Plasma Physics and Applications

Prof. MV Sunil Krishna

Department of Physics

Indian Institute of Technology Roorkee

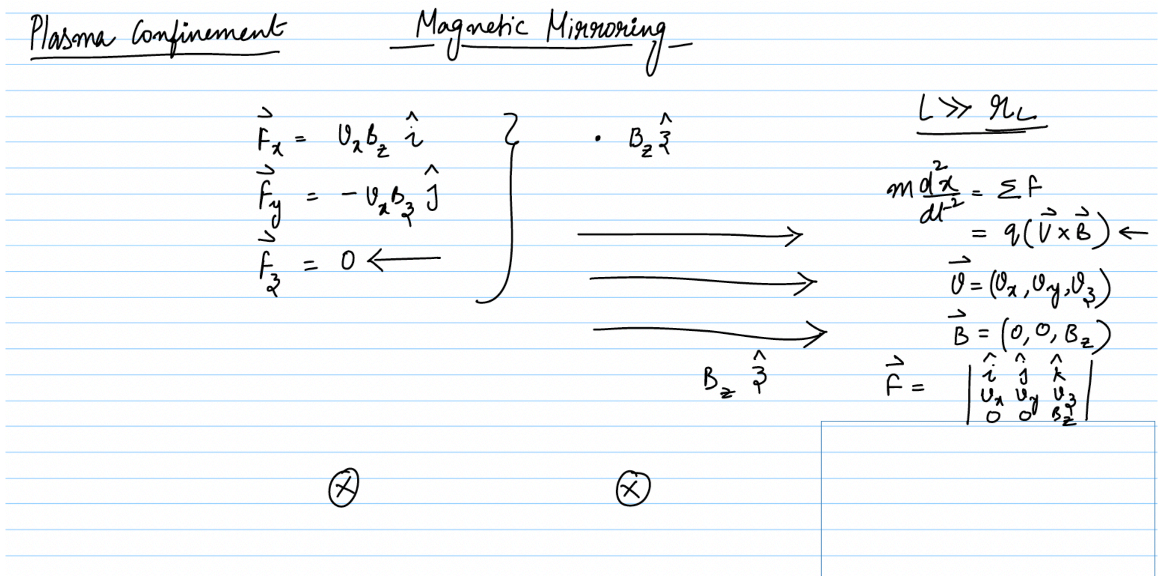
Week – 05

Lecture 23: Magnetic Mirroring

Hello dear students. In this lecture we are going to learn about a very important topic which is called as the magnetic mirroring. So, this is discussion is leading towards the confinement of plasma. So, we are going to understand how we can confine plasma which is plasma confinement. So, we have to trap plasma in a particular arrangement or in a chamber or anything. So, this is facilitated by a process called as magnetic mirroring.

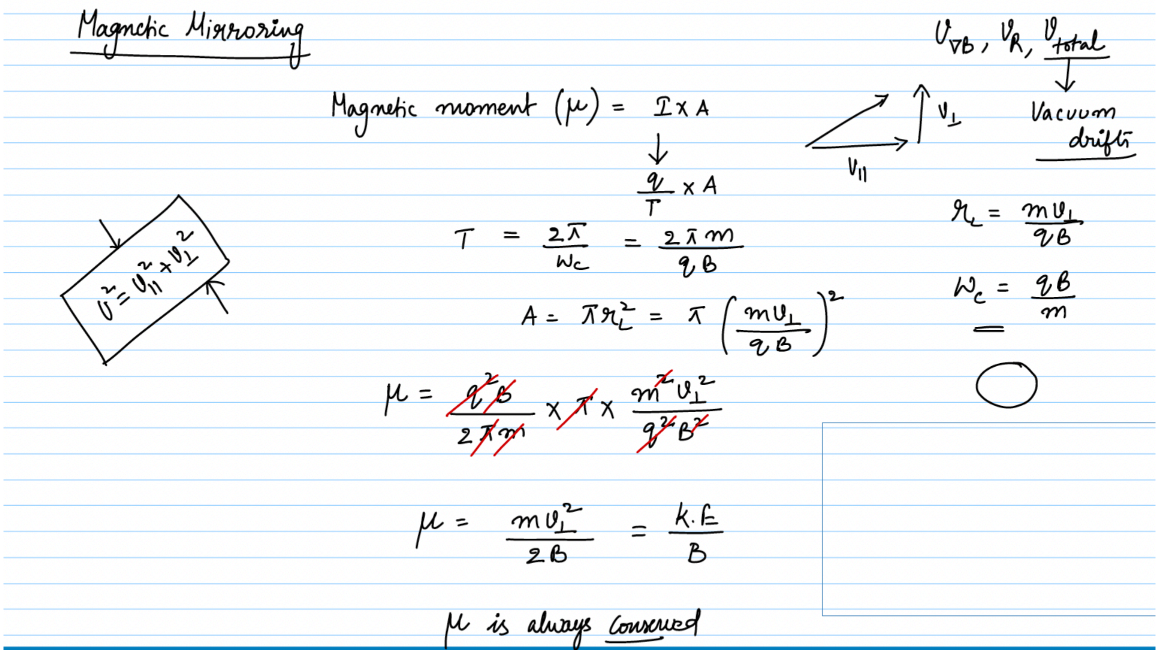
What have we learned so far in our discussions is that when the plasma is subjected to a magnetic field the plasma particles will gyrate and the moment you start imposing inhomogeneity in the magnetic field we wanted to restrict it to a particular scale. So, the length of inhomogeneity or the scale of inhomogeneity was considered in proportion to let us say the radius of gyration. So, if R L is the radius of gyration of the particle in the plasma which is basically due to the magnetic field itself. Now, we wanted this length of inhomogeneity to be greater than this R L.

