Plasma Physics and Applications

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Week – 06

Lecture 27: Motion in Time Varying Magnetic Field

Hello dear students. In today's lecture, we will try to understand particle motion in a time varying magnetic field. So far in our discussions, we have seen how particle which is actually a plasma particle moves in the presence of a static electric field, static magnetic field, a combination of electric and magnetic fields. We have seen how a space varying magnetic field with a gradient will affect the particle in this trajectory and what is curvature drift and how the magnetic mirroring can be helpful in trapping a particle. So, in today's class, we will try to vary magnetic field with respect to time that means dB by dt will not be 0. So, magnetic field will vary as a function of time.

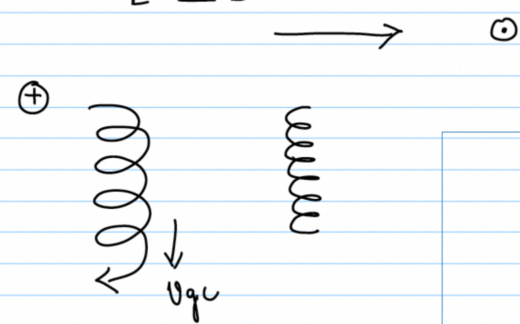
So, in order to understand or appreciate how this is different from the earlier scenarios, we will try to look back in the basic E cross B drift when you have an electric field and a magnetic field. We have derived the expression for the drift velocity in this case which is E cross B by B square. So, this is the expression for the E cross B drift. So, if you look at this expression, there are few messages, there are few outcomes of this is that the drift velocity Vd is independent of the charge.

So, the Vd will be same for electron and ion independent of charge. So, both the charges will be affected in the same way. Second is independent of mass. So, there is no mass dependency. So, if you have electric field and magnetic field in perpendicular direction, the velocity that is attributed because of these two fields on a particle will be same for electron as well as ion.

$\vec{E}$ & $\vec{B}$ $\qquad \vec{V}_D = \frac{\vec{E} \times \vec{B}}{B^2}$ $\qquad \frac{dB}{dt} \neq 0$

1) e^-, ion (independent of charges)
2) (m) is not dependent
3) V_D is average flow of particle
4) V_D $\quad |E| < B$

$E \perp B$



So, both the particles will move with the same velocity. So, this velocity actually M is, so this velocity is an average velocity of the flow of the plasma. So, this drift velocity Vd is average velocity of average flow of particle or plasma. So, if the plasma is non neutral, the electric field produces a current density perpendicular to both electric and magnetic fields. And if it is a weak plasma, the electrons and ions move differently because of their collisions with the neutrals.

This motion leads to the formation of currents inside the plasma. And most importantly, this expression for Vd, the drift velocity is valid when the electric field is the magnitude is less than the magnetic field. Now, this is one situation. So, considering the electric field to be perpendicular to B, if you have let us say the electric field in this direction and the magnetic field is out of the page, then a positive charge will move in this way. And a negative charge will move like this.

So, this moment of the guiding center is in this direction. So, when you have electric and magnetic fields perpendicular to each other. So, what happens is in the first half the particle will gain energy and increases the V perpendicular component and hence the radius increases and in the second half the particle loses energy because it is opposite to the direction of the electric field. So, velocity decreases and as a result, the radius will decrease. So, this difference in the radius between the first half and the second half is basically what you refer to as the drift.

The negative electron also gains energy opposite to the direction of electric field and thus moves in the same direction but rotates exactly opposite to the ions. Now, the characteristic parameters of this gyration are the RL which is the radius of gyration which is mV perpendicular by qB and omega c the frequency of gyration is qB by m. We know these parameters very well. So, for instance when the velocity is same but if they have different masses, so the radius of gyration RL if you have higher mass the radius of gyration will be larger and if it is a lighter mass the radius will be smaller. So, as a consequence the drift of the velocity of guiding centers, the guiding centers drift per cycle will be very small.

