Plasma Physics and Applications

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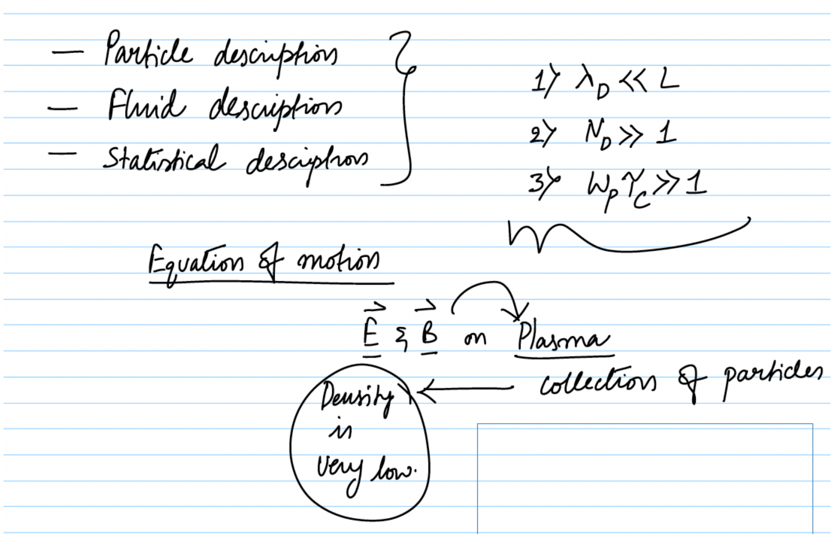
Week – 03

Lecture 12: Single particle motion in Uniform Electric Field

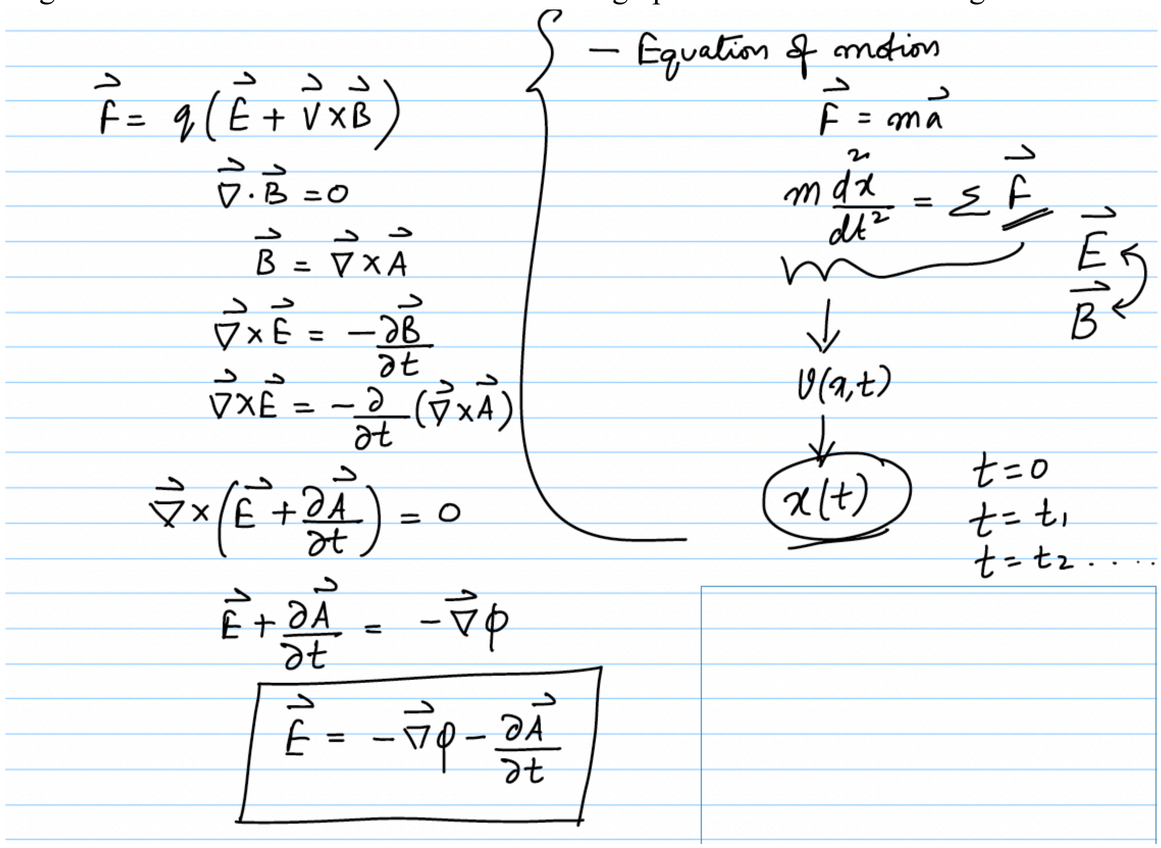
Hello dear students. In today's class we will try to understand the single particle theory of plasma. So far in our discussions we have understood what is plasma? How is plasma different from other states of matter? What are the defining characteristics of plasma? We have understood what is plasma criteria which enables us to differentiate between an ionized gas and an actual plasma. We have understood what is plasma frequency? We realized plasma frequency is nothing but plasmas attempt to regain or to retain charge neutrality. As an example of corrective behavior, we have seen what is Debye shielding and we have derived the form of potential of Debye's electric field. We have realized the mathematical form of the Debye's potential and how this potential ceases to exist beyond the Debye's length.

