Plasma Physics and Applications

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Week – 04

# Lecture 16: Single-Particle Motion Under Uniform Magnetic field - IV

Hello dear students. In this lecture, we are going to continue our discussions on single particle theory of plasma according to which we consider the plasma as a single particle and we subject it to a variety of electromagnetic fields. So far in our discussions, we have seen how a particle will be influenced by an isolated static uniform electric field where we have realized that the particle will be linearly accelerated in the direction of electric field. Then we considered a magnetic field which is static and uniform that means, there are no changes with respect to temperature or space. We have realized the particle will execute a circular motion around an imaginary guiding center . After that we have realized that the electron and ion that means, the negative and positive charges will not rotate in the same direction rather they will exactly move in the opposite circular directions .

So, the equation which governed these motions was derived in the last class. We will try to explore some other possibilities or inferences of these equations. So, if I remember correctly, so we have x square plus y square is equals to p perpendicular square divided by omega square which is equals to rl square . So, this is the equation of a circle .

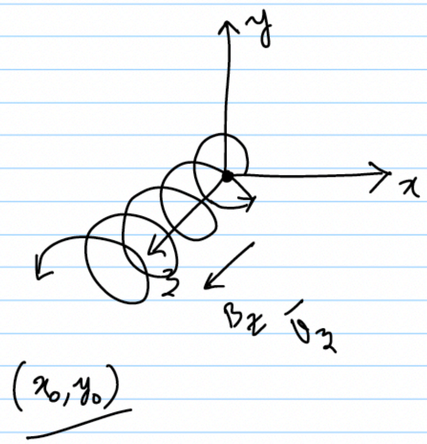
So, the particle is moving in such a direction and from here we also derived what is rl, rl is nothing but the radius of gyration which is m v perpendicular by q and omega the frequency of gyration is qb by m. So far it is straightforward. Now, if we look back at the velocities vx, vy and vz we can refer vx and vy to be constituting v perpendicular they are not equal as such. So, vx square plus vy square is equals to v perpendicular square . So, they are perpendicular to what? They are perpendicular to the applied magnetic field.

$$x^2 + y^2 = \frac{v_\perp^2}{\Omega^2} = r_L^2 \implies r_L = \frac{m v_\perp}{qB} \; ; \; \Omega = \frac{qB}{m}$$

$$v_x \,\&\, v_y = (v_\perp) \; ; \; v_z = v_{\parallel}$$

$$m \frac{dv_z}{dt} = 0$$

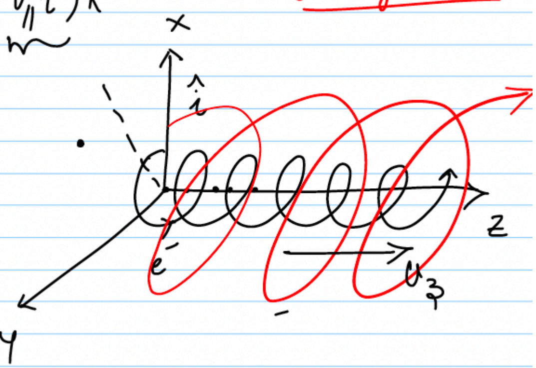
$$v_z = k$$




So, they are the components of velocity actually which are basically influenced by the presence of the magnetic field whereas the z component of the velocity vz can also be referred to as v parallel and this equation m dvz by dt will be 0 which means the z component of the velocity will not change because it is parallel to the external applied magnetic field and by nature magnetic field the force which arises out of the magnetic field will always be perpendicular to the velocity and the magnetic field. So, the guiding center now let us say I was talking about the guiding center. So, we realize that in a coordinate system we take like this in the right handed coordinate system the charged particle will move circularly like this. Now, this imaginary center of this circle is called as the guiding center. So, the particle is gyrating around this imaginary point.

$\rightarrow r_g = x_0\hat{i} + y_0\hat{j} + (z_0 + v_\parallel t)\hat{k}$ ← Movement of guiding center

$r_L = \frac{mv_\perp}{qB}$



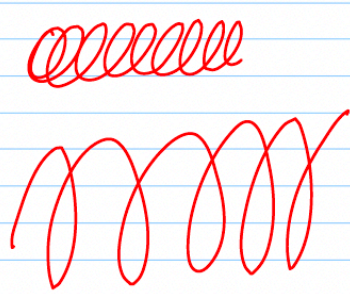

$\rightarrow$ $m\uparrow\ r_L\uparrow\ \Omega\downarrow$

$\otimes\ \vec{B}$ separate charges (or) not

So, let us say we say that the guiding center has a position coordinate x naught y naught. So, here in this case it is origin actually just for the sake of convenience or to represent how the electrons and ions will rotate circularly in the opposite directions we have taken the origin to be the reference, but we can take any point x 0 y 0 to be the guiding center. So, we can say that the particle moves in a helical path. So, it has a velocity vz. So, that is what it is here it is very important let us say.

