Plasma Physics and Applications

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Week – 05

Lecture 25: Magnetic Mirroring - III

Hello dear students. We are trying to understand what is magnetic mirroring which is a process in which we can confine plasma between two points. A quick recap of the things that we have learned so far is that when we have a magnetic field and its gradient parallel to each other we have realized there will be a non-zero force which is acting on the particle in the opposite direction to the direction of the magnetic field. So, which is something that we have not seen in the case of particle motion in a magnetic field. So, the last expression that we have derived is Fz the z component of the force is minus half m v perpendicular square by B dou B by dou z. So, dou B by dou z indicates that the gradient in the magnetic field since we know that the magnetic moment generated by an individual particle is half mv square by 2B or mv square by 2B which is the kinetic energy by the magnetic field or the perpendicular kinetic energy by B.

So, if you use that we can write Fz as minus mu dou B over dou z or we can write a generalized expression as z component of the force is minus mu times the parallel gradient along the direction of the magnetic field. Now this does not lead us anywhere this does not explain why the particle will be trapped between two points. So, we have not so far discussed anything about the two points but what we have seen is that if you have a magnetic field like this in which we prefer to call it as a bottle configuration in which the field lines are converging into a smaller area from point A to point B. So, here it is important to see how this non-zero force along the z direction along the negative z direction allows the particle to come back.

$$F_z = -\frac{1}{2}\frac{m v_\perp^2}{B}\frac{\partial B}{\partial z}$$

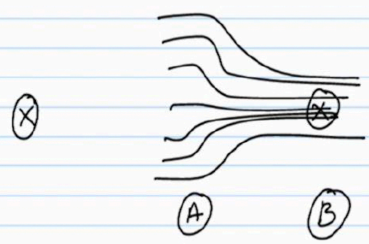
$$\mu = \frac{K.E}{B} = \frac{W_\perp}{B}$$

$$\vec{B} \parallel \vec{\nabla} B$$

$$F = -\mu \frac{\partial B}{\partial z}$$

$$F_z = -\mu \frac{\partial B}{\partial z}$$

$$\boxed{F_z = -\mu \vec{\nabla}_\parallel B}$$



$$\underline{m \quad a}$$

$$m\frac{dv_z}{dt} = \mp q\, v_\perp \frac{r_L}{2}\frac{\partial B_z}{\partial z}$$

$$\rightarrow m v_z \frac{\partial v_z}{\partial t} = \mp v_z q\, v_\perp\, r_L/2\, \frac{\partial B_z}{\partial z}$$

$$\frac{d}{dt}\left[\frac{1}{2} m v_z^2\right] = \mp \frac{v_\perp v_z q}{2}\, \frac{m v_\perp}{qB}\, \frac{\partial B_z}{\partial z}$$

So, let us say we start by conserving the energy we will write the force as m dvz by dt is equals to minus plus cube v perpendicular RL by 2 dou Bz by dou z. How did I get this expression? This is the original expression after simplifying this we got this expression Fz but I am writing it for a purpose. So, m dvz by dt the z component F is equals to m here mass times the acceleration. Acceleration is nothing but the rate of change of velocity is minus plus cube by m v perpendicular RL by 2 dou Bz by dou z. That means the particles of different charges will have different directions.

As per this formula. Now let us say we multiply this equation with vz m vz dou vz by dou t is equals to minus plus vz cube v perpendicular RL by 2 dou Bz by dou z. So, the reason that I have multiplied with vz is that I will be able to write the left hand side as the rate of change of kinetic energy. So, if I take a derivative what will I get half times m times 2 vz dou vz by dou t. So, 2 2 gets cancelled we will still have the same formula appearing on the left hand side which will be equal to minus plus v perpendicular is the perpendicular component of the velocity and we have vz anyway times Q by 2.

