Plasma Physics and Applications

Prof. MV Sunil Krishna

Department of Physics

Indian Institute of Technology Roorkee

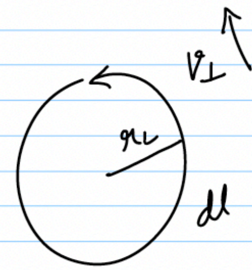
Week – 06

Lecture 28: Motion in Time Varying Magnetic Field - II

Hello dear students. We are discussing the particles motion or particle trajectory in time varying fields. So, we have considered time varying magnetic field to begin with and we have realized if the magnetic field is varying with respect to time, then the rate of change of velocity or the rate of change of energy delta of half m V perpendicular square can be written as minus Q integral over a surface dou B by dou T T s. This expression gives you the rate of change of energy or change in energy as a function of variation of magnetic field with respect to time. So, here s is the surface area enclosed by one orbit. So, the understanding is the particle is gyrating and we are trying to account changes that will happen within one gyro orbit because we have to fix the limit of time over which we will integrate and this interval of time is chosen such that it represents the duration of time the particle will take to cover or to make one circular orbit.

So, let us say you have this particle which is moving like this and this is the R L the radius of gyration R L and V perpendicular is there. V perpendicular is responsible for V perpendicular square is equal to V x square plus V y square. So, V perpendicular is responsible for the circular motion, V parallel is along the direction of magnetic field. Now the direction of the particle can be given by the Red Hand rule by considering or by pointing the fingers in the direction of velocity and this is d L along this d L is there.

$$\boxed{\delta\left(\frac{1}{2}mv_\perp^2\right) = -q\int_S \frac{\partial B}{\partial t}\, ds}$$



$$\delta\left(\frac{1}{2}mv_\perp^2\right) = -q\dot{B}\cdot\pi r_L^2$$
$$= |q|\,\dot{B}\cdot\pi r_L^2$$

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \pm q\dot{B}\pi r_L^2 \leftarrow$$

$$r_L = \boxed{\frac{mv_\perp}{qB}} \longrightarrow \quad \omega_c = \frac{qB}{m}$$

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \pm q\dot{B}\pi\left(\frac{v_\perp}{\omega_c}\right)^2$$
$$= \pm \cancel{q}\pi\dot{B}\,\frac{v_\perp^2}{\omega_c}\cdot\frac{m}{\pm\cancel{q}B}$$

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \frac{\frac{1}{2}mv_\perp^2\cdot 2\pi\dot{B}}{B\,\omega_c}$$

So, we can write delta of half mV perpendicular square is equal to minus Q times dou B by dou t is B dot dot pi R L square. This represents the surface area or we can write mod Q B dot dot pi R L square. Now this B dot dS will be greater than 0 or less than 0 or for different charges it is better that we write in a more generalized way delta of half mV square is plus minus Q B dot by R L square. Now R L can be written as V perpendicular by omega C, R L is mV perpendicular by Q B, omega C is Q B by m. So, this is Q B by m, this one is 1 by omega C.

So, R L can be written as V perpendicular by omega C. So, we will write delta of half mV perpendicular square is equal to plus minus Q B dot pi R L is V perpendicular by omega C whole square or we can make some small algebraic simplification putting Q as it is, pi as it is, B dot as it is and V perpendicular by omega C times 1 by omega C is m by plus minus Q B. This Q gets cancelled and delta of half mV perpendicular square is equal to we can rewrite this expression so that it can be identified easily with known physical parameters. So, we have mass velocity perpendicular square we can write half mV perpendicular square we are introducing a factor of half. So, it should be compensated as a 2 pi B dot is equal to under the denominator we have B omega C.

