Plasma Physics and Applications

Prof. MV Sunil Krishna

Department of Physics

Indian Institute of Technology Roorkee

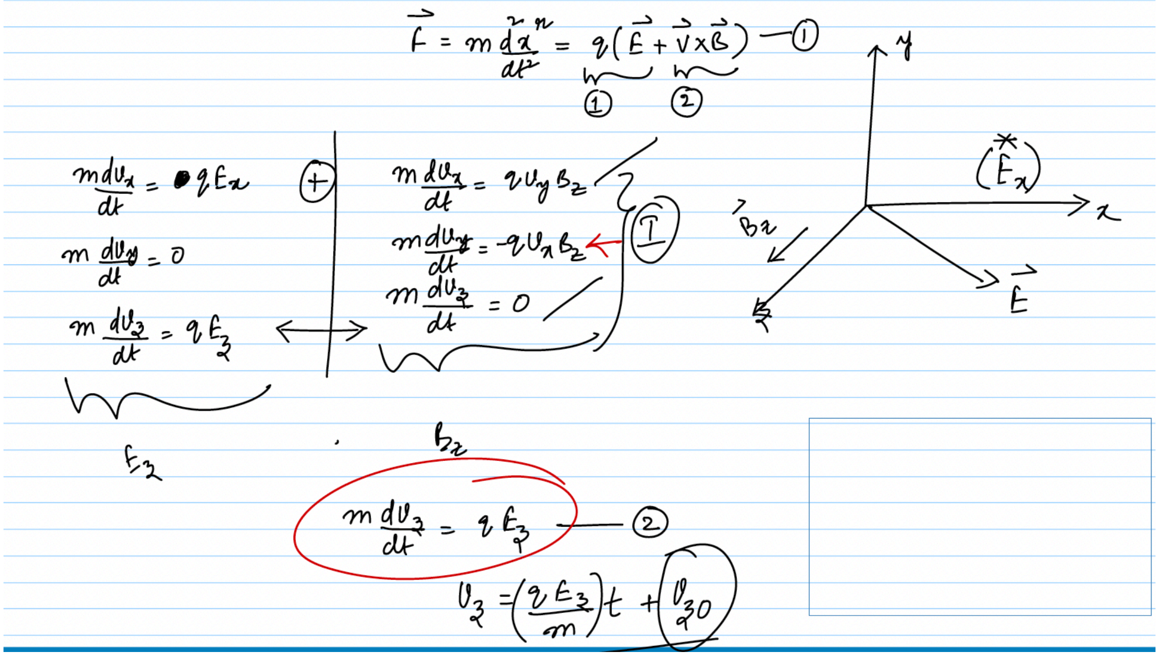
Week – 04

Lecture 17: Motion in Perpendicular Electric and Magnetic fields

Hello dear students. We will continue our discussion on understanding the particle motion in a variety of electric and magnetic fields. So far we have studied the motion of charged particle in the presence of static uniform electric field and magnetic field. So, in today's class we will try to understand how will the particle move in the presence of a perpendicular static uniform electric and magnetic field. So these two fields are perpendicular to each other. So, let us say we consider a coordinate system like this and let us say this is x, y, z take the electric field to be in this direction and the magnetic field to be in this direction.

Now the response of charged particle to a perpendicular component of electric field in the presence of the magnetic field that is the basic outline of the topic. So, the particles velocity and electric field being parallel to each other and at times being perpendicular to the velocity will make how the particle will behave. So, the perpendicular component of electric field will exert a force in the direction of magnetic field. So, if the particle is having some velocity the perpendicular component of the electric field will exert a force in the direction of magnetic field because these two fields are perpendicular to each other.

So then what happens electron will move freely in the direction opposite to the electric field so by nature and ion will move freely in the direction of the electric field. So, the charged particles are now separated because of the electric field. So, they will produce an electric field which will try to cancel the original electric field because the direction of the electric field thus produced by the charge separation will be opposite the original electric field. So, as a result no electric field no resultant electric field will exist and as a result no further forces will act along the electric field. So, let us say we consider the motion of particle in perpendicular electric and magnetic field configuration the equation where we start our discussion will again be the same equation F is equals to Q times E plus V cross V the Lorentz force.