$$r_L = \frac{m v_\perp}{qB}, \quad \omega_c = \frac{qB}{m}$$

$$r_L \uparrow \; m \uparrow \; v_\perp \uparrow$$

$$\vec{F} = \underbrace{\vec{E}q}_{E} + \underbrace{q(\vec{v} \times \vec{B})}_{B} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{F} + q(\vec{v} \times \vec{B})$$

$$F = \vec{E}q$$

$$\vec{E} = \frac{\vec{F}}{q}$$

$$\vec{v}_D = \frac{\vec{E} \times \vec{B}}{B^2}$$

$$\vec{v}_G = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2}$$

If you have a lighter mass particle, so the average drift of the guiding center per one cycle it will be smaller. However, this is compensated equally by the larger frequency of the because the mass being very small will increase the frequency. The number of cycles per second will be very large and the drift per cycle the guiding centers movement per one cycle will be small but the frequency being larger it will be averaged out. But if you have a situation where particles have same mass but different energies that means mass is same but V perpendicular will be different because of the difference in the kinetic energy. So then both particles will have the same gyration frequency because the gyration frequency is independent of the perpendicular component of velocity.

What will happen is slower one will have a smaller radius because of V perpendicular directly proportional to RL and hence it is present in the direction parallel to the electric field for a shorter period of time because of the smaller radius and thus the energy that it can gain per cycle will be very small. So, in addition to this E cross B drift we can write an expression for generalized drift. So, let us say if we remember the Lorentz force equation F is equals to Eq plus q times V cross B or we write it as q times E plus V cross B. So, this is the force due to electric field and this is the force due to the magnetic field. Let us say if you have a situation in which the particle is experiencing a force that can be of any type.

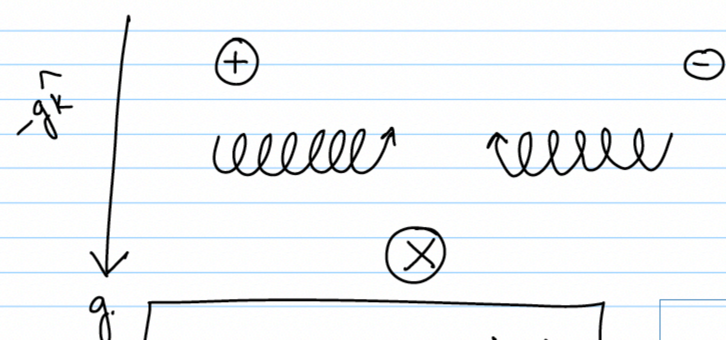
Let us say this is the force and it is also in the presence of a magnetic field then we can write it like this. So, in this situation the expression for drift so F is Eq. The electric field because it is a charged particle electric field can be written as the force experienced per unit charge. So, by substituting this into Vd, Vd is E cross B by B square. So, if I do that then you have 1 by q E is F cross B by B square.

$$\vec{F} = m\vec{g}$$

$$\vec{V}_g = \frac{m\vec{g} \times \vec{B}}{qB^2}$$

$$\vec{F} = \frac{1}{q}\frac{\vec{F} \times \vec{B}}{B^2}$$

$$\vec{F} = \vec{E}q \qquad V_D = \frac{\vec{E} \times \vec{B}}{B^2}$$



$$\boxed{\vec{J} = n(m+M)\frac{\vec{g} \times \vec{B}}{B^2}}$$

So, this is let us say we call it as Vg this is the generalized expression for the drift velocity in the presence of a force F and a magnetic field. So, as an example we can take if the charged particle is in a gravitational field that means its mass is making the particle to move towards a particular direction because of the gravity then the force is F is equals to mg. So that means that for this type of force the drift velocity let us call it as Vg the gravitational drift velocity this can be mg cross B by qB square. Before we do we write this the generalized expression is 1 by q F cross B by B square. If you look at this expression this expression is valid even if you substitute F is equals to Eq you will get the same Vd the drift velocity which is E cross B by B square.

If you have electric field then the drift velocity will not depend on charge or mass of the particle, but the generalized expression for the drift velocity seems to be dependent on charge that means the direction of force will be in one direction for positive ions and in another direction for the electrons. But when you bring in a force like gravitational pull the expression seems to be dependent on the mass as well as the charge. So if you have the gravitational pull in this direction which is minus g k cap then the positive ions I am drawing the trajectory for positive ions will drift like this and the negative ions or electrons will drift like this. And for reference I forgot to write it in the beginning the magnetic field is like this which is into the page. So, in this situation you can use the simple cross product rules or the figuring out for figuring out the directions of this drifts and this is how it will look like.