So, this is the kinetic energy this is B dot B or delta of half mV square is equals to kinetic energy by B times B dot 2 pi by omega C. Change in the kinetic energy seems to be dependent on B dot kinetic energy and B as well. Now, want to find out the rate is the change if you want to find out the rate at which this is happening or with respect to time with respect to the for one gyro orbit things are now or still with respect to one gyro orbit. So, we say that delta of W perpendicular divided by delta T is delta W perpendicular by tau C which is equals to Q pi R L square times B dot divided by tau C. We have just used this one.

$$\boxed{\delta\left(\frac{1}{2}mv^2\right) = \frac{KE}{B}\cdot\frac{\dot{B}\,2\pi}{\omega_c}}$$

$$\frac{\Delta W_\perp}{\Delta t} = \frac{\Delta W_\perp}{\tau_c} = \frac{q\pi r_L^2\dot{B}}{\tau_c} = I\pi r_L^2\dot{B}$$

$$\frac{\Delta W_\perp}{\Delta t} = \underbrace{I\cdot\pi r_L^2}\cdot\dot{B}$$

$$= \mu\dot{B}$$

$$\boxed{\frac{dW_\perp}{dt} = \mu\frac{dB}{dt}} \text{ --- (a)}$$

$$\mu = \frac{W_\perp}{B}$$

$$W_\perp = \mu B$$

$$\frac{dW_\perp}{dt} = \mu\frac{\partial B}{\partial t} + B\frac{\partial\mu}{\partial t}$$

Of course after this we modified this expression to look something like this but if you are trying to find out the rate at which the kinetic energy is changing we started with the same expression Q pi R L square B dot by tau C. Now this Q by tau C charge per unit time charge divided by time gives you current times pi R L square B dot. This is delta W perpendicular divided by delta T is I times pi R L square times B dot. We know that current multiplied by area gives you magnetic moment mu B dot or when limit delta T tends to 0 we can write it as dW perpendicular divided by dt is mu dB by dt or dou B by dou t. So, this expression tells you how the changing magnetic field changes the kinetic energy of the particle.

Now with reference to something that we know already we know that the magnetic moment mu is W perpendicular divided by B or W perpendicular is mu B. dW perpendicular by dt is mu let us say dou B by dou t plus B dou mu by dou t. So, from this let us say we call A from this we know mu times dou B by dou t is nothing but dW perpendicular by dt. So, this can be replaced we can write dW perpendicular by dt is dW perpendicular by dt plus B dou mu by dou t. So, since these two things are same we can simply infer that B times dou mu by dou t is 0.

It is actually 0 that means the magnetic moment is a constant. The rate of change of kinetic energy is already compensated with the changing magnetic field. So, that means there is no provision that you can expect the magnetic moment to change when you have a time varying magnetic field. So, this is of course we are again referring back to the adiabatic invariance where when the particle is gyrating if it is gyrating faster in smaller orbit the magnetic moment generated will be equal to gyrating slower in larger orbits. We will now look at one very important consequence of this magnetic moment being a constant in the presence of a time varying magnetic field.

So, if you consider the flux we know that flux per unit area is the magnetic field. So, we can write the flux as B times area or B times pi rl square. So, from the definition of rl we can write pi times V perpendicular square divided by omega c square. Using some formula we can write V perpendicular square m square divided by q square b square. We have used omega c is equals to q b by m.

$$\cancel{\frac{dW_\perp}{dt}} = \cancel{\frac{dW_\perp}{dt}} + B\frac{\partial \mu}{\partial t}$$