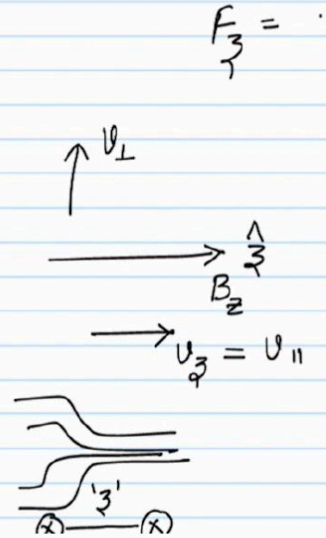
$$\frac{d}{dt}\left[\frac{1}{2} m v_z^2\right] = \mp \frac{m v_\perp^2}{2B_z}\frac{\partial B_z}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$\frac{d}{dt}\left[\frac{1}{2} m v_z^2\right] = \mp \frac{m v_\perp^2}{2B_z}\frac{\partial B_z}{\partial t}$$

$$\frac{d}{dt}\left[KE_\parallel\right] = \mp \frac{m v_\perp^2}{2B_z}\frac{\partial B_z}{\partial t}$$

$$\frac{d}{dt}\left[KE_\parallel\right] = \mp \frac{KE_\perp}{B_z}\frac{\partial B_z}{\partial t}$$

$$\boxed{\frac{d}{dt}\left[KE_\parallel\right] = \mp \mu\frac{\partial B_z}{\partial t}}$$



I have expanded RL the definition of RL mv perpendicular by Qv. So, here Q gets cancelled and we can write it as the rate of change of kinetic energy is equal to minus plus mv perpendicular square divided by 2 Bz dou Bz by dou z into dou z by dou t. So, how did I get dou z by dou t because you see here you have vz that is appearing here. What is vz? vz is dz by dt the rate of change of position or along z axis with respect to time. So, now I can write dz gets cancelled so d by dt of half m vz square is equals to minus plus mv perpendicular square by 2 B into dou Bz by dou t.

Now the direction along which the magnetic field is defined is this one this is Bz. So, this direction is v perpendicular and this direction is vz which is nothing but v parallel. So, this component of the kinetic energy or not component actually this is this part of the kinetic energy is the parallel kinetic energy. So, we will write d by dt of kinetic energy parallel is equals to minus plus mv perpendicular by 2 B dou Bz by dou t or we can write d by dt of Ke parallel is equals to minus plus Ke perpendicular by B dou Bz by dou t. So, the rate at which the particles energy changes along the gradient of the magnetic field seems to be dependent on how the magnetic field changes with respect to time.

$$KE = KE_{\parallel} + KE_{\perp}$$

$$\frac{d}{dt}\left[KE_{\parallel} + KE_{\perp}\right] = 0 \qquad \mu = \frac{W_{\perp}}{B}$$

