Plasma Physics and Applications

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Lecture 30: Motion in Time Varying Electric field - II

Hello dear students. We will continue our discussion on understanding the particle motion in a time varying electric field. So, far we have derived this expression where the polarization drift Vp is written like 1 by q B square times T by dT of e cross B by B square cross B. So, we again have this triple vector product on the right hand side which can be resolved like this or d by dT of e dot B times B minus B square e. So, the electric field and magnetic field are perpendicular to each other. So, this term right away becomes 0.

So, as a result we have Vp is equals to minus m by q B power 4 times d by dT of B square e or Vp is equals to m by q B square times e dot let us say plus minus. So, this is called as the polarization drift velocity. So, the polarization drift velocity is a consequence of the time variation of the electric field. So, this particular term arises only because of the time dependence of electric field.

So, we can rewrite this expression using the gyration frequency omega c as q B by m into this expression and we can write Vp is equals to plus minus 1 by omega c B times e dot. So, if we take this expression ahead Vp is equals to plus minus 1 by omega c B e dot. What you have to understand is this one this velocity depends on the mass of the particle as well as the charge of the particle or the polarity of the charge which means if you have a collision less plasma and if this collision less plasma is subjected to electric field, the electric field will separate the charges that means it will create some polarization. So, an additional electric field will be set up which is exactly kind of opposite to the original electric field. So, there will be some polarization.

$$\vec{U}_p = \frac{m}{qB^2}\frac{d}{dt}\left[\frac{\vec{E}\times\vec{B}}{B^2}\times\vec{B}\right]$$

$$= \frac{m}{qB^2}\frac{d}{dt}\left[\cancel{(\vec{E}\cdot\vec{B})\vec{B}}^{\,0} - B^2\vec{E}\right]$$

$$\vec{U}_p = -\frac{m}{qB^4}\frac{d}{dt}B^2\vec{E}$$

$$\boxed{\vec{U}_p = -\frac{m}{qB^2}\dot{E}}$$

Polarization drift Velocity

$$W_c = \frac{qB}{m}$$

$$\boxed{\vec{U}_p = \pm\frac{1}{W_c B}\dot{E}}$$

So, this situation is something similar to the well known expression d is equals to epsilon naught e plus p which we must have studied in earlier classes. So, the polarization drift now when the charges are separated it leads to the formation of currents or we can also expect some current density because of the separation of charges. So, if you consider a simple neutral I mean quasi neutral overall neutral and a collision less plasma the polarization drift leads to the formation of the current density. This leads to the formation of current density. So, we know the familiar expression J is equals to N e V d or in this case we will write N e V p.

So, this can be written as N e V pi minus N e V pe. Then we can write N e M i by q B square plus N e M e by q B square times e dot. So, do you see this expression or it is more appropriate to write this as M by q B square e dot or minus M by q B square times e dot. So, we can simplify this expression as N e M i plus M e times e dot by q B square. What is this? This is the polarization current density J p or if we assume the charge to be the same we can write N times M e plus M i times e dot by B square this is equal to the current density.

So, what is N? N is the number of particles charged particles per unit volume. We can define the total number of particles per unit volume times the mass of each particle the density can be written as N times M e plus M i this is not charged density this is just density. So, the current density can be written as J p is equals to rho by B square times e dot. So, using this we can get an expression for the plasma dielectric constant. We know what is a dielectric constant.

$$\vec{U}_p = \pm \frac{1}{\omega_c B} \dot{E} \qquad \underline{m, q}$$

$\Longrightarrow$ Current density.

$$\vec{J} = ne\vec{V}_d = ne\,v_p$$

$$= ne\,v_{p_i} - ne\,v_{p_e}$$

$$= \left(\frac{ne\,m_i}{qB^2} + \frac{ne\,m_e}{qB^2}\right)\dot{\vec{E}}$$

$$\vec{J}_p = ne\,(m_i + m_e)\,\frac{\dot{\vec{E}}}{qB^2}$$

$$\vec{J}_p = n\,(m_e + m_i)\,\frac{\dot{\vec{E}}}{B^2}$$

$$\rho = n(m_e + m_i)$$

So, generally from the Maxwell equations we know that del cross B is mu naught J plus mu naught epsilon naught dou E by dou t or we can write it in terms of H capital H as del cross H is equals to J plus epsilon naught dou E by dou t. So, this equation represents the variation of magnetic field with respect to the electric field or induced by the electric field. Now we are discussing this because dou E by dou t is positive or non-zero in this case which actually gives out or gives rises to the polarization drift. Since we are discussing polarization drift so it is very important to consider the consequences of polarization drift on the using this Maxwell equation. So, if we have to write this expression for the plasma then we have to make suitable modifications.

