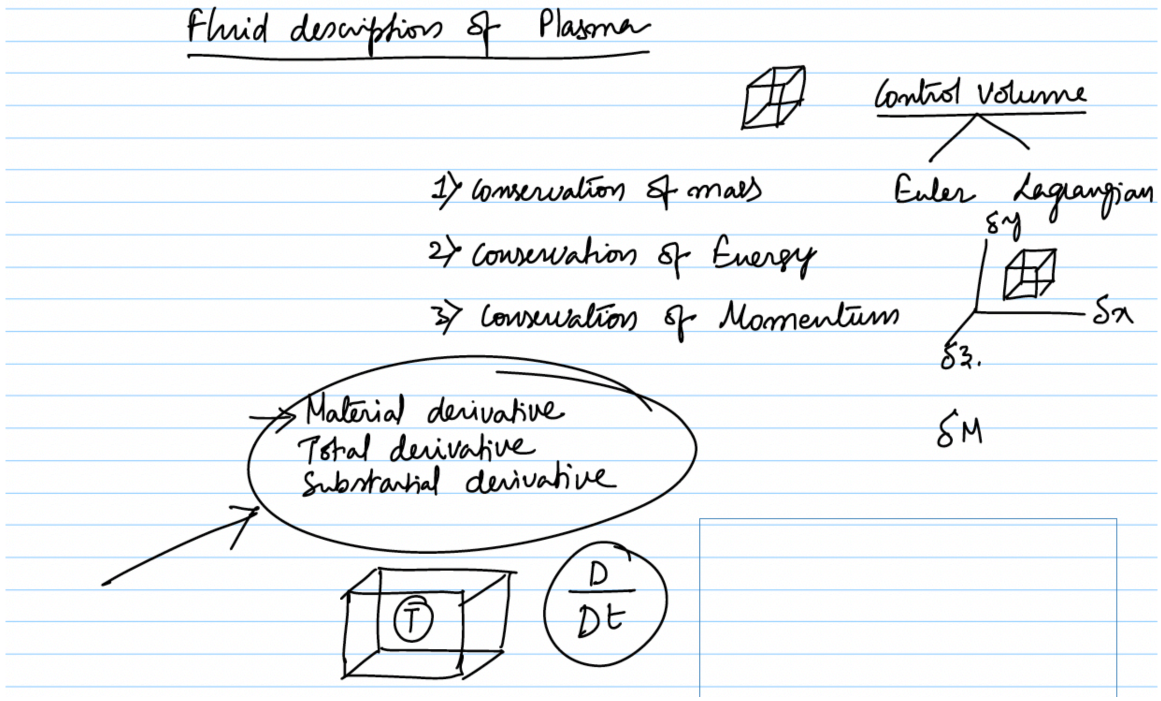
Now most importantly in plasma or in general situations the electromagnetic fields are not prescribed. What do you mean by prescribed? You are imposing you are constructing a magnetic field or electric field and then depending on this construct you are trying to understand how the particle will move. So, this is what is called as the prescribed way of doing things. So, in this class we will try to understand how is this different when you consider plasma as a fluid not as a single particle rather a collection of many particles where individual behaviour of a single particle does not matter only the collective behaviour in the entirety of the plasma only matters.

So, we will try to understand the macro physical aspects of the plasma by considering it as a moving fluid. So, we know very well that plasma given any volume when kept in a volume it will not occupy the entire space like if you put a gas in a chamber it will occupy the space. So, unlike that plasma has a different characteristics. So, when you say it is a fluid so we have to understand the electromagnetic fields in plasma are not prescribed rather they are determined. What do you mean by that? They are determined in the sense plasma is a collection of electrons and ions.



So, when these particles move or gyrate they will generate currents and they will generate electric field or magnetic field. Now these fields that are generated by the movement of plasma will at the same time will also guide the movement of plasma. So, the plasma has to move in such a way that the electric and magnetic fields created should be consistent or should allow the plasma to move as it was moving. So, try to understand this very carefully. So, that means that this problem of understanding plasma being a gas with charged particles needs to be self-consistent in two different ways.

What are the two different ways? The first one is the particles the charged particles the charged particles generate they generate electric field and magnetic field as they orbit. We are trying to build a self-consistent picture for plasma and we are moving away from the prescribed electromagnetic fields and we are trying to understand how the electromagnetic fields are determined in a self-consistent way. So, first one is the charged particles they generate electric and magnetic fields as they orbit. Why do they orbit? As they experience an electric field or magnet they will orbit according to the fields condition. Now, the field will cause the particles the resulting field the electromagnetic field will cause the particles to move in these orbits or we can go ahead and say that the resulting field of this charged particle movement should continue or should allow the particles to move in the same way as they were moving.



