Course Name: Newtonian Mechanics With Examples
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