So, we do not want things to change within one gyro radius. And we have started with a gradient magnetic field in which the magnetic field strength is changing with respect to space and then we imposed what is called as a curvature magnetic field. So, any magnetic field any realistic magnetic field will always have a gradient and will always its field of field of lines will always be curved. What does it mean? So, they have to converge at the point. So, as to make del dot B is equal to 0 all these things have to be true.



So, in today's class we are going to extend the gradient and curvature drifts and we are going to learn how a particular type of magnetic field or a particular configuration of magnetic field can help us trapping the plasma between two points. So, the plasma cannot pass these two points it has to be confined between these two points. So, that is the basic discussion of today's class. So, if you are familiar with all the earlier classes if we have to write an equation of motion we have to write M d square x by dt square is equal to sum of forces. So, in this case if the particle is experiencing only a magnetic field an isolated magnetic field it has to be Q times V cross B.

So, in our earlier discussions we have taken V to be Vx Vy Vz and if I have taken the magnetic field B to be having a component only Bz. So, the force F is V cross B which is let us say if I take ijk as the unit vectors Vx Vy Vz 0 0 Bz. So, I can write this as let us say the components Fx as Vx Bz minus 0 or Vz into 0. So, this force will be along i cap and Fy or it is just Fy a component of the force. So, it is just a scalar component.

