Plasma Physics and Applications

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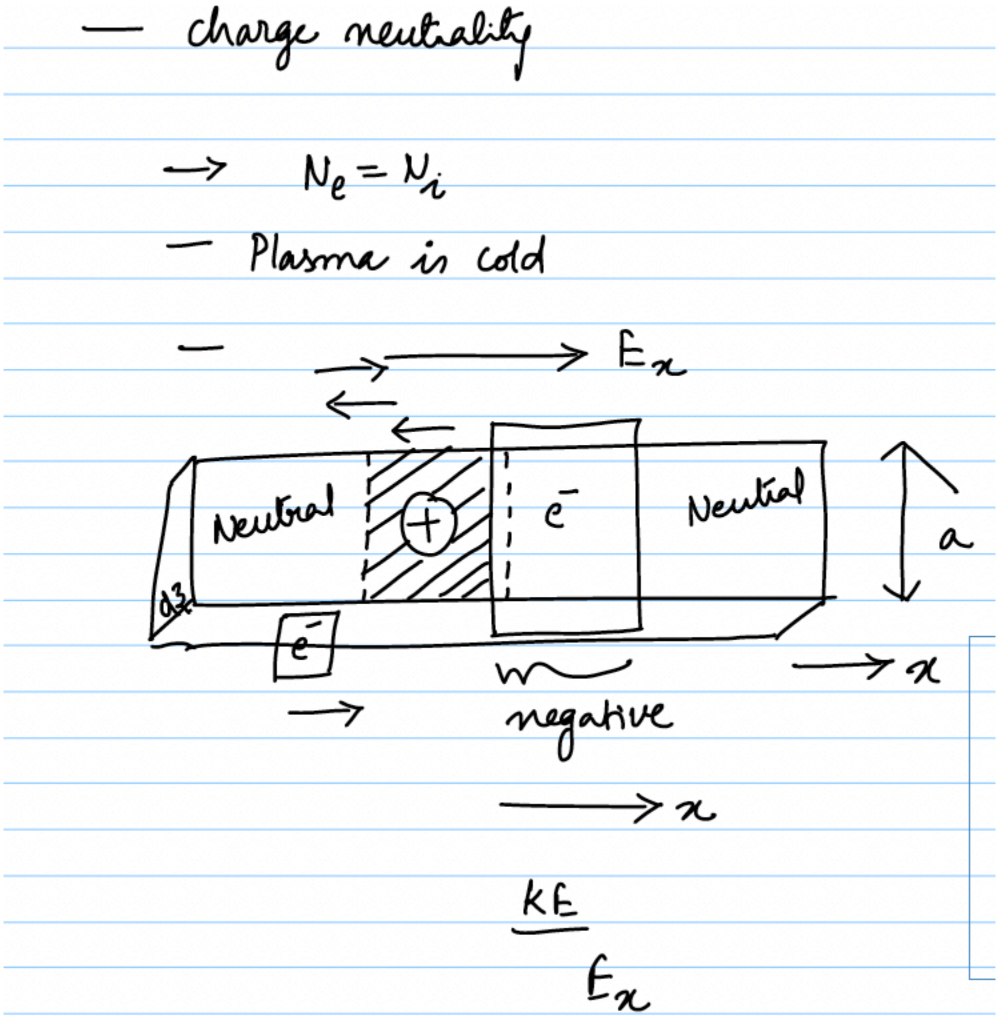
Week : 01

Lecture 03: Plasma Oscillations

Hello dear students. In today's lecture, we will look at a very fundamental concept in plasma physics which is plasma oscillations. So far in our discussions, we have understood what is plasma, how plasma can be produced and what are the defining characteristics of plasma. So, one of the most fundamental characteristic of plasma is that it tries to establish charge neutrality. What is charge neutrality? Although charged particles are present in the plasma, they are generally assumed to be of equal in number and overall plasma is electrically neutral. So, this we call as the quasi neutrality of plasma.

So, the fundamental property of the plasma is its ability to remain electrically neutral and plasma oscillations are the means by which plasma attains or plasma establishes charge neutrality. So, what are plasma oscillations? Let us say we take a plasma in which there are ideally equal number of electrons and ions wherein number of electrons is equal to number of ions. Ions represent the total population of negative charges and ions represent the positive charges. So, we are just considering only the positive ions.