$q,\ m$

$r_L = \frac{mv_\perp}{qB}$, $\uparrow\Omega = \frac{qB}{m\downarrow}$



$\mu = I \times A$ ($I\uparrow$, $A\uparrow$)

So, the vx and vy component will make the particle to move in the circular path, but as the velocity along the other direction the parallel direction of the magnetic field let us say for instance this is the magnetic field direction let us say bz. So, vz is the velocity which is along this direction the component of velocity. Now, this will remain a constant it will not change that means, it will not change with respect to time. So, there is no acceleration as long as the magnitude of velocity is concerned along this direction, but the vx and vy component they are not constant. So, they are sinusoidal in nature we have seen the form of velocity.

So, these two components will add up and give you this circular motion, but because of the non zero component of velocity along the magnetic field. So, this will not be a circle always at the same point rather the particle will gyrate like this for example. The idea is the electron will gyrate like this the ion will gyrate exactly in the opposite direction that is the basic idea. So, this guiding center x0 y0 what happens to this guiding center as the particle moves the guiding center is also moving. Now, when the velocity vz is a constant we can without denoting all the directions of the coordinate system let us say I have a particle which is moving in this way.

$$\mu = I \times A$$

$$= \frac{q}{T_c} \times \pi r_c^2$$

$$\mu = \frac{q}{2\pi/\omega_c} \times \pi r_c^2$$

$$\omega_c = 2\pi\nu$$

$$\nu = \frac{\omega_c}{2\pi} \rightarrow T = \frac{2\pi}{\omega_c}$$

$$\mu = \frac{q\omega_c}{2\pi} \times \pi \left(\frac{m v_\perp}{qB}\right)^2$$

$$\mu = \frac{q}{2\pi}\frac{qB}{m} \times \pi \left(\frac{m v_\perp}{qB}\right)^2$$

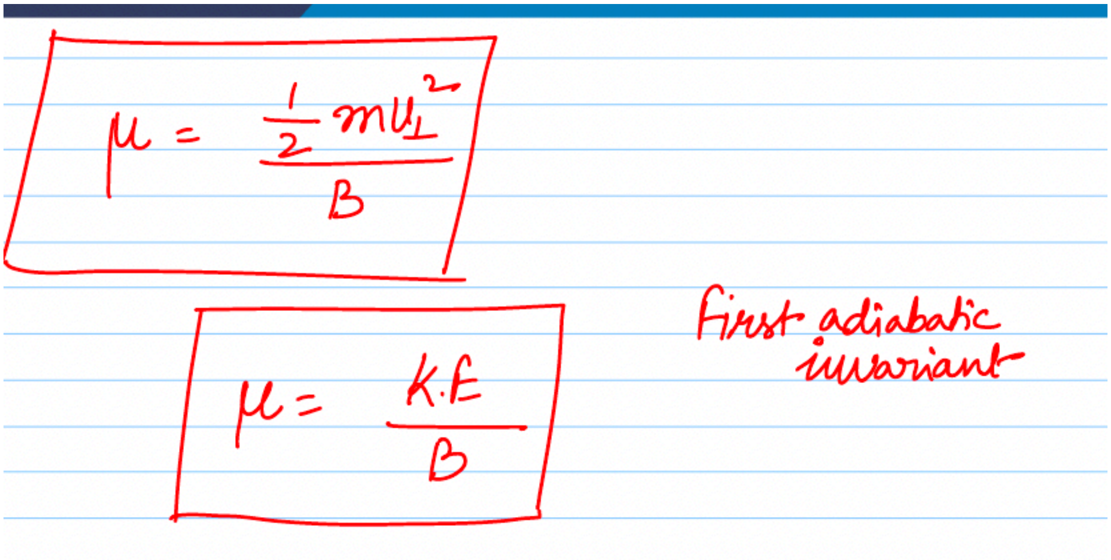
$$\boxed{\mu = \frac{\frac{1}{2} m v_\perp^2}{B}}$$

So, if you look in the three dimensional picture you will realize that this is a circle with a constant radius all the way maybe I am not able to draw it such that it appears to be a constant radius, but in the three dimensional picture. So, if you rotate it like this out of the plane you will realize that the particle is gyrating outwards. Now, these points the loci of all the points let us say this is the guiding center here these are the guiding center. Now, the point that I am trying to make is this circular path is because of vx and vy and as the particle has this velocity this helical path comes out because of the vz. Now, as long as there is no acceleration along vz the distance between these two points or the pitch of this trajectory that is the distance between this and this the pitch will remain a constant.