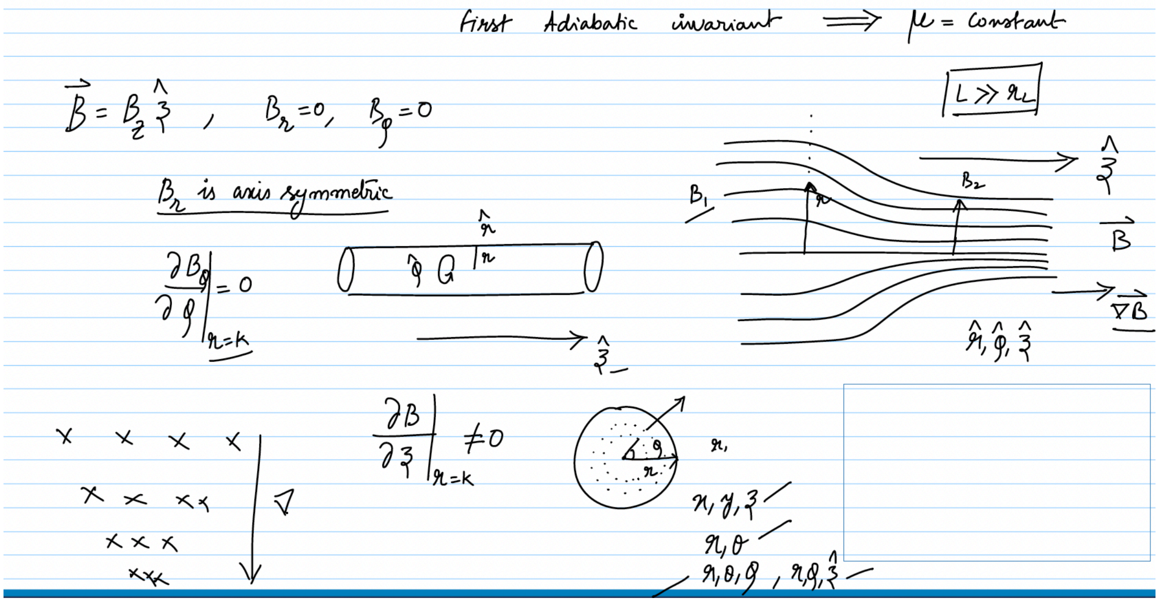


So, if I have indicated the direction then it makes sense. Fy is minus Vx Bz along j cap and Fz is simply 0. So, this gives us what will be the force that will act upon a particle which is passing through a magnetic field which is aligned only along the z cap direction. Now, the message that I have to convey here is that in this configuration the magnetic field is not experiencing any force along the z axis or the direction of the magnetic field itself. So, if the magnetic field is in this direction let us say we assume this to be the z direction this is z cap for example and there is no gradient in the magnetic field.

So, this is the magnetic field which is along z cap. Now as the particle is going is having a velocity which is along Vx Vy Vz despite of that because of this V cross B term one very important conclusion that we draw here is that there is no force along the z axis that is M d square x by dt square is simply along z axis will be simply 0. Now, the reason that I have brought here is that we will derive something which is drastically different which is going to be drastically different from this Fz is equal to 0 and depending on that we will try to understand what is magnetic mirroring and how magnetic mirroring can help to trap plasma between two points. So, what are these two points and how plasma is going to be trapped between only these two points is going to be the topic of discussion for today's class. Now, let us try to understand some basic details about the magnetic nature of plasma.

So, let us say so far we have derived what is so the topic is magnetic. Considering the fact that the audience of this course are from a diversified background of mathematics and physics I have always tried to do the derivations step by step so that it will be easy for you to follow and understand the underlying physics of these mathematical steps.

So, we will also do this in today's class as well. So, let us say so far we have derived expressions for V del B what is V del B the gradient drift velocity, Vr the curvature drift velocity, V total or the total drift velocity this we have also referred to as vacuum drifts. We have done all this.



Now, let us see what is the magnetic moment of a particle. What is magnetic moment? Magnetic moment is usually denoted by Mu which is nothing but current times area. So, we have known RL which is the radius of gyration is mv perpendicular by qB the omega c the gyration frequency as qB by m. Now, if we have to evaluate the magnetic moment or get an expression for the magnetic moment we will see the current is nothing but q by T charge by time times area is the area of the circular loop in which the current is flowing. So, current we have to have a measure of the time period or the time.

Time is nothing but let us say we can get the time by 2 pi by omega c which is nothing but using omega c from here we can write 2 pi m by qB. What is this? This is the time. So, in order to evaluate what is the magnetic moment we have T is equals to 2 pi by omega c which is 2 pi m by qB and A is pi RL square circular loop if the particle is gyrating. Now the idea is what is the idea? So, the particle is not the particle is gyrating along the magnetic field line it is a charged particle after all it constitutes some current and since it is moving in a circular path in a closed path it will generate some magnetic moment it will create some magnetic moment. The measure of the magnetic moment is current times the area.

So, area is pi RL square so we know pi RL we know which is mv perpendicular square by qB. So, this square is something that appear that should appear outside. So, putting this all together mu is equals to q square p pi 2 pi m times pi times m square v perpendicular square divided by q square B square or if we cancel out all the terms what will be left with is mu is equals to mv perpendicular square by 2B or you can also write it as kinetic energy by the magnetic field. So, the magnetic moment of a particle is equals to mv perpendicular square by 2B. Now from the earlier discussion we know that the magnetic moment is always conserved when we were discussing the particle motion in a magnetic field itself we realized that if you increase the magnetic field strength the radius of gyration would go small.

So, as a result the net magnetic moment will always be conserved when the magnetic field is changed. So, this is a known concept where the magnetic moment is always conserved. If you do not want the magnetic field to do any work or to change the energy of the particle then this needs to be followed all the time. So, magnetic moment has to be conserved. Now if you see this equation if the magnetic moment has to be conserved despite the changes in the magnetic field strength B what do you expect the only other variables here are m so for a given particle the mass will not change mass will remain a constant.