So, it will be del cross H is equals to it is a J prime plus J p plus epsilon naught dou E by dou t. So, J prime is the current density due to other sources. It can be anything and J p is the current density because of the time varying electric field. So using the expression that we have obtained del cross H can be written as let us write J prime. J prime is current density due to other sources and J p is the polarization polarization current density or J prime plus which is rho by B square dou E by dou t plus epsilon naught dou E by dou t or we can rearrange the terms on the right hand side with del cross H is equals to J prime plus 1 plus rho by B square epsilon naught times dou E by dou t.

Let us just for discussion del cross H is equals to J prime current density due to other sources plus epsilon naught times 1 plus rho by epsilon naught B square times E dot. Now this one if it is in a medium or if it is not in vacuum it is ideal that we use the permittivity of epsilon naught is the permittivity of free space. If it is not in vacuum we can use epsilon which represents the permittivity of the plasma or the medium. That means that this product epsilon naught times 1 plus rho by epsilon naught B square should be equal to epsilon. What does it mean? It means epsilon naught times all of this must be equal to epsilon r.

$$\boxed{\vec{J}_p = \frac{\rho}{B^2}\dot{\vec{E}}}$$

Plasma Dielectric Constant

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = J + \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = J' + J_p + \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}' + \frac{\rho}{B^2}\frac{\partial\vec{E}}{\partial t} + \epsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \vec{J}' + \epsilon_0\left[1 + \frac{\rho}{B^2\epsilon_0}\right]\frac{\partial\vec{E}}{\partial t}$$

J' : current density due to other sources

$\vec{J}_p$ = Polarization current density.

We define the relative permittivity as the ratio of epsilon by epsilon naught with respect to the permittivity of free space. If that has to be followed then we will write the relative permittivity as 1 plus rho by epsilon naught B square. Plasma with a time varying electric field and in the presence of a normal magnetic field behaves like a dielectric with effective permittivity epsilon is equal to epsilon naught into 1 plus rho by epsilon naught B square. What is epsilon? Epsilon is effective permittivity. Epsilon naught is permittivity of free space.

Epsilon r is relative permittivity. If you have studied any fundamental courses on electrodynamics, all of these terms will be very much familiar to you. Let us grab the context of this formula or this mathematics. The context is simple. When you have a time varying electric field, the drift velocity Vp appears to be a function of charge, the polarity of the charge of the particle and the mass.

That means that when a collisionless plasma is subjected to this type of field, it will separate the charges. All the particles with positive charge will be given a polarization velocity in one direction and all the negative particles will be exactly opposite. That means there is a charge separation. This charge separation leads to current or current density. We have got the expression for the current density as this JP is equal to rho by B square times e dot.

$$\vec{\nabla} \times \vec{H} = \vec{J}' + \underbrace{\epsilon_0 \left[1 + \frac{\rho}{\epsilon_0 B^2} \right]}_{\epsilon} \dot{\vec{E}}$$

$$\underbrace{\epsilon_0 \left[1 + \frac{\rho}{\epsilon_0 B^2} \right]} = \epsilon$$

$$\epsilon_0 \epsilon_r = \epsilon$$

$$\epsilon_r = \left[1 + \frac{\rho}{\epsilon_0 B^2} \right]$$

$$\boxed{\epsilon = \epsilon_0 \left[1 + \frac{\rho}{\epsilon_0 B^2} \right]}$$

ϵ: effective Permittivity
ϵ_0: Permittivity of free space
ϵ_r: Relative permittivity.

Now, since the time varying electric field has many other consequences, it is important to bring the Maxwell equation into consideration. Here by substituting what we have from the earlier treatment, we realized that this plasma in the presence of a time varying electric field and a magnetic field behaves like a dielectric with effective permittivity given by this expression. Let us mark this expression because this is a very important formula that you should remember. Epsilon is equal to epsilon 0 times 1 plus rho by epsilon 0 B square. Now, which parameter in this describes plasma? It is the rho which describes the plasma.

Where did we define rho? This is the rho. This is the parameter or variable which carries information about the plasma that we are trying to understand. In essence, we can simply say that plasma in the presence of a time varying electric field behaves like a dielectric with permittivity as given by this. We can see this as simply as the factor by which the electric field between the charges is decreased in comparison to free space or vacuum. For plasma, we have a typical plasma with number densities as 10 to the power of 14 Gauss per centimeter cube and a magnetic field of 10 to the power of minus 4 Gauss.