And after that we have discussed how plasma can be treated as a fluid or a particle or a statistical entity. Plasma description can be made in three different ways. One plasma can be treated as a collection of particles which we call as the particle description of plasma. We can also treat plasma as a fluid in which we call this description as the fluid description where we treat the plasma as just a fluid, as a conducting fluid or we can also use statistical methods to describe the plasma.



Now depending on the situation we have to make an appropriate choice of this particular description. In addition to this, it is very important to recollect the plasma criteria here. The plasma criteria is given by these three conditions. Number one is lambda d should be much less than capital L where lambda d is the Debye's length and capital L is the length scale of the plasma system and number two. What it means? The condition number one means the Debye's length should be within the length scale of the plasma system.

It is reasonable to assume that. We cannot extend the Debye's length beyond the length scale of the plasma system. The second condition is Nd should be much greater than 1. What this means is the total number of particles in the Debye's sphere should be much larger than 1. And the third condition is omega p tau c should be much greater than 1.



What is omega p? Tau c is the plasma frequency, tau c is the collision time within which collisions will happen. It can be called as the relaxation time also. These three are the plasma criteria. If an ionized gas obeys all these three conditions, you can call it as plasma otherwise no. Going back to the particle description of plasma, when you have very small number of particles per unit volume, let us say electrons or ions charged particles per unit volume or when you simply say that the density of the charges is very low.

So, we can then take the single particle trajectories are relevant. We can say that the particle trajectories can be considered. The collective behaviour of the plasma becomes not so important. Plasma sometimes behaves like fluid and sometimes behaves just like a collection of particles. In order to understand how to describe plasma in various situations, it is very important for us to understand how single particle will behave in the presence of different types of electric and magnetic fields.

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) + mR$$

Electromagnetic forces

(mass) gravitational etc

Single Particle Theory

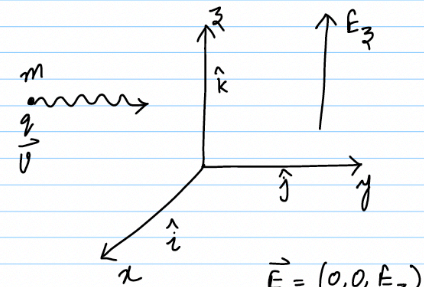
$v \ll c \implies$ Non-relativistic case

$$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

We have realized that plasma is an ionized gas which can be influenced by electromagnetic fields. Now, this ionized gas may sometimes have very less number of particles or very rare verified ionized gas. So, in that situation, we can treat plasma as a collection of particles and we should know how this collection of particles will behave in the presence of electric or magnetic fields or a combination of both. So, if you want to understand anything what you require is an equation of motion. What are you trying to understand? You are trying to understand what will be the influence of electric field and magnetic field on plasma.

Uniform Static Electric field

$$\frac{\partial E}{\partial z} = 0 \; ; \; \frac{\partial E}{\partial t} = 0 \; , \; \vec{B} = 0 \quad \text{(I)}$$



$$\vec{E} = (0, 0, E_z)$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{F} = \vec{E} q \quad \text{— (II)}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

Using $\vec{v}$ in (II)

$$m \frac{d\vec{v}}{dt} = \vec{E} q \quad \text{— (1)}$$

$$m \frac{d}{dt}(v_x \hat{i} + v_y \hat{j} + v_z \hat{k}) = q E_z \quad \text{— (2)}$$

Now what is plasma? In this case, it is a collection of particles and this collection can be such that the number of particles per unit volume is very high or low. But we have taken a special case in which this collection of particles is such that the density of the charged particles is very low. When the density of the charged particles is very low, we will try to understand how the electric field and magnetic field will act on the plasma or how plasma will behave in the presence of this electric and magnetic field. So, depending on the density of the charged particles, plasma can behave as a fluid that means collectively as a fluid or it can be seen as a collection of individual particles. So, we are referring to the second case where we are treating it to be an individual collection of particles and how the particles will behave.