So, the only other parameter with that you can tweak is the v perpendicular the perpendicular component of the velocity. So, what is the perpendicular component of velocity if you take v to be this so this becomes the parallel component v parallel and this becomes the perpendicular component. So, ideally we should be able to write v square is v parallel square plus v perpendicular square. So, all this is basic vector algebra so if you have studied some plus 2 level vectors you should be able to understand what is going on here. So, mu is always conserved the magnetic moment is always conserved.

So, under the situation when the magnetic field will change the only parameter which can compensate the change in magnetic field so as to result in the net magnetic moment to be conserved becomes the perpendicular component of the velocity. Now but we know that the velocity is not just perpendicular component rather it is a sum of the vector sum of the perpendicular plus parallel component. So, if you say that when the perpendicular component of velocity is changing so as to keep the total velocity constant why the total velocity should be constant the energy should be conserved at all times. The parallel components takes the effect or undergoes an opposite change so as to conserve the total velocity. So, something is very clear here a gyrating particle in a magnetic field generates a magnetic moment this magnetic moment seems to be dependent on the strength of the magnetic field and on the perpendicular component of the velocity.

So, if the magnetic field is increased the perpendicular component of velocity should decrease so as to keep mu constant and this is the entire reason why magnetic mirroring is actually a reality or how the magnetic mirror actually works is basically due to this magnetic moment being conserved. So, this conservation or this invariance of the magnetic moment is generally called as the first adiabatic invariant what does it mean? This implies that the magnetic moment mu is always a constant. So, the magnetic moment is conserved only when the magnetic field is not changing with within one gyro orbit this is also very important. So, you have to you cannot make a more general case so you have to confine the magnetic fields in homogeneity beyond one gyro orbit. So, things are not changing as long within the small distance over which one gyration is happening.

So, things are changing the magnetic field is of course changing but it is changing over the scales which are larger than the gyro orbit. So, if you denote the length of inhomogeneity as capital L so this has to be much larger than RL. So, this is something that we have already learned we have already many times we have seen the consequences of this. Now let us talk about the magnetic mirroring. So now we have equipped ourselves with two important concepts which are one when the magnetic field is along a particular direction and if it is not changing with respect to space or time we realized the magnetic field will not be able to have a component of force along the same direction.

So, the fz will become 0 as long as the magnetic field is along the z direction. The second thing is when the particle is gyrating in a magnetic field along a field line the magnetic moment that is thus generated will remain a constant and this type of invariance is called as the first adiabatic invariant. Now let us talk about the magnetic mirroring. So, for the case of magnetic mirroring we take a special configuration of magnetic field in which the magnetic field lines are converging along a particular direction. So, if you go like this so we draw like magnetic field lines.

So, what do you see here you see that the magnetic field lines are converging into a region along one particular direction. Now what is the geometry or what is the preferential coordinate system for this kind of a configuration it is basically cylindrical coordinate system. So, how does it work we take for reference we take a cylinder like this is for reference to understand the coordinate system that is valid for this type of magnetic field configuration. So, this is this axis becomes the z cap the z direction and this is the r and the increasing magnitude of r from the center of the cylinder denotes the r cap direction. And this angle so this angle like this angle so you start from the center at one particular value of r you go from 0 to 360 this denotes the direction of phi cap or the let us say the anti-clockwise direction in this point of view is the phi cap.

So, you have what are the coordinates we have r we have phi we have z the unit vectors along these directions are r cap phi cap z cap. So, I hope you have understood this very clearly the radius is from the axis of the cylinder towards the outer surface z is along the length of the cylinder and phi cap or the angle phi you take a cross section of this and you see if this is the axis and this becomes the radius and in order to identify any point here at one particular radius. So, what is this? This is the angle phi. So, any point can be understood any point let us say in this cross section can be understood by a distinct value of r the radius away from the center and where exactly is the point? The point is at an angle phi with respect to this let us say. This is true as long as you are talking about cross section of this cylinder but when you want to identify or name or put a coordinate for any point along the cylinder then you will first thing is you have to know where this point this intersection is going to come that is going to be the value of z and then within at that particular value what is the radius and what is the angle.