So, any changes in the local charge separation, these ions and electrons are distributed. The total charge let us say electron density times the total number of electrons, the times the total number of ions will be equal. This is what we call as the charge neutrality or quasi neutrality. So, any change in the local neutrality resulting from the charge separation, if you want to create separation of charges, if you want to take all the electrons and separate them from that particular space results in the creation of electric fields and these electric fields will try to pull back the electrons to their original position. And when these electrons are coming back, when the electric field is acting on them and trying to bring back those electrons to their original position, then the oscillations will result because of the inertia of the electrons, they will not be able to come immediately and occupy their initial positions, they will always overshoot the initial positions and as a result some oscillations will develop.



And in the absence of any damping, these oscillations can continue for a very long time and these oscillations are called as the plasma oscillations. Now, for example, so let us say we take a portion of this filled with electron and we try to move it away, then because of the lack of electrons here, an electric field is created and this electric field's role is to drive the electrons back into their original position. So, we can briefly discuss the plasma oscillations in today's lecture. More detailed discussion on plasma oscillations will be covered when we are discussing waves in plasma in subsequent units. So, consider a steady state plasma, what is steady state? When things are not changing with respect to time, we call the total state as the steady state.

So, in which the total number of electrons is equal to the number of ions. So, there are neutralized by an equal number of ions, electrons equal to the number of ions. We assume that the plasma is very cold, this is number assumption 1 and plasma is cold. So, we are going to use this statement quite often, plasma is cold, cold plasma. What does it mean? It means that there are no thermal motions of the atoms or ions.

— Damping. $\Longrightarrow$ **Plasma Oscillations**

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

E_x

$$m\frac{d^2x}{dt^2} = qE_x$$

$$\oint_S \vec{E}\cdot\vec{ds} = \frac{Q}{\epsilon_0}$$

N_e

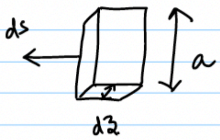
$$Q = A\,x\,N_e q_e$$

N_e → no of carriers per unit volume

q_e → charge per e^-

x → distance

A → Area cross section



$$\oint \vec{E}\cdot\vec{ds} = \int E\,ds\cos\theta$$

So, electrons or ions are not moving from one place to another place as a result of the temperature that is available ambient temperature. So, there are no collisions obviously. So, in a situation when the thermal motions are not present, we can call this plasma as a cold plasma or we say that the thermal motions are negligible. Let us say now we perturb the system by picking up some electrons and moving them away from their initial position to some other position. So, the displacement of electrons away from a region produces an electric field and a positively charged region will form where electrons are now being displaced away from.

So, it will look something like this. So, let us say we have we consider a strip of plasma. So, before this perturbation everything is neutral, everywhere it is neutral, everywhere it is assumed to be equal number of electrons and ions and effectively there is no charge. So, now what we do is from this region we take out some electrons and move it to this region. So, here it is neutral and here again it is neutral.

$$E_x\, a \Delta z \cos(180) = E_x\, a \Delta z.$$

$$E_x\, a \Delta z = \frac{Q}{\epsilon_0} = \frac{a x \Delta z N_e q_e}{\epsilon_0}$$

$$E_x = \frac{-x N_e q_e}{\epsilon_0}$$

$$m\frac{d^2x}{dt^2} = -q_e\left(\frac{x N_e q_e}{\epsilon_0}\right)$$

$$m\frac{d^2x}{dt^2} + \frac{q_e^2 N_e x}{\epsilon_0} = 0$$

Now we have taken some electrons and perturbed the system by moving away these electrons from one place to another place. So, this region is of course negative overall and this region is clearly positive. Let us denote the direction as this is x direction and let us say we the thickness of this strip. So, we call the thickness of this strip as a and the directions are denoted by this x y. Now when electrons are displaced it leaves behind a net positive charge where ions are majority charge carriers.

So, obviously an electric field would be set up immediately and the direction of electric field would be this e x because of the displacement of electrons from this original position to this position. In this region the electrons and the majority charge carriers the total charge is negative and the region that was left behind is totally positive and as a result we have an electric field in the x direction. Now immediately when there is an electric field the obvious consequence is that you will expect the electrons to move in a direction opposite to the electric field and compensate this extra positive region thereby trying to rebuild the charge neutrality. This charge separation results in the formation of electric field which tries to move the electrons and ions exert force on electrons to go back. Now the electrons are much lighter in comparison to the ions.