This is a very important conclusion that we have seen today. What is it? Let us say we relook at it number one as long as the particle is in a uniform static magnetic field it will execute a circular trajectory circular path it will move in a circular path. So, the direction which this executes the circular motion will depend on the charge. That means, both the charges will move exactly in opposite direction to each other. The second thing is the non-zero component of the velocity which is uninfluenced by the magnetic field will make the particle move in a helical path.

So, this helical path is a result of vx, vy and vz. And now if it is moving in a helical path the pitch of this particle will remain a constant as long as vz is a constant. But we have seen in the case of electric field if there is an electric field which is acting on it. We have realized that F is equals to E then electric fields magnitude times E. So, we have seen that the velocity will be linear that means, it will linearly increase.

So, at different instances the particles per unit second the distance travelled by the particle will be increasing in a linear series that is what the point is. Now, we will bring the electric field on to this and then we will see how the pitch will change, but for now this is the most important conclusion the pitch will remain a constant and the particle will execute a helical path because of this. Now, about the x0 y0 which is the guiding center let us say. Using x0 y0 we can write the position of the guiding center because the guiding center is also moving. So, the position of the guiding center can be written as Rg is x0 along i cap plus y0 along j cap plus you see the particle is the linear motion of the particle is not in the direction of x nor in the direction of y it is in the direction of z.




So, that means, x0 will remain as it is the point on the x coordinate y coordinate will also remain a constant, but it will the particle is moving along the z direction. So, the guiding centers position on the z axis will change with respect to time. So, it will look something like this. So, this is z0 plus v parallel t times k cap. I hope you are able to understand this.

Let us say we draw it for an easier understanding. So, we say we want to have a right handed coordinate system that we are familiar with. So, this becomes x cap x y and this is z. So, this is x this is y x y and so, this is the z axis. Now the angle phi will always be taken with respect to the positive x axis.

So, phi. Now the electron like I said before electron will rotate such that angle phi will increase. So, electrons trajectory will be like this. Now you see you have to imagine this in the three dimensional plane that means, the circle that I have drawn is actually in the x y plane. So, if you are looking from this direction the z direction you will see a circle in the only in the x y plane. So, x minus x y minus y the whole surface that you get in this x y plane you can see that you have drawn a circle on the x y plane.

So, if you see if this is the plane if you are looking directly perpendicular to this x y plane you will see that I have drawn a circle that is it. What is the direction of the circle? This circle is in the counter clockwise direction drawn in the moment is in the counter clockwise direction along this circle. So, as long as there is no z component of the velocity. Now the moment it has a z component of the velocity the particle because the particle tries to come back to the same point where it started, but the velocity along the z direction will not allow the particle to come back rather it will move like this. So, the particle will move will continue to move this way.

So, you have to forgive the way I have drawn it, but the message is conveyed. So, this is the v z and the circular motion like I said many number of times is because of square root of v x square plus v y square. Now let us talk about the guiding center. Guiding center is of course, this at this point this is the guiding center. So, here it is a guiding center after the point particle moves here the guiding center is also moving along with it.

So, is the guiding center changing the x coordinate and y coordinate? No it is not it is along the same axis. Let us say if you want to have a better feeling of this maybe I can start the particle to be from some other point in this plane let us say this point this is x 0 y 0 let us say. And at this point if I draw a line the particle position x naught and y naught will remain a constant, but it will its position along the z axis along the z direction will change. How is it changing? Because it is moving with a constant velocity v parallel the position at any point s is equals to u t distance is equals to v parallel times t. So, that so how far has it deviated from the z naught direction? So, this is z naught for this case this is z naught how far is has it deviated away from z naught? The distance is the initial distance plus the amount of distance that the particle travel as a result of the velocity.

So, at this point if you take t is equals to t t and at this point if you take t is equals to 0. So, the distance that is travelled between these two points s is equals to u t or v t whatever. Now, since there is already z naught the initial coordinate is there. So, there at this instant t what is the distance away from that initial point that will be z naught plus u t that is what I have done here.