So, let us assume for this discussion let us assume the magnetic field along the z axis and the electric field to be in the along the y axis. We will try to understand how the motion will be different in with respect to the uniform magnetic field. Let us say we have a particle which has no initial velocity along the y direction what have we assume we have assumed the electric field to be along E and magnetic field to be along the z axis. Now since the particle has no initial velocity along the y direction there will not be any force because of the magnetic field why because the velocity of the particle is 0. Because the you see this magnetic field can only act in the presence of a nonzero velocity.

$$v_z = \frac{q}{m} E_z t + v_{z0} \qquad (3)$$

$$\frac{dv_x}{dt} = \frac{q}{m} E_x \pm \frac{q}{m} v_y B \qquad (4)$$

$$E_y = 0$$

$$m\frac{dv_y}{dt} = 0 \quad \Big| \quad m\frac{dv_y}{dt} = -q v_x B_z$$

$$\frac{dv_y}{dt} = -\frac{q}{m} v_x B_z$$

$$\frac{dv_y}{dt} = \mp \Omega_c v_x \qquad (5)$$

So, velocity is 0 that is why there will not be any force. But the moment electric field is there electric field will cause the particle to move along the y direction and the particle will gain some velocity. Now the magnetic force will start to act on the particle because the particle has now a nonzero velocity. Magnetic field will have its effect and deflect the particle sideways that means it will try to move the particle in a circular path. The moment it has some velocity it will try to now we are talking about the y direction velocity so obviously the z and x direction velocities will constitute the circular motion will be responsible for the circular motion.

$$\left.\begin{aligned}
&(a)\quad m\frac{dv_z}{dt} = qE_z \\
&(b)\quad m\frac{dv_x}{dt} = qE_x \pm qB_z v_y \\
&(c)\quad m\frac{dv_y}{dt} = \mp qB_z v_x
\end{aligned}\right\} \quad (I)$$

Time derivative of I(b)

$$\frac{d^2 v_x}{dt^2} = 0 \pm \frac{qB_z}{m}\frac{dv_y}{dt}$$

$$\frac{d^2 v_x}{dt^2} = \pm \omega_c \frac{dv_y}{dt}$$

$$\frac{d^2 v_x}{dt^2} = \pm \omega_c \left(\mp \frac{qB_z}{m}\right) v_x$$

$$\boxed{\frac{d^2 v_x}{dt^2} = -\omega_c^2 v_x}$$

So, velocity the particle the magnetic field will try to move the particle in the side direction. So, the velocity of the particle will now vary and will come back to 0 because of the effect of the magnetic field. It will not allow the particle to accelerate in a linear way along the y direction because of this the particles velocity will again come to 0. But the electric field will again act will again start influencing the particle and under the influence the whole process is again repeated. So, this trajectory of the particle becomes what is called as a cycloid.

So, particles with an initial velocity the only difference if we have a particle which has some initial velocity to begin with is that the trajectory of the particle will not be a cycloid rather it will be a tortuoid. Since the particle already has a velocity along y direction under the influence of magnetic field the particle will of course gyrate and the gyro radius is in the positive y axis get enhanced and the velocity does not change to 0 as it touches the x axis. The particle will move something like this. Now let us try to derive some equations which will represent this movement. Now let us construct the coordinate system.

Let us say we now start with say x, y, z we take the electric field to be in this plane E which is in the x z plane and the magnetic field is along the z direction. So, it becomes B z B of course is along the z direction. So, I have kept B z. So, the electric field is in the x z plane and the magnetic field is along the z direction. Now, how do we start? We start with F is m d square x by dt square is equals to q times E plus v cross B.

Now ideally I can take the position to be r rather than x dealing with the three dimensional coordinates. Now one thing is simple we have dealt with parts of this right hand side individually. For example, when we were discussing the isolated static uniform electric field we considered F is equals to E q and v z was or v along the direction is simply E E by m d. That means we have dealt with the uniform electric field separately and we also dealt with the second part which is the uniform static magnetic field separately. Let us say we have studied this first and then we have studied this part.