Then the value of epsilon will be approximately of the order of 10 to the power of 4 which means the dielectric constant of plasma can be very high. So, this is very high dielectric constant. In essence, we can simply remember that the plasma can be treated as a dielectric material and given that it is a dielectric, it is very important we study the propagation of waves in this dielectric medium. Waves we will take up after like some lectures, wave propagation in plasma or different types of waves. How do they travel through plasma or what kind of modes are permitted in the plasma? So with this, we have covered all different possible situations of particle experiencing electric and magnetic fields.

$$\vec{V}_E = \frac{\vec{E}\times\vec{B}}{B^2} \qquad \vec{E}\times\vec{B} \text{ drift}$$

$$\frac{B_0}{B_1} = \sin^2\theta \quad \text{Mirror Condition}$$

$$\vec{V}_F = \frac{1}{q}\frac{\vec{F}\times\vec{B}}{B^2} \qquad \text{Generalized drift due to } \vec{F}$$

$$\vec{V}_{\nabla B} = \frac{m v_\perp^2}{q}\frac{\vec{B}\times\nabla\vec{B}}{B^2} \qquad \text{Gradient drift}$$

Magnetic moment $\quad \mu = \frac{m v_\perp^2}{2B} = \frac{W_\perp}{B}$

$$\vec{V}_g = \frac{m}{q}\frac{\vec{g}\times\vec{B}}{B^2} \qquad \text{Gravity drift}$$

Mirror force $\quad F_\parallel = -\mu\nabla\vec{B}$

$$\vec{V}_R = \frac{m v_\parallel^2}{q}\frac{\vec{R}_c\times\vec{B}}{B^2} \qquad \text{Curvature drift}$$

$$\vec{V}_R + \vec{V}_{\nabla B} = \frac{1}{q}\left(m v_\parallel^2 + \frac{1}{2} m v_\perp^2\right)\frac{\vec{R}_c\times\vec{B}}{B^2 R_c^2} \qquad \text{Vacuum drift}$$

$$\vec{V}_p = \frac{m}{qB^2}\dot{E} \qquad \text{Polarization drift}$$

We can say that we can sum up all the discussion that we had on the particle drifts. We will write all those expressions just so that we can remember them or we can appreciate the difference of these things. So, we will start with the E cross B drift Ve is E cross B by B square. What is this? The name for this is E cross B drift. Now, this is a constant you see E cross B both of them are not varying with respect to time.

They are also homogeneous. So, this is a constant term that means that appears on the right hand side. A more general case let us say we call it as Vf which is generalized drift which we write as 1 by Q times F cross B by B square. The name for this is generalized expression for drift or generalized drift due to a non zero force F that is it. Then we define what is V del B which is mV perpendicular square divided by Q times B cross del B by B square. This is a more important drift which is called as the gradient drift.

Drift due to gravity is m by Q G cross B by B square. This is where you see gravity and the magnetic field of earth together. This is called as the gravity drift. V R the curvature drift mV parallel square divided by Q times R C cross B by B square.

This is the curvature drift. So, why these things appear? All of this is already discussed in earlier videos. You can go and watch them. But this is the flow that we have maintained so far. We discussed how the E cross B drift will develop and how we can generalize the expression for generalized force. Then we put some inhomogeneity in space onto the magnetic field and then we put some curvature which comes together actually into the magnetic field.

And then we learned a mechanism by which we can combine these two expressions of gradient and curvature drifts and we write in the simple expression which is called as the vacuum drifts V R plus V del B is 1 by Q mV parallel square plus half mV perpendicular square times R C cross B by B square R C square. What is this? This is the vacuum drift. Then we have the time varying things V P is the polarization drift which is m by Q B square times E dot this is called as the polarization drift. And in addition to all these drifts we also seen something called as magnetic mirroring which is represented by this mirror condition which is B naught by B 1. B naught is the weak field and B 1 is the strongest point into the gradient is sin square theta.

This condition is known as the mirror condition. This tells you when the mirroring will happen and we also learnt that the magnetic moment of particle is mV perpendicular square by 2B or which can also be written as the kinetic energy divided by the magnetic field. And this is the magnetic moment. And then we learnt a very important mirror force. So, the mirror force is a very unique thing because this is responsible for bringing back the particle into the weak field F parallel is minus mu times del B.

You see del B is there so F parallel has to be existent. So, this is the conclusion of the particle drifts. We have all these drifts just so that I can summarize all these things I have written these things. We have the E cross B drift, generalized drift, the gradient drift because of the increasing magnetic field as you proceed towards a magnetic field. Then you have gravity drift curvature is when you have because the magnetic field lines have to be converging from somewhere and they have to diverge from somewhere.

So, this convergence and divergence should be accommodated only at the expense of some curvature in the magnetic field lines. So, that is when you have a gradient drift you always have a curvature drift. So, we learnt how to combine these two things and this vacuum drifts. And recently we have covered what is the polarization drift and some other conditions which are relevant for magnetic mirror. So, with this we will stop this discussion about particle theory of plasma.

In the subsequent lectures we will try to understand how plasma can be treated as a fluid instead of a particle and what is the benefit of treating plasma as a fluid in comparison to particle theory. Most importantly try to understand the complexity of these things because so far we have been able to discuss very few tailor made fields electric fields and magnetic fields. And in reality the situation could be very complex and it can be drastically different from this simple magnetic and electric fields that we have taken. It can just be a combination of many things. But this is where we have to start and we have to understand how each and every different type of field will affect the particles trajectory. Thank you.