The point is in the presence of a generalized force the drift velocity will be charge dependent and for the case of gravitational force if it is the drift would be depending on mass as well as charge of the particle. Now this is in a nutshell what we can say is that the magnetic field most important conclusions of this entire discussion is that magnetic field will never be able to affect the energy of the particle. The velocity component which is along the direction of magnetic field will remain constant. If you put an electric field perpendicular to this direction then the other two components of velocity will be affected by the electric field and for the case of a gravitational field. If there is a drift in which gravity force gravitational pull is combined with the magnetic field of the earth itself then the particles will experience a drift like this.

Slowly varying Magnetic field (w.r.to time)

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

$E \perp B$

$dv_{\parallel} = 0$

$\rightarrow$ Induced emf will change the K.E of particle

$dv_{\perp}$ will change due to $\vec{E}$

'l'

$$v_{\perp} = \frac{dl}{dt}$$

$$m\frac{dv_{\perp}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$m\frac{dv_{\perp}}{dt} \cdot v_{\perp} = q(\vec{E} \cdot \vec{v}_{\perp}) + q\,\cancel{(\vec{v} \times \vec{B}) \cdot \vec{v}_{\perp}}^{\,0}$$

$$\frac{d}{dt}\left(\frac{1}{2}mv_{\perp}^2\right) = q(\vec{E} \cdot \vec{v}_{\perp})$$

Now this drift is allowing particles to be separated this is the most important aspect just like what we have seen in the gradient and curvature drifts. This type of configuration will allow the charged particles to be separated because the force the positive charges are experiencing a force in one direction and the negative charges are experiencing a force in other direction. So, the particles will be separated that means you are creating some current you are creating some current density. So, the net current density that can be created or generated out of this charge separation can be written like this. So, J is a vector J is N times M plus M G cross B by B square.

So, this is the current density due to gravitational drift and earlier we had seen what are the characteristic features of E cross B drift and how this can expression can be generalized by considering any force capital F and this force F being the gravitational force facilitates the charge separation. Now coming to the topic of today's discussion we have to see how the magnetic field will affect the particle or how a time varying magnetic field can affect the particles movement. So, at this point of time we know very well that magnetic field will not be able to change the kinetic energy of the particle. So, now let us allow the topic is slowly varying magnetic field with respect to time. Now what we do is we allow the magnetic field to vary slowly with respect to time, but we know very well that no matter what the Lorentz force will always be perpendicular to the magnetic field and the velocity of the particle.

$$\frac{d}{dt}\left(\frac{1}{2}mv_\perp^2\right) = qE \cdot \frac{dl}{dt}$$

$$\tau_c \qquad \omega_c = 2\pi\nu = \frac{2\pi}{\tau_c} \Rightarrow \tau_c = \frac{2\pi}{\omega_c}$$

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \oint_0^{2\pi/\omega_c} qE \cdot \frac{dl}{dt}\,dt$$

$$\delta\left(\frac{1}{2}mv_\perp^2\right) = \oint qE \cdot dl$$

$$= q\int_S \vec{\nabla} \times \vec{E}\,ds$$

$$\boxed{\delta\left(\frac{1}{2}mv_\perp^2\right) = -q\int_S \frac{\partial B}{\partial t}\,ds}$$

So, as a result the net work will always be 0 and magnetic field no matter what cannot impart any energy to the particle. So, there cannot be change in the kinetic energy change in the energy, but this is going to change. So, our understanding is as long as the particle is experiencing only electric field it can get linearly accelerated we have seen that in the very first discussion of particle drifts. But when it is a magnetic field the z component of the magnetic field which is assumed to be parallel to the velocity Vz will remain unaffected Vx and Vy will continue to execute the circular motion. When you have an electric field and magnetic field things change you come across what is called as drift which is the velocity of movement of the guiding center or how the guiding center the pitch brought in by the guiding centers movement at different intervals of time will be different.