I(c)

$$m\frac{d^2 v_y}{dt^2} = \mp q B_z \frac{dv_x}{dt} \qquad \omega_c = \pm \frac{qB}{m}$$

$$\frac{d^2 v_y}{dt^2} = \mp \frac{qB_z}{m}\frac{dv_x}{dt} = \mp \omega_c \frac{dv_x}{dt}$$

$$\frac{d^2 v_y}{dt^2} = \mp \omega_c\left(\frac{qE_x}{m} + \frac{qB_z}{m} v_y\right)$$

$$= \mp\left(\frac{q}{m}\omega_c E_x \pm \omega_c^2 v_y\right)$$

$$\omega_c = \frac{qB}{m}$$

v_x, v_y, v_z

$$\frac{d^2 v_y}{dt^2} = \left(-\frac{\omega_c^2}{B_z}E_x - \omega_c^2 v_y\right)$$

$$\frac{d^2 v_y}{dt^2} = -\omega_c^2\left(\frac{E_x}{B} + v_y\right)$$

Now can I be wrong if I say that the resultant motion of a particle in a situation where we have electric and magnetic fields both of them will be a combined motion of 1 and 2. So, if I take it let us say if I recall both the sets of equations we will write m dv x by dt is 0, m dv y by dt is again 0 and m dv z by dt is q E z. Now similarly when we have only the magnetic field we can write m dv x by dt is q v y v z m dv y by dt is minus q v x v z and m dv z by dt will be 0. Now you have to understand the magnetic field is along the z axis in according to these set of equations and according to these set of equations the electric field is also along the z axis. Now we have equations for isolated electric field, isolated magnetic field.

Now let us combine both of them for a perpendicular E and B field configuration. The z component of the velocity will not be influenced by the magnetic field as it is parallel to the magnetic field. So we can say that these two equations combined will remain the same because dv z by dt is 0 according to the first equation. So we will write q E z let us say we call this equation as 1, this as rho 1 and this as 2. What are we doing? We are trying to combine both the sets of equations for understanding the motion of particle.

What does equation number 2 suggest? If you integrate this equation we will get something like this v z is q E z by m of course m is there. Since we are integrating you will get time on the right hand side plus a constant which you are going to obviously call as the initial velocity. So what does it suggest? It suggests that there will be linear acceleration along the z axis or z component of the electric field. So you have the electric field in the x z plane so you can obviously resolve this electric field along z axis or along x axis. But the point is you will have the velocity v z only influenced by the electric field so which is q by m E z t plus v z 0.

Now let us look at the other components of velocity. Let us say we consider the x component of velocity. Now since the electric field can be resolved to have components along x as well as z direction. So we have taken care of z direction of the electric field and this is what we have got. The z direction of the electric field will accelerate the particle along the z direction but it will be uninfluenced by the magnetic field which is along the same direction.

This is what it is. This is the constant velocity. But when you look at the x direction of the electric field you resolve this E to have a component E x. This is just a component this is a scalar component. If you have an E x then what will happen is this E x will accelerate the particle along the x direction but the velocity will also be affected by the magnetic field. So if you want to combine these two things this will no longer be just 0 you are trying to project this geometry of electric and magnetic field onto this equation.

So this will be q times E x. So now the equations cannot be just combined just like that. So we will write m dV x by dt will be q by m no it is q E x plus minus q by m V y b that is what we have here dV x by dt q by m V y b z and q by m E x. If I have written q by m for the magnetic field part I should remove it here and I should write it like this. Now the plus minus what is the purpose of this plus minus? The one thing is sure the magnetic field will only be able to turn the particle in a particular direction and this direction depends on the charge of the particle.

So as to account this I will keep plus minus so that positive charges will be in the positive direction and negative charges will be exactly in the opposite direction. So what is the purpose of plus minus? It is facilitating the direction of the circular motion. What else is there about it? So similarly we can have the third equation what is it? You see V x we have taken care of V z we have taken care of V y. Is there any component of electric field along the y direction? No it is not there. So E y is of course let us say we write it as equation number 3 and 4.

E y is 0. So obviously m d V y by dt is 0 for the electric field and m d V y by dt is q minus q V x. V x V z. So we have to combine these two things. So we will write d V y by dt is minus q by m V x V z or more conveniently we want to still have the plus minus factor. So this will become minus plus because there is already a minus and q V by m is omega.