$$\frac{d}{dt}\left[KE_{\parallel} + \mu B\right] = 0$$

$$\frac{d}{dt}(KE_{\parallel}) + \frac{d}{dt}(\mu B) = 0$$

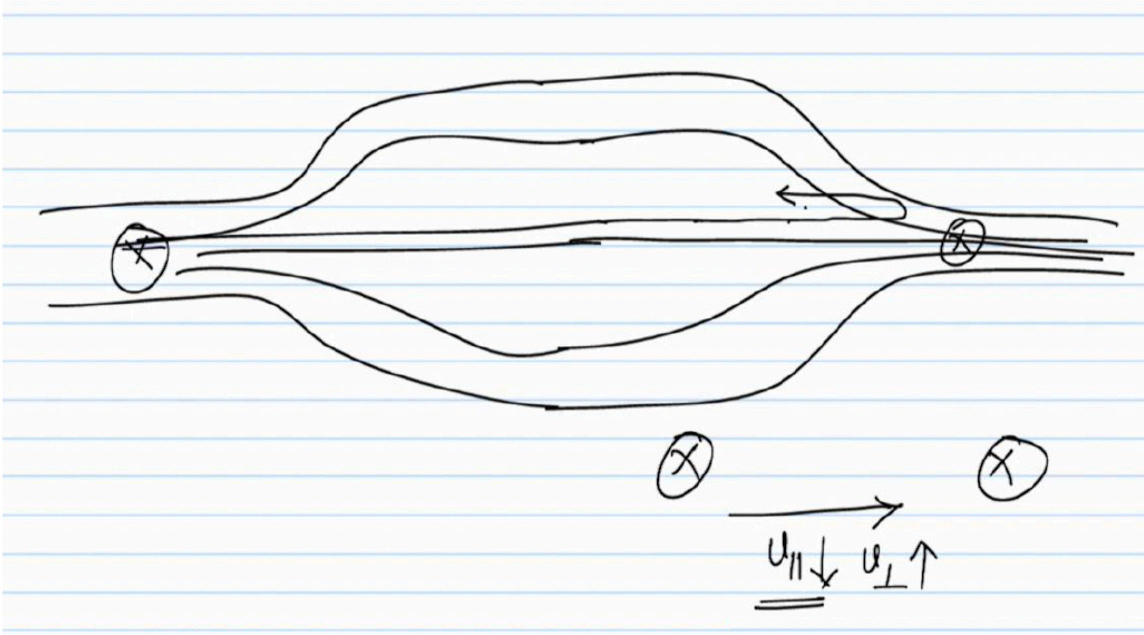
$$-\mu\frac{\partial B}{dt} + \mu\frac{\partial B}{\partial t} + B\frac{\partial \mu}{\partial t} = 0$$

$$B\frac{\partial \mu}{\partial t} = 0$$

$$B\frac{d\mu}{dt} = 0 \Rightarrow \mu = K$$

So, how is the time coming into this picture? You have a configuration like this a bottle configuration in which the magnetic field is being convergent. So, where is the variable time defined here or if the particle is travelling from this point to this point it is covering the distance of z. As the particle is travelling so the gradient that the particle is experiencing as it travels is represented in the units of time. So, the passage of time brings along the change in the magnetic field either you call it as the rate of change of magnetic field as it as the particle travels along the z axis or you may call the rate at which the particle will experience different magnetic fields as it travels I am referring to the evolution of the magnetic field with respect to time. So, time can be thought of an implicit variable which is not directly present but as the things go on time becomes a part of the entire discussion.

But one thing is for sure if at a given point the magnetic field is not changing that is the most important thing at a given point. But the magnetic field only changes when the particle is having a velocity and the particle is moving along a particular direction. So, what is the consequence? The consequence is very simple actually. So, we will write the rate of change of parallel kinetic energy Ke parallel is equals to minus plus mu dou B by dou T. This is not new we have already had this sort of expression but we never had the kinetic energy that is appearing on the left hand side.



So, the earlier form of this expression is in terms of the force along z axis. But now we have seen that see whatever happens the kinetic energy is changing and who is compensating for this change of kinetic energy? The magnetic field or the rate at which the magnetic field changes or this rate is actually at the expense of the particle traveling along the particular direction simple. Now if the kinetic energy is changing you cannot expect the energy to come down. So, because it is a magnetic field after all it will not do any work it will not attribute it will not increase or decrease the energy. So, the energy should remain a constant.

So, the total energy kinetic energy has to be conserved. And if you write the total kinetic energy Ke is equal to Ke parallel plus Ke perpendicular. Now when this is increasing the kinetic energy perpendicular has to decrease so as to keep the total kinetic energy as constant as simple as that. So, we will write it as d by dt of Ke parallel plus Ke perpendicular should remain constant. Or in terms of things that we know d by dt of Ke parallel plus Ke perpendicular is mu B is equal to 0.

Why Ke perpendicular is mu B? We have actually derived this expression W perpendicular by B. So, this is the kinetic energy. So, the notation is slightly different I am not using Ke perpendicular but W perpendicular is the perpendicular kinetic energy. So, which is equal to mu B. So, I will write d by dt of Ke parallel plus d by dt of mu B should be equal to 0.

So, d by dt of mu B is mu times dB by dt plus B times d mu by dt. So, this is of course

0. So, we can straight away write it as d by dt of so we can write this as d by dt of Ke parallel plus d by dt of mu B is equal to 0. Now let us go back. Let us go back that what is the rate of change of parallel kinetic energy? It is mu dou B by dou t.