$$m\frac{dv_x}{dt} = 0 \quad \Rightarrow v_x = k$$

$$m\frac{dv_y}{dt} = 0 \quad \Rightarrow v_y = k.$$

$$m\frac{dv_z}{dt} = qE_z \quad \Rightarrow v_z = \frac{q}{m}E_z t + k$$

(3)

$t = 0 \; ; \; v_z = v_{z0}$

Using

$$v_{z0} = \frac{q}{m}E_z(0) + k$$

$$k = v_{z0}$$

$$\boxed{v_z = \frac{q}{m}E_z t + v_{z0}}$$

$t \uparrow \quad v_z \uparrow$

$\oplus \quad q = + \quad v_z = \frac{q}{m}E_z t$

$q = -q \quad \ominus \quad v_z = -\frac{q}{m}E_z t$

So, in dense plasma when the density is very high, particles couple due to the electromagnetic forces and there will be effective interaction with the electromagnetic fields. When the plasma is very dense, these individual particles will couple by the means of electromagnetic forces and this will combinedly interact with the external electromagnetic fields. But in the case of highly rarefied plasmas where the density is very less, particles do not produce sufficient electric and magnetic fields by themselves to be influenced by the external electric fields particles in total. So, now we are going to talk about individual particles or single particles. So, that is why we are going to call this as a single particle theory and we are going to see how the individual particle will behave in the presence of electric and magnetic fields.

So, like I said you want to understand how this particle will behave in the presence of electric and magnetic fields. What do you mean by behave? We want to understand in the perspective of science that how this particle will move, whether the particle will continue to move in the same direction with the same velocity or not, whether the particle will modify its trajectory due to the application of electric field or magnetic field or a combination of both or different types of electric and magnetic fields. So, this is all what you mean by behaviour of the particle. Now in physics this behaviour or this term can be addressed by what is called as an equation of motion. An equation of motion tells you everything about the particles movement, particles velocity, particles acceleration and how the particle will be moving as a function of time.

So, this equation of motion is nothing but F is equals to mass times acceleration or you write M d square x by dt square is equals to let us say sum of forces. So, you see this is the second order differential equation. Now if you know the analytical form of forces that will appear on the right hand side and if you are able to solve this equation or you integrate this equation once to get the velocity as a function of position and time, you integrate it twice, you will get position as a function of time. So, you are getting position as a function of time. So at any value of time let us say t is equals to 0, t is equals to t1, t is equals to t2 and so on.

Once you know this analytical expression of x of t, you will be able to find the position at any subsequent values of time. So, to be able to deduce this information, all this information is going to be referred as the behaviour of plasma or particle. Now in this case, we are considering the position x of a single particle and on the right hand side the force can be basically due to the electric field or it can be due to the magnetic field alone or both of them. That is one possibility, you can have an electric field to begin with, you can have a magnetic field to begin with, you can have a combination E and B and then you can have an electric field which is changing with respect to time. You see it is a different story.

You can have an electric field which is changing with respect to space. Similarly, you can have a magnetic field which is changing with respect to time or space and so on. So, the combinations can be many or the configuration of electric and magnetic fields can be many. Now in this class or the following lectures, we are going to understand how the plasma as a particle is going to interact with these different configurations of electric and magnetic fields. Now the plasma is going to be treated as a particle and this particle can be an electron or an ion.

Now let us say to begin with, we can write the force F when you have electric and magnetic field, you can take the general expression of Lorentz force which is Q times E plus V cross B, where E is the electric field, V is the velocity of the particle, B is the magnetic field. Now in electrodynamics, it is very convenient to write electric field and magnetic field in terms of scalar and vector potentials. So, if you recall your earlier courses on electrodynamics or maybe Maxwell equations directly, we know that del dot B is equal to 0. So the magnetic field B can be written as del cross A. What is A? A is the vector potential.

We know that del cross E is minus dou B by dou t. So, we can write del cross E is equal to minus dou by dou t of del cross A. So we can write del cross E plus dou A by dou t is equal to 0. So, E plus dou A by dou t is minus del phi. Electric field can always be written as the negative gradient of potential.

So, electric field E can be written as minus del phi minus dou A by dou t. We are just getting our tools which will be used subsequently in our discussions. So, this is the form of electric field that can be used. Generalized expression for the force, the particle can experience F is equal to Q times E plus V cross B.

We add additional term MR. So this one, the first term on the right hand side is represent all the forces which can come into existence or which can act on the particle which are electrodynamic in nature or electromagnetic forces. This one are the forces by the virtue of the particles mass. Basically mass is the factor here such as the gravitational force etc. So, is there any other type of force which can be thought of? Maybe at this moment no. So we have all the different types of forces which can influence the particles trajectory.