See V x d V y by dt. This is one equation. So this is equation number 5. The equations which are representative of charged particles movement are what are those equations? This one and then this one and this one. So can we write them together so that we can easily understand? So the equations which represent the particles motion in the perpendicular E and B field configuration are these. So it will be m d V z by dt is q E z m d V x by dt is q E x plus minus q E x q B z V y and m d V y by dt is minus plus q B z V x.

Let us say we call this as roman one. So we take double derivatives of these equations. We say d square V x by dt square. This square V x by dt square will be you have to charge is a constant, the electric field can be assumed to be a constant. You have to do it by assuming E x to be constant in time.

That means static electric field. So this will be 0 and plus minus m is there q B z by m d V y by dt. So d square V x by dt square is plus minus omega c, omega c is the angular frequency q B by m is omega c, omega c times d V y by dt. Now we will use d V y by dt here from this equation we will substitute on the right hand side. Then we will write d square V x by dt square is plus minus omega c d V y by dt is minus plus q B z by m times V x. So this is multiplication here within the bracket.

So d square V x by dt square minus plus multiplying plus minus will result in a minus and q B by m being omega c omega c square because q B by m is omega c, omega c multiplied by omega c is omega c square times V x. Now we have a very interesting equation d square V x by dt square is minus omega c square V x. Let us see the other equation. What have we done so far? taken a derivative of let us say this is a b c time derivative of time derivative of roman 1 b. Let us say time derivative of roman 1 c will be m d square V y by dt square is equals to minus plus q B z d V x by dt.

This is what we have. So we have to now substitute d V x by dt from roman 1 b. We will do that m d square V y by dt square is equals to minus plus or rather what I will do is I will take this mass on to the other side and I write q B z by m just one step for the sake of clarity. So, we will write d square V y by dt square q B by m is omega c. So, minus plus omega c d V x by dt. So we will bring d V y V x by dt from the equation 1 b.

This is q Ex plus q B z V y q Ex plus q B z V y if I correct there will be a mass in the denominator at q B. Now we have something interesting. So, this will be simplified as minus plus q by m omega c Ex plus plus minus omega c square V y. Why you ask q B by m is omega c omega c is plus minus q B by m q B by m is plus minus omega c taken the same correspondence omega c is already there here. So, it will multiply and I will have omega c square and d square V y by dt square is taking this minus plus inside I will be able to write q by m omega c is q B by m q by m is omega c by B that means I have to write omega c square by B z Ex you have a minus here.

You have a plus minus here multiplying minus plus will give you minus. This will these two things again minus omega c square V y. So, do not be confused with the algebra I will just tell you what I have done so far. I substituted for V x by dV x by dt into this I have taken this omega c inside you see here there is an omega c already existing which gave you omega c square and here omega c is not there but q by m is there. So, q by m can be written as omega c by B from this which gives you omega c square by B and Ex is still there.

About the plus minus and minus plus q B by m so we write omega c is plus minus q B by m. The role of this plus minus is to indicate the direction of the circular motion. I have used plus minus where it is not there let us say q B by m I have replaced for omega c. So, this plus minus has to be brought along. So, that is what I have done and this is a minus plus which is available outside.

So, here q by m omega c when converted to omega c will give you a plus minus minus plus multiplied by plus minus will give you a minus again minus plus multiplied by plus minus will give you a minus. So, we can summarize this equation as d square Vy by dt square as minus omega c square times Ex by Bz plus Vy. You look at this equation this equation needs a lot of understanding. It says the rate of change of velocity d square the second rate the d square Vy by dt square has some angular frequency component. But in addition it has this information about the electric field also.

What it means for you to understand that we have to go back to the preliminary equations. So, along the x direction see along the z direction it is only electric field which is acting and along the y direction it is only the magnetic field which is acting. But along the z direction you have the net acceleration dependent on the magnitude of electric field and the magnetic field. So, that is why in your equation on the right hand side you have both.

You see this factor Ex by B. So, Ex by B is in the units of velocity. The velocity Vy is being added by an additional velocity which seems to be a function of the electric field and the magnetic field. So, we will stop it here today. In the next lecture we will write this equation in a slightly modified form and ultimately we have to seek solution. Until and unless we know what exactly is Vx what exactly is Vy what exactly is Vz we will not be able to tell any inference about the particles motion.

So, that we will do it in the next lecture. Thank you.