So, the electric field may not be able to influence the ions to go, but since electrons are very light they will be getting some velocity out of the electric field and they will try to go back. The electric field e acts to reduce the charge separation and electrons are accelerated towards their initial position. Now as the electrons are accelerated by the means of electric field they will acquire some kinetic energy. So, they will acquire some kinetic energy which is basically driven by this electric field e x and their inertia because of the mass makes them always overshoot this position where they are supposed to come back. So, that means that they will actually try to come and sit here, but as a result of their initial velocity and the inertia they will overshoot.

$$\frac{d^2x}{dt^2} + \left(\frac{N_e q_e^2}{m_e \epsilon_0}\right) x = 0$$

$$\omega^2$$

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

$$\omega_p^2 = \sqrt{\frac{N_e q_e^2}{m_e \epsilon_0}}$$

$$\omega_p = \sqrt{\frac{N_e q_e^2}{m_e \epsilon_0}}$$

rad/s

Now immediately when the electrons of all the electrons are going towards this direction immediately again here an electron abundant region is created and electric field will exactly be opposite. Now the electrons are again pushed back in this direction. Now in the absence of electrons are moving to the left and immediately an electric field which is opposite to the initial electric field will be set up and electron will be forced to move exactly in the opposite direction because the electric field is strong and it can influence a very light particle such as an electron. So, this process will go on like this. So, until unless they come and sit in the original position this process keeps going on.

Now if the electrons have to come then there should not be any damping. So, if there is damping what will happen? There could be collisions with electrons and ions with each other and the plasma media, but in a very weak plasma you will not expect this collisions to happen. So, if there is damping the electrons may not always go as a result they may come and because of the collisions they can settle down in a place and you can expect something like that, but in a medium which is very very weak that means the number of charged particles or particles per unit volume is very very less then the chances of collision will become even more critical and as a result the electrons will always be going back and forth around this position resulting in what are called as the oscillations. Of course they are going back and forth this is a periodic motion. So, these motions or these oscillations are called as the plasma oscillations.

Now we have to get the characteristic frequency of these oscillations that is the basic task. So, the oscillations are referred to as plasma oscillations. Plasma oscillations are result of plasma trying to maintain the charge neutrality that is the basic purpose of this discussion. What are plasma oscillations? Number one, plasma has the ability to maintain overall charge neutrality despite any perturbation in the charge concentration locally. In the process the electrons will oscillate around their main position and these oscillations are called as plasma oscillations.

Effectively plasma oscillations are the means by which plasma retains or maintains charge neutrality. Now let us obtain an expression for the frequency of these oscillations for the frequency of plasma oscillations. Now the moment you talk about oscillations you should know the restoring force that is acting. Restoring force is something that tries to restore things or tries to bring back or tries to nullify the effect of perturbation. Now the restoring force depends on the number of electrons that are being displaced because the number of electrons will only decide the strength of the electric field that will be created.

It will obviously drive back the electrons and so on. The inertia of the electron depends on its mass. The oscillatory frequency depends on the mass of electron. We will come to that. Let us say Ex is the electric field that is generated then we can write in equation of motion m d square x by dt square is E Ex.

The force ma is the force. E is the charge of electron Ex is the electric field that is set up. Let us say we write it conveniently as instead of E Ex we write it as q times Ex. q is the charge of electron. Let us say we use the Gauss law and apply it for the closed rectangular shape that we have drawn in the last slide.

What is Gauss law? Gauss law tells you that del dot E is equal to rho by epsilon naught. We write it in the integral form integral over the closed surface E dot dS is the total charge enclosed divided by epsilon naught. What is q? Is the total charge contained in the closed surface and if the equilibrium density is maintained the total number of charge carriers is n e. Then the total charge q can be written as A x n e q e. What is A? A is the area, x is distance, n e is number of carriers per unit volume and q is charge per electron or carrier.

Number of charge carriers multiplied by the charge is equal to the total charge. But since you have defined n e as the total number of electrons per unit volume we have to multiply with volume which is area times distance. This A x will cancel out the volume that appears in n e and you get the total charge as n e multiplied by q e. What is area? We say that this is cross sectional area in the x direction. So, we can write integral the closed integral E dot dS.

What is dS? dS is the surface. So, how do we define it? We take this volume now this is A and this will be dS. E dot dS is E dS cos theta. So, the direction of the area element is defined perpendicular to the area at the point on the surface. This is simple we know all these things.

Now using all this we can write. So, this one this can be this is of course A and how do we draw it? This one is dZ. So, E x dS cos theta. Let us say we write E x A delta Z cos of 180. See this is your slab and this is your coordinate system x y and z.

You call this as a measure. So, dX dY is going to be giving you J when you multiply with delta Z you have this which will be E x A times delta Z. So, since we know from the Gauss law E x A delta Z is equal to q by epsilon naught. We can write q as area times x times delta Z small a is the length along the y direction. x is the length along the x direction delta Z is the depth of this slab A delta Z n e q e divided by epsilon naught.