Under all these assumptions, we will now treat plasma as a single particle and we will impose various different configurations of electric as well as magnetic fields. This is a very important topic in plasma physics. Before we go to the advanced topics, it is very important for us to understand the single particle theory of plasma. This is the first step in understanding the collective behavior of plasma. All of this becomes valid only when the velocity is much less than C or velocity is less than C.

We are referring to the non-relativistic case. We are not going to look at the limits when the velocity of the particle is nearing the velocity of the light because you have mass which is m0 by square root of 1 minus V square by C square. What have we done so far? We have realized plasma descriptions can be in three different ways and we have picked up the first case in which plasma is very rarefied. That means plasma density is very low where the particles cannot come together and create electric and magnetic fields, collective electric and magnetic fields which can then go on and interact with the external electric and magnetic fields. So each particle is on its own and this particle trajectory subjected to different types of electric and magnetic fields is now the topic of discussion.

This discussion is called as the single particle theory of plasma. Now let us take the first configuration of electric field which is a uniform static electric field. Let us consider our coordinate system like this. Let us consider an electric field in the positive z direction like this and let us imagine a particle with a charge q and a mass m having a velocity and moving in this direction. It has a charge q, mass m and a velocity V.

The electric field E has only one component, we have 0, 0 and E z. By the means of saying uniform, we are saying that the electric field is not changing its magnitude as a function of the distance. It is constant along a particular direction and by the means of static, we say that dou E by dou t is 0. These two conditions are mathematical representation of the fundamental nature of the electric field that we have taken. And in this picture, we do not consider the magnetic field to be present.

That means B is equal to 0. Now if you bring the Lorentz force, we start from the Lorentz force because it is a second order equation or equation of motion. Only our chance is to solve that equation and get position as a function of time by what is called as the behaviour of the particle. So, F becomes q times E plus V cross B. Since the magnetic field is 0, F is simply E q. Now let us consider the velocity of the particle to be three dimension.

Let us say V is V x i cap plus V y j cap plus V z k cap. If you substitute this, there is velocity anyway. Let us say we call this set of conditions as 1. Let us say this is roman 2.

Now using velocity in 2. Where is velocity in V? You do not see a velocity in V. But when you write force as m dV by dt, we have E q or m d by dt of V x i cap plus V y j cap plus V z k cap is equals to q times E z. It is a component on the right hand side. E z is a component. Let us say we call this equation as now 1, this as 2.

Now comparing the components on either sides of the equality, we will be able to write m dV x by dt is 0, m dV y by dt is again 0, m dV z by dt is q E z. Just compare. So, E z, this is probably in the z cap direction. So it is i cap, j cap, k cap direction. So that means what is the meaning of these three equations? Let us say we call this set of equations as 3.

m dV x by dt is equal to 0, m dV y by dt is equal to 0, m dV z by dt is equal to q times E z. So straightforward you can say that along the x direction velocity will be a constant, along the y direction velocity would be a constant. That means there is no acceleration along the x and y direction. So if you integrate this equation V z, we will be able to write V z is equal to q by m E z t plus a constant. So let us say at t is equal to 0, we assume the velocity V z to be V z 0 using this condition using t is equal to 0, we will write V z 0 is equal to q by m E z times 0 plus the constant k.

So that means k is equal to V z 0. So using k is equal to V z 0, we will write V z is equal to q by m E z t plus Z 0. Due to the presence of electric field along the z direction, the velocity of the particle along the z direction is influenced by this. So you have this initial velocity before it sees the electric field or before it senses the presence of electric field and then you have a linear acceleration along the z direction. Now more importantly we have to understand that velocity of the particles along x and y directions remains constant, it will not change. It is only along the z direction, velocity changes linearly with respect to time.

So as you increase time, the velocity is changing linearly by a factor q by m E z. As the time progresses, the velocity will increase. So we can say that when time is increasing, the V z is also increasing, a linear increase with respect to time. Now let us talk about the polarity of the particles. Let us say we have a positive charged particle and a negative charged particle.

So if you put q is equal to positive, what will happen? We will get V z is q by m E z t. Let us say for instance we take that velocity to begin with positive and q is equal to negative. If it is negative, so V z is equal to minus q by m where q is equal to minus q for an electron E z t. So what do you see here? You see that for positive charged particles, its positive charged particles velocity along z is increasing linearly in the positive z direction. But for negative charges, the velocity is increasing linearly but exactly in the opposite direction.

So it is safe for us to say that the positive charged particles will move along E z and negative charged particles will move opposite to E z. So if you subject plasma which is a collection of both positive and negative charges, what would happen? If you do that then you realize that all the positive charges will move in one direction and all the negative charges will move in another direction. That means you are trying to separate charges. The separation of charges will lead to the formation of electric field. So we will try to understand more consequences of this result in the next lecture. Thank you.