So, this behaviour has to be self-consistent that means one has to be we have to have this reinforcement of these two conditions going hand in hand only then we can expect plasma to be a stable entity and most importantly all of this positive feedback or reinforcement should happen in a time varying fashion that is most important. So, with time time varying fashion this is the most important thing or the most difficult thing. Now, let us say if you are trying to understand plasma as a fluid what will you do when you are when you are deviating away from understanding plasma as a particle what will you do? Let us say for example we will start with a single particle. Now, the particle is of course kept in an electric field or in a magnetic field so the particle will gyrate something like this. As it gyrates it will generate a magnetic field.

So, let us say we call this as the secondary magnetic field for example. So, this magnetic field should now allow the particle to move or should allow the particle to continue moving as it was moving earlier. So, this is the complicacy that I am talking about. So, what I can do is if I consider a single particle since I know what will be the electric field that can be generated out of this particles movement given the velocity and the kinetic energy and etc. I can also make a measure of the magnetic field that will generate and with respect to these two I can be able to predict how this particle will move in a time varying fashion.

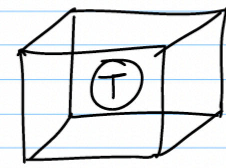
$\delta x, \delta y, \delta z$ — δt

T: Temperature

$x(t), y(t), z(t)$

$T(x, y, z, t)$

$$\delta T = \left(\frac{\partial T}{\partial x}\right)\delta x + \left(\frac{\partial T}{\partial y}\right)\delta y + \left(\frac{\partial T}{\partial z}\right)\delta z + \left(\frac{\partial T}{\partial t}\right)\delta t$$



$$\frac{\delta T}{\delta t} = \left(\frac{\partial T}{\partial x}\right)\left(\frac{\delta x}{\delta t}\right) + \left(\frac{\partial T}{\partial y}\right)\left(\frac{\delta y}{\delta t}\right) + \left(\frac{\partial T}{\partial z}\right)\left(\frac{\delta z}{\delta t}\right) + \left(\frac{\partial T}{\partial t}\right)$$

$$\lim_{\delta t \to 0} \frac{\delta T}{\delta t} = \frac{DT}{Dt}$$

So, what will be the position of the particle or the radius of the particle or any associated physical parameters that will be generated as a function of time for this particle if it is one particle. Now, as it can become complicated we can have many particles. So, approximately we can consider a typical of 10 to the power of 12 particles or electrons per unit volume then you can imagine how complicated this entire situation can become. So, that means that you this is going to be highly computationally intensive and highly demanding but at the end it will almost be an impossible task because each particles current or the magnetic field that is generated will be superimposed on the other particle and the direction of velocity of all these particles put together will almost make it an impossible thing to achieve. So, now that we are trying to deviate away from the particle theory of plasma we are able to realize that if we have to build a self consistent picture for understanding plasma particle theory may not be an appropriate candidate.

So, what do we do? We what we do is we can move away and say that generally a more acceptable theory to understand plasma behaviour could be based on treating the plasma as a fluid. So, what has been found is surprisingly nearly 80% of the plasma phenomena observed in real experiments can be explained by a simple and crude model which is called as the fluid description of plasma. This is the fluid description of plasma. The basic difference that we have to understand is we no longer deal with individual particles individual electrons and ions. What we rather do is we will look at the overall behaviour or macroscopic behaviour of the plasma considering it as a single entity.

So, this is just like fluid mechanics where the identity of individual particle is neglected and only the motion of fluid element is taken into account. So, in plasma the motion of these charges make everything complicated actually that is the basic reason. So, fluid model can be a very good candidate up to 80% of the plasma behaviour can easily be understood. And if you fail that then the kinetic theory of plasma is a more involved or mathematically robust theory to understand plasma behaviour. Even if that fails then we have to go back and get to the task of tediously understanding individual trajectories of the particles which can be like I said very complicated and at the same time computationally almost impossible.

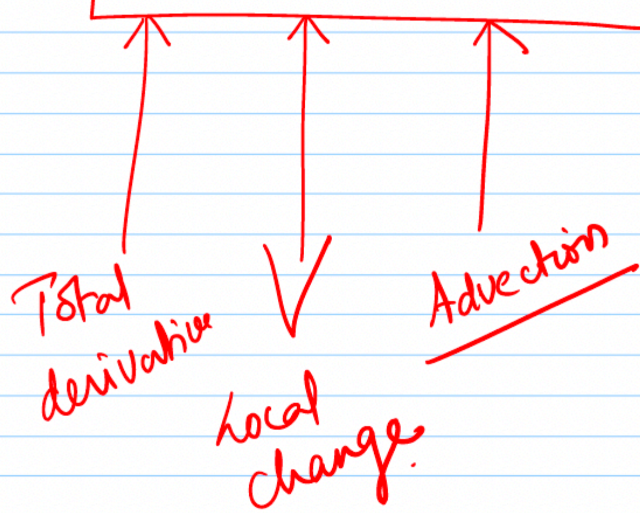
But people have been able to do it not to very large densities of plasma it may be up to 10 to the power of 3 or 10 to the power of 4 up to that people have been able to do it. So, this is the basic understanding of plasma as a fluid or what is the reason that you have to discuss plasma behaviour using the fluid model. So, when you are now you no longer bothered with individual particles nor that you are bothered with the electro dynamical effects but what you are trying to see is the entire plasma as a fluid. So, it is obvious that you have to equip yourself with the fundamental laws of fluid dynamics how things are dealt with in fluid. So, as long as it is a rigid body we know that the body or the object has a mass and you write your equations on that objects mass involving the mass.