So, you need these 3 coordinates at the same time to be able to identify one point at one point within this cylinder. Now this is why do we generally prefer different set of coordinates we sometimes prefer to work in a coordinate system like something like x, y, z sometimes we prefer coordinate system which is r theta sometimes we prefer r theta phi sometimes we prefer r phi z. We know all these coordinates one is the polar coordinates the Cartesian coordinates the spherical coordinates the cylindrical coordinates all this because we choose the coordinate system such that you take a coordinate system such that the physical process or the phenomena is invariant along one direction as such is invariant along one direction. So, what it facilitate in terms of mathematics is that it will reduce the number of equations. So, something some physical parameters physical quantity will remain conserved along that axis so you do not have to worry about it you can think of the other directions along which the physical quantity is varying.

So, this reduction in number of variables or the number of equations is basically facilitated by the choice of coordinate system and the geometry of course. So, the first answer that should come is the depending on the geometry of the system that you are trying to understand you take a coordinate system which also has that sort of geometry. So, this is the configuration itself is see ideally these magnetic field lines are not starting from here they are starting from somewhere and they are converging into a particular small area. So, the geometry itself is so clear here that you have to take cylindrical coordinate system. Now this is the basic the magnetic field configuration.

Now the magnetic field is going in this direction. So, let us say we have the magnetic field along z cap itself what do we mean the magnetic field is directed or the direction of the magnetic field is along z direction. So, we write what do we write we write the magnetic field B is Bz which is along 0 what does it mean that means that Br is 0 and B phi is 0. Br is 0 what does it mean you see here at any distance r the magnetic field is constant and as long as you do not start the bending of magnetic field lines there is no component of r. So, Br is not existing it is 0 but when you think you take this distance r you measure the magnetic field at this point as something let us say B1 you go to the same distance at this point the magnetic field B2 will be different.

So, that is the basic essence of the geometry of the magnetic field that you have considered. Now B phi is 0 now Br is now most importantly we have to say Br rather than saying Br is 0 itself let us say Br is axis symmetric Br is axis symmetric what does it mean? So, for a given value of r for a given value of r you take any value of phi the magnetic field will be constant like this. So, along all along the loci of these points the magnetic field will be a constant. So, we have I think we have spent enough time to understand the configuration itself let us see the consequences of these configuration. So, as the B magnetic fields is axis symmetric we can write that d B phi by d phi is when you say axis symmetric for a given value of r for a given value of r for any value of phi the magnetic field is constant.

So, it has to be let us say we have 3 variables in at hand. So, let us say we write dou B phi by dou phi at a given value of r is equals to constant is can you make a guess what would be it would be 0 or it would be a constant it would be 0 why because the magnetic field at a given value of r for various values of phi is non 0 but it is not changing. So, the derivative thus becomes 0. So, this is basically defining all the characteristics of the magnetic field the magnetic field itself. But what happens as you go along z axis it slightly changes as you go along z axis for the same value of r you will find you will find different magnetic fields.

That means dou B the change in magnetic field at different values of z for the same value of r is non 0 you see you have a gradient the magnetic field at a given distance r is assumed to be V1 if you take the same distance at a different point along the z axis or at a different value of z the magnetic field will be different. That means the magnetic fields change with respect to z at the same value of r is not a constant. So, there is a gradient. So, the gradient is of course visible straight away it is there. Now most importantly something that you have to you must be able to understand by now is that the gradient is in this direction.

So, this is the direction of del B the gradient is always points towards the increasing direction of a physical quantity. So, you have let us say you have something like. So, gradient always points in this direction this is the direction of gradient that is by definition it is like that. So, magnetic field is also along z axis and the gradient is also along the same axis. So, now we have a situation in which we have gradient which is parallel to the magnetic field.

How is this different? If you ask you have to go like a couple of lectures before and you will realize that when we were discussing the gradient magnetic field the particle drift in a gradient magnetic field it was the magnetic field was perpendicular to the gradient or the gradient was chosen to be perpendicular to the magnetic field. But now here we have a totally different situation where the magnetic field is in the in one direction and the gradient is also in the same direction. Now having framed all the basic details about the magnetic mirror mirroring and the type of geometry that you require to facilitate magnetic mirroring we will continue this discussion in the next class so as to see how the magnetic mirroring actually works. Thank you.