So, this is from the earlier expression we can write it as this one is mu dou B by dou t plus mu dou B by this is minus dou B by dou t plus B dou mu by dou t is equal to 0. So, these 2 things get cancelled what we left with is B dou mu by dou t is equal to 0. If you are just wondering what did I do? So, the rate of change of parallel kinetic energy is equal to the magnetic moment multiplied by the rate of change of the magnetic field with a minus. So, that I have used here and d by dt of mu B using chain rule I have expanded this got these 2 terms and it so happens that this term gets cancelled with the first term that is appearing as a result of the chain rule. So, as a result we have B d dou mu by dou t or d mu by dt we have only one variable.

So, we can write it as B d mu by dt is equal to 0. What does it mean? It means that the magnetic moment will remain a constant. So, we defined the adiabatic invariant as the process in which the magnetic moment is held constant. But what we have seen is that in this whole process of magnetic mirroring a non-zero force appears on the opposite direction. So, this non-zero force along the negative z axis can be written in terms of the rate of change of magnetic field along the positive z axis.

So, somehow the rate at which the parallel kinetic energy is changing for the particle as it moves further into the magnetic field seems to be connected to the rate at which the magnetic field strength is changing with respect to time with respect to the trajectory of the particle. Now we have a measure of the rate of change of parallel kinetic energy with respect to the rate of change of magnetic field. That is this expression that relation is this expression. Now then we did a simple calculation simple rearrangement of the terms in which we have got if the parallel kinetic energy is of course changing then it has to be compensated by the perpendicular kinetic energy. That means if the parallel velocity of the particle is changing it has to be compensated at the expense of changing perpendicular kinetic energy.

So, these two things go hand in hand V perpendicular V parallel. So, these two things compensate each other at all times just to keep the kinetic energy the total kinetic energy as a constant. Because the total kinetic energy has to be considered because it is the magnetic field of cloud magnetic field does not change the energy or does not do any work as such because V is perpendicular to B. So, if the kinetic energy is conserved as a consequence you will expect the magnetic moment to be constant. Now let us bring all of these things together.

Let us say we have this magnetic field configuration I have to draw it just so that. Now we have these two points. What happens is as the particle is moving further into the strong magnetic field region its V parallel will decrease. So, from the earlier discussion is very clear. So, as the particle is moving into the strong magnetic fields its V parallel will keep on decreasing the kinetic energy part will keep on the kinetic energy part along the parallel direction will keep decreasing.

At the same time the perpendicular component will keep increasing. So, V perpendicular will increase. So, at a particular point a threshold is reached where V parallel is made completely zero because of the very strong magnetic field there. So, what happens at that point the entire energy of the particle has to be perpendicular energy but not the parallel energy. So, particle can no longer go into the positive z axis because its velocity is zero along the direction.

So, the particle only has the perpendicular component of the velocity. So, now the particle due to its so let us say if it is going at a velocity it slowly becomes very slow the magnitude of the velocity will keep decreasing and at a point when the magnetic field is sufficiently large the particle will bounce back. So, this concept is called as the magnetic mirroring. Now about the trapping of plasma what you do is you keep another bottle kind of arrangement here in which the magnetic field is converging. So, what happens after some time so between these two points the particle will be trapped.

So, this arrangement is called as the magnetic bottle or magnetic mirroring. So, how the magnetic mirroring or the confinement of plasma is facilitated if you ask it is facilitated due to the fact that with the increasing magnetic field strength the particles parallel component will keep decreasing and this decreasing parallel component of kinetic energy has to be compensated with increasing perpendicular kinetic energy and at a point the magnetic field becomes so strong that the only the perpendicular component will survive bringing the particle back into the into the weak field region. So, this is something about the magnetic mirroring we will there are a few more aspects to this magnetic mirroring and we will also try to solve some numerical based on this idea so that the concept becomes very clear. So, we will continue this in the next lecture. Thank you.