So, you will think that all the forces will be acting towards the centre of mass of this object. But now since it is a fluid you cannot pinpoint and say that this is where the fluid is concentrated the fluid is everywhere within that enclosure whatever the fluid is a dispersed entity or a diffused entity. So, in that case what we do is we do not pinpoint and say this is the fluid rather we will say that we will consider a volume of the fluid and say that we will apply our law onto this volume and we will try to understand the behaviour the physical characteristics of this particular volume what happens within the volume. And we can say that the fundamental fluid dynamics is basically nothing but conserving the mass, the conservation of mass, the conservation of energy, third conservation of momentum. So, in fluid we always apply conservation over a very small parcel of fluid and we take the budget of physical parameters or physical quantities over this volume.

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\cdot\frac{Dx}{Dt} + \frac{\partial T}{\partial y}\frac{Dy}{Dt} + \frac{\partial T}{\partial z}\frac{Dz}{Dt}$$

$$\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}u + \frac{\partial T}{\partial y}v + \frac{\partial T}{\partial z}w$$

$$\boxed{\frac{DT}{Dt} = \frac{\partial T}{\partial t} + \vec{U}\cdot\vec{\nabla} T}$$



So, what is this volume called as? So, this is an element of fluid actually this is a parcel or this is a packet of fluid and this element or the volume is called as the control volume. This control volume can be of two different types or two different kinds one a Eulerian control volume another Lagrangian control volume. So, what happens what is the difference between these two control volumes is that in the Eulerian reference or control volume it is actually fixed to the coordinate axis it is basically a parallel pipe with sides delta x delta y and delta z. So, it is fixed with respect to the coordinate system it is not going anywhere. So, this is delta x this is y z.

Mass momentum and energy budgets will depend on the fluxes caused by the flow of the fluid through the boundary of the control volume. Why do I speak of budget? Budget is nothing but conservation how much is coming in and how much is going out. If these two things are same it will be conserved otherwise we have to look for what is causing this difference of incoming versus outgoing as simple as that. Now coming back when we consider a Eulerian control volume the basic idea is that it is fixed with respect to the coordinate system it is not going anywhere. But the budget of physical parameters physical quantities is calculated by understanding how much is getting into this volume and how much is going out of this volume that is the basic idea.

So, in the Lagrangian control volume on the contrary what you do is the idea of the this control volume itself is in infinitesimally small mass of particles let us say delta m you consider this delta m mass to be the control volume and you what you do is you see where this mass is going how this mass is behaving with respect to time and how the velocity of the object of this mass everything related to this. So, the basic difference is the Eulerian control volume is fixed with respect to the coordinate system and you expect the entire fluid to kind of go through it and you calculate the budget and in the Lagrangian control volume you identify a small mass of particles and you name it as delta m and you say that this mass is going from one point to another point. So, this is the basic idea. Now what you are we are trying to understand is we are trying to equip ourselves with the basic laws of fluid dynamics because just like particle theory of plasma in particle theory of plasma we had only one equation which is the Lorentz force equation and this equation was able to tell us what will be the position of the particle as a function of time under the effect of these different electric and magnetic fields. But in fluid dynamics we have to see how this fluid as a whole behaves this electrified fluid behaves as a function of time that is the basic difference.

So, in fluid dynamics what you we do is we use a derivative called as material derivative also called as the total derivative can also be called as the substantial derivative. What is it? So, when you are trying to understand the movement of this control volume or movement of fluid through this control volume both the things Eulerian and Lagrangian. We have to understand if it is the measurement of physical parameters inside this small volume that you are actually trying to accomplish then it can change. Let us say if you are trying to measure the temperature inside this the measurement of temperature inside this can change because of two things. One let us say the fluid is passing through the volume with time the temperature of fluid that is getting in may differ and as a result you may see a variation of temperature inside this volume.