I hope you have followed it. It is very simple from simple plus two level kinematics you can understand all these things. The message that I wanted to convey is rather more profound than this. Let us try to draw the trajectory. Now, you try to do many examples based on this. You try to construct right handed coordinate system in different ways and try to see how you can draw the trajectory of a particle within that particular coordinate system.

Now, this is for an electron. So, this is for an electron. Electron will move such that it is moving away from the positive x axis. You remember this. Now, let us say the radius we will come back to this R L is m v perpendicular by q B. That means the radius will be small for lighter particles when the mass is very small.

That is what we have seen here m v perpendicular by q B. But when you consider ions the radius will be larger. So, as m the mass of the particle increases R L will increase. How about omega? Omega is q B by m.

Omega will decrease. This is a beautiful relation which takes care of many things in plasma physics. We will explore this at multiple stages in our discussions. So, the mass is inversely proportional to the frequency but directly proportional to the radius. Now, if I ask you to construct the trajectory of the particle in for an ion.

Now, it will be exactly opposite. So, it will be opposite but will it move in the same direction in the positive z axis along the positive z axis or not. So, if I write the trajectory this figure will become very messy, but the particle will rotate like this in. So, maybe I can draw it with a different color. It will be easy for distinguishing. So, particle will move in larger radius, but still helical path.

Now, the question is you try to think about this question I will answer the question anyway. Will the magnetic field P be able to separate charges or not? This is a very important question that I want you to think and find out the answer before I tell you the answer. Now, coming back to this gyration or what do you say the guiding center. So, the motion what is this equation R g giving you? It is giving you the movement of guiding center.

This is the movement of the guiding center. Massage particle will move in a helical path in the presence of external magnetic field. So, you always keep that in mind how the particles trajectory can be drawn within a given coordinate system. One more very important conclusion is so, Q is there the mass m is there Q is just the charge. So, we take both the charges at least in the magnitudes to be the same electron and a singly ionized species. So, the charge is same polarity is different, but charge is same.

Mass is for the electron it is very small for the ion it is very large. So, R L is m v perpendicular by q B omega is q B by m. So, for an electron the radius would be smaller like this and frequency would be larger. Smaller mass means larger frequency that means, electron of course, it moves in a small circles, but it will move very fast the frequency will be very large. But for an ion the radius will be large and frequency would be very small.

The number of revolutions that the ion can complete in a given amount of time will be small. So, a direct consequence of these things can be understood by bringing in what is called as the magnetic moment of the particle. What is magnetic moment? Magnetic moment is the it is measured as let us say current times the area or magnetic moment in the context of a bar magnet can be written as a measure of the magnetic pole strength and the distance between the poles. So, that is the moment. So, it tells you how strong the magnetic field is that is generated out of that magnet.

But in the case of current driven magnetic fields where a current carrying wire is creating a magnetic field, the magnetic moment is the amount of current multiplied by the enclosed area. So, we can write it like this. So, magnetic moment is i times magnetic moment mu is i times A. If q is the charge and tau c is the characteristic time for completing this circular motion and if R L is the radius we write pi R L square.

Current is charge per unit time. Where is the measure of time? Time is this amount of time that it requires to complete that circular motion times within that circular motion it covers a closed loop of area pi R L square. So, we can write the magnetic moment as mu is equals to q by 2 pi by omega c times pi R square. Omega c is 2 pi mu, mu is the frequency omega is the angular frequency, mu is omega c by 2 pi inverse of frequency is the time that is associated with that motion. So, time is 2 pi by omega c this is what I have written here. So, mu is equals to q omega c by 2 pi into pi into m v perpendicular by q b whole square.

magnetic moment mu is equals to q by 2 pi omega c is q b by m times pi m v perpendicular by q b square. So, after cancelling out all that we will have half 1 m is remaining inside m v perpendicular square by b is remaining half m v perpendicular square divided by b. What is this? This is the magnetic moment half m v perpendicular square by or we can write magnetic moment as the kinetic energy the perpendicular kinetic energy by magnetic field. So, this magnetic moment decides what is called as the first adiabatic invariant. So, whether the particle is moving with higher angular frequency, but with smaller radius of gyration or if the particle is moving with higher radius and smaller angular frequency the magnetic moment generated will be like this.

So, how the information about the magnetic moment can be useful in proving the first adiabatic invariance or what are the consequences of this first adiabatic invariant? We will discuss that all of that in the next lecture. Thank you.