So, do not be confused. So, this product is written as capital E in the earlier slide. We are still in the same picture. So, now we can write E x the electric field magnitude is minus x times n e q e divided by epsilon naught. Now, we can use this in the equation the force equation which is m d square x by dt square is equals to minus q e times x n e q e by epsilon naught. I will just go back and tell you what I have done.

The charge is A x n e q u. So, if we go given a slide before that. This is A is along the y direction as per this along the x the distance is measured in x and since you are not drawn this the other dimension here. This becomes this one is dZ or delta Z. So, that is what I have used here. The total charge is area times x times n e q e.

So, area has A times delta Z and the other dimension is x. Since I have used x here so, area is now small a times delta Z. Now what we will do is we will write this equation as m d square x by dt square plus q e q e square n e x divided by epsilon naught is equals to 0 or d square x by dt square plus n e q e square by m epsilon naught times x is equals to 0. So, since we already made an assumption that the electric field will only be able to drive these light particles the electrons. The mass that is under consideration here is obviously, the mass of electron.

So, if you look at this equation this equation looks very familiar representing a harmonic motion with a characteristic angular frequency omega square. Where is this omega? So, always remember this expression d square x by dt square plus omega square x is equals to 0. This equation represents the simple harmonic motion with a characteristic frequency omega or what I am trying to say is whatever that appears to be multiplying x is in the dimensions of omega square. So, we can write omega square we will put a suffix p to indicate plasma omega p square is n e q e square divided by m e epsilon naught. So, this is the characteristic frequency of the oscillations.

What is the purpose of these oscillations? These oscillations are a means to establish quasi neutrality of plasma. So, this frequency is also called as plasma frequency. We will derive the plasma frequency using a totally different approach maybe in the subsequent classes, but omega p is square root of n e q e square by m e epsilon naught. The units are radians per second and this frequency is called as the plasma. The characteristic frequency of plasma oscillations is called as the plasma frequency.

What have we neglected in this simple picture? We have neglected that there are no thermal motions of course, that is we have said that plasma is cold. There are no thermal motions of course, and then we have also made one more very important assumption that there is no damping. If there is damping these oscillations will not sustain, these oscillations will not continue beyond a small amount of time because the damping will take care of these moving electrons and it will settle it. The damping or the collisions with other electrons and we also made one more assumption saying that the plasma is very weak in the sense the number of particles is very less that implies that there can be very little number of collisions. At this point of time itself we can probably define the first plasma criteria.

If there are collisions if there is this frequency omega p which is indicating the characteristic oscillation frequency of these electrons. That means electrons are oscillating with this frequency. Now if we define a time period tau p or tau n whatever it is tau n let us say we call it as tau n not directly related to the plasma frequency listen to this carefully omega p is the characteristic frequency which is telling you that oscillations are there and these oscillations are happening at this speed per second these many oscillations are there. And we define what is called as tau n which is the mean free time between two collisions. So now you are removing the collisions just so that the oscillations will persist or oscillations will exist to begin with but if there are collisions of course.

Then these collisions will obviously stop this oscillations. So, if there are collisions of electron and the mean free time between two successive collisions is tau n then just think about these two scales. So within tau n time the collisions will happen so that gives you some idea of the time that it requires for the electrons to encounter another collision and then you have another characteristic frequency or time which is telling you how fast the oscillations are happening or if you write it in terms of time then it will tell you within this particular time how many oscillations how many cycles can happen. Now for the plasma to sustain or for plasma to exist you would obviously expect that the characteristic frequency or the time that is represented by omega should be much larger than the collision time. You would expect that let us say before the collision happens the plasma oscillations should be substantially large.

So if you relate these two things we can simply write omega p should be much greater than 1 by tau n. So collisions can happen of course but collisions will happen after the plasma oscillations have taken place. So this is because both of these so frequency is 1 by time. So if I am comparing these two time scales I have to do like this. So otherwise I have to convert this frequency into time and then compare these two times time scales but I have done it by inverting the time.

So both of them are in the same dimensions. Now you would expect that the plasma frequency is so high in comparison to the collision in free time or we can write omega p tau n much greater than 1. So this is the first plasma criteria. In order to call an ionized gas as plasma it should fulfill this criteria. So this is about plasma oscillations and characteristic frequency which is called as a plasma frequency the derivation and one conclusion of that is that it gives you the first plasma criteria. So invariably this should be followed whenever you are trying to distinguish an ionized gas and a plasma.

So in the next class we will try to understand what is called as the Debye shielding and how Debye shielding can also suggest plasma criteria. Thank you.