The second type of variation that can be brought in into this is that let us say the control volume itself is moving. Now as it is moving to a colder place what happens it will start telling you a different value of temperature. So, that means in order to account for both these variations variation with respect to the changes in the in the position as well as variations with respect to time. With respect so these two things have to be separated variation with respect to time that means the fluid volume is there itself but with time the fluid that goes through it is changing. So, as a result the physical parameter changes or if the control volume is being displaced from one place to another place.

So, the position the variation is now with respect to the position of this control volume.

So, in order to accomplish a comprehensive picture of the variability of physical quantities inside the control volume we define this material derivative or total derivative. So, this will include the variations with respect to space as well as with respect to time. Let us see how we do it. So, this is basically a Lagrangian control volume d by dt this is called as the total derivative capital D by dt.

So, in order to derive this expression or this relation let us say we define a variable called as temperature. T is temperature inside the control volume. So, when the fluid parcel or the control volume is flowing following the position of the fluid as a function of time. So, we can write the position x is a function of time because it is changing y is a function of time z is also a function of time. Now the temperature that you are trying to measure inside the volume let us say we draw it also for the sake of better understanding obviously this is the temperature inside.

Like I said this temperature can change because of two different things it can change due to the control volume being kept at different physical coordinates and the control volume experiencing different temperatures as a function of time. So, now the temperature can be a function of position x y z as well as time. So, basically we are trying to account the change in temperature as we follow the motion of the fluid along the direction of the fluids movement. So, if the parcel moves an incremental distance from x y z t to let us say by a distance incremental distance of delta x delta y delta z and this incremental displacement in position happens over an incremental time delta t. So, the temperature change that can be observed within this control volume delta T could be the partial change of temperature with respect to x dou by dou T of 2 x into delta x and partial change of temperature with respect to y into delta y plus partial change of temperature with respect to z into delta z plus partial change of temperature with respect to time.

Let us say we divide this whole quantity with respect to time delta T by delta t whatever if the parcel is moving from one place to another place it is happening over a period of time. So, when you talk when you include only time the variations with respect to space are implicitly covered when you say the main variable is time you understand if the parcel is going from let us say like this it is right here sorry if the volume is here as long as the volume is here the fluid is coming and going like this let us say for example. So, at T is equals to 0 and T is equals to 10 what happens different fluid of different temperature comes and goes. So, there will be a change in delta T there will be a change in temperature delta T will be not equal to 0 or you take the same time interval T is equals to 0 to T is equals to 10 and you expect that this parcel by some buoyancy is rising across this landscape and at T is equals to 10 there is no movement in the fluid we can make it simple. We say that the fluid is at rest it is what it is static stratified but the at

T is equals to 10 the parcel is now experiencing a different air or different fluid around it.

So, there can be some delta T which is which will not be equal to 0. So, what this total derivative does is that it accounts for changes which happen with respect to position as well as with respect to time. But ultimately both these things both these scenarios have to happen over a period of time. So, it is obvious that we take a derivative or we divide this with respect to time then we will capture both the different types of variations in a single equation. So, what will happen the equation will simply look like this dou T by dou x delta x by delta T plus dou T by dou y delta y by delta T plus dou T by dou z delta z by delta T plus dou T by dou t or delta T.

So, delta x by delta T the rate at which position changes with respect to time along x direction is nothing but the velocity along x direction which is called as u you can you can call it as u or before that we can get rid of this delta T by imposing that limit delta T tends to 0 we can write delta capital T by delta small t as using the limit we can write d capital T by d small t is dou T by dou t temperature plus dou T by dou x into d x by d t plus dou T by dou y d y by d t plus dou T by dou z d z by d t or d t is plus. So, we have this d x by d t is u v w are the velocity components along the three directions we can simplify it by using the gradient and say that dou T by dou t will remain as it is plus u dot del T. So, this can be written using the gradient like this. What you see here is this is the total change in temperature which happened over a period of time that is equal to the rate of change of temperature which happened only because of time that is one situation which can exist all by itself also it is not necessary that the position or change with respect to position should always be there. But another thing is the advection this term is called as the advection.

So, this refers to change in the properties change in the physical quantities at a fixed location due to replacement of original air or original fluid with a new fluid. So, the advection the fluid volume is not going anywhere rather the fluid volume as it as new fluid or the fluid that is there inside is replaced the change in the physical quantity is given by this advection term. So, this one the first term is called as the total derivative total derivative total change. This one is the local change and the third parameter is called as the advection. So, now we have understood what is the importance or the necessity of the fluid theory of plasma and what are the complexities if you can continue using the particle theory of plasma.

And then in order to proceed further into the understanding of plasma as a fluid we need to equip ourselves with the fundamental laws of fluid dynamics and the total derivative or the understanding of total derivative is very important to understand plasma as a fluid. We will continue this in the next lecture. Thank you very much.