$$\Longrightarrow B\frac{\partial \mu}{\partial t} = 0$$

$$\boxed{\mu = k}$$

$$\phi$$

$$\frac{\phi}{A} = B$$

$$\phi = B\cdot A = B\cdot \pi r_L^2$$

$$= B\cdot \pi \frac{v_\perp^2}{\omega_c^2}$$

$$= B\pi \frac{v_\perp^2 m^2}{q^2 B^2}$$

$$= \frac{2\pi m}{q^2}\cdot \frac{\frac{1}{2}mv^2}{B}$$

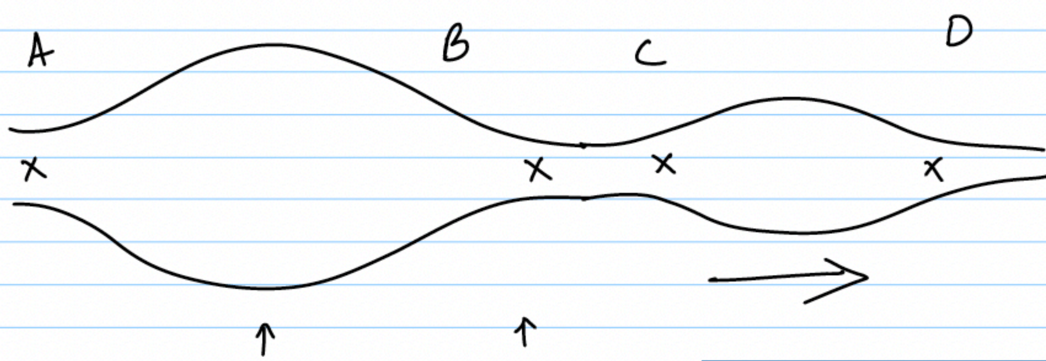
That means then we can write it as 2 pi m by q square times half m v square by b. So, the flux is you see this is the kinetic energy this is the magnetic field. So, the ratio is magnetic moment we know already 2 pi m by q square which is 2 pi m by q square times the magnetic moment. What is this? This is the flux. So, if the magnetic moment is to remain a constant the flux will also remain a constant as per this expression.

Now what have we understood? So, the flux also remains a constant the magnetic moment remains a constant, but the energy of the particle is changing. So, the only thing that is changing is the energy of the particle. So, it is possible that you create a setup in which you are changing the magnetic field with respect to time and this setup will make the particles accelerate. The particles will be energized. So, this setup is actually being used in many places where we can energize plasma.

So, for example a simple setup of this adiabatic compression of plasma the process by which you can energize plasma using time varying magnetic field. So, this is called as the adiabatic compression of plasma. So, we know that when you have a mirror confinement we already discussed and we have also done some numerical problems based on it. When we have a mirroring arrangement a bottle arrangement that we say where the magnetic field is having a gradient towards both the sides. So, at the highest points or at the at certain points when the magnetic field is strong enough the particle will simply bounce back and come back to the weaker region.

$$\boxed{\phi = \frac{2\pi m}{q^2} \cdot \mu}$$

Adiabatic Compression of plasma



And if you have the setup something like this where you have the gradient established at both sides then it may be possible to confine this plasma within this arrangement. So, this is one method to trap plasma. So, that is the whole point of discussing time varying magnetic fields after discussing the mirroring magnetic mirroring. So, if you have a mirror something like this where the middle portion is representing a weaker magnetic field and the both ends are representing very strong magnetic field. The arrangement simply facilitates trapping of particles between these two mirror points and depending on the ratio of magnetic field strengths between let us say here and here we can trap the particle or for certain pitch angles the particle will simply be lost.

So, we also discussed what is called as the loss cone where the velocity if it is within this cone the particle will be trapped otherwise it would not be trapped all that. Now coming back we are now discussing what is called as adiabatic compression of plasma wherein time varying magnetic field can increase the energy of the particle can change the energy of the particle. So, using this concept in addition to the mirroring it is possible to energize plasma to very high energies. So, this is this method or this technique is called as the adiabatic compression of plasma. So, what we do in this is that we have a setup like this and then we have another mirror like this.

Let us say we have these two points A, B, C, D. What happens is now in this mirror configuration you pulse the magnetic field between these two points between A and B where the time varying magnetic field is energizing plasma as it moves and you also have these mirrors maintained so that the plasma is the heated plasma is existing within this mirror. Now once you achieve some amount of energy some threshold of energy it may be possible that you change this magnetic field and allow the particles to proceed into this next a mirror and then again between the coils let us say C and D it may be possible to pulse the magnetic field and further compress or further energize the plasma. So, this method is called as the adiabatic compression of plasma which is basically used to accelerate plasma to very high energies. Now we have discussed so far time varying magnetic field.

In the next lecture we will try to understand particles movement in a time varying electric field.