But a time varying magnetic field will change all of this how because the time varying magnetic field the moment you call you make the magnetic field to vary with time it will generate a electric field. So, how is it how can we prove it we can start from the very basic Maxwell equation. So, which is del cross E is minus dou B by dou t del cross E is equals to minus dou B by dou t what does it mean? The rate at which the magnetic field changes with respect to time is equal to the negative of the del cross E. So, when E is perpendicular to B. So, electric field perpendicular to B what will happen V parallel will remain constant V perpendicular D will change or will remain will be non zero due to the electric field.

So, this electric field is secondary that means if this electric field is generated by the magnetic field. That means now we have a provision where the magnetic field is creating an electric field and this electric field is now acting on the particle and accelerating the particle. Let us see how all of this can be understood with the help of some mathematics. So, electric field will now change the kinetic energy no doubt kinetic energy of the particle in a perpendicular direction. So, actually this induced EMF is because of the changing a magnetic field will increase the energy of the particle.

So, induced EMF will change the kinetic energy of particle. Now let us say we take length let us say we have a small length L this is the length the particle travels along. So, particle is travelling along a length L. Now since the particle is moving perpendicular to the field itself. So, we can say that the perpendicular component of the velocity V perpendicular is rate of change of position.

So, dL by dt and we will assume that the parallel component being will be unaffected by the magnetic field the perpendicular component will be picked up by the electric field. So, in the presence of electric field where is this electric field coming from this is from the time variation of magnetic field. So, in the presence of E and B fields we can write M dV perpendicular by dt is Q times E plus V cross B. Let us take a dot product of this expression with V perpendicular from the right. So, which is M dV perpendicular by dt dot V perpendicular is equals to Q times E dot V perpendicular plus Q times V cross B dot V perpendicular.

M dV by dt dot V can be written as d by dt of half M V perpendicular square. This is d by dt of half M V perpendicular square which will be Q times E dot V perpendicular. You see this V cross B dot V perpendicular this has to be 0. Divergence of curl will be 0. So, this can be rewritten as d by dt of half M V perpendicular square is equals to Q E dot dL by dt.

So, what is this expression is telling you that the rate of change of kinetic energy is dependent on the electric field charge and velocity. Now, we have a reason to believe that the kinetic energy will change, but this kinetic energy is not directly because of the magnetic field, but it is through the effect of magnetic field or through the time variation in the magnetic field. Now, if the particle is moving along a particular direction and if it takes if because there is magnetic field. So, V x and V y will execute circular motion. Let us say the particle is gyrating in the presence of this magnetic field and if it is making a circular orbit and it takes nearly tau c amount of time for it to complete one orbit with a frequency omega c, then omega c is 2 pi nu which is 2 pi by tau c.

Then we can write that time as 2 pi by omega c. Now, let us take this to be one orbit. This is the amount of time that it takes for the particle to complete one orbit. If you look at this relation, we are accounting for the rate of change of kinetic energy and that seems to be dependent on this. L is the distance that it travels over a particular time.

So, let us say we take this L to be the distance the particle will travel in one orbit which is actually a circular orbit or a circular path. We can try to find out what will be the measure of change that happens within this one orbit. So, we can integrate this expression within one gyration orbit or the time that is going to take. So, we can say delta of half mv perpendicular square will be equal to closed integral 0 to 2 pi by omega c and QE dot dL by dt. So, you see the limits of integral are representing the amount of time that it takes for the particle to complete one orbit and within this one orbit, we know how much distance it travels that is represented by small l.

You can use the simple vector algebra delta of half mv square is equal to closed integral QE dot dL using the Stokes law. We can write Q times integral over a surface del cross E times dS and from the Maxwell equation we know del cross E is equal to minus W by dou t. So, using that Q times integral over a surface dou B by dou t dS. What is this? All of this is equal to delta of half mv perpendicular square. So, this is a very important expression relation which tells how the changing magnetic field is responsible for the change in the kinetic energy.

So far our earlier discussions have confined ourselves to believe that the magnetic field will never be able to change the kinetic energy but through the electric field we are now able to see some change in the kinetic energy. We will continue this discussion in the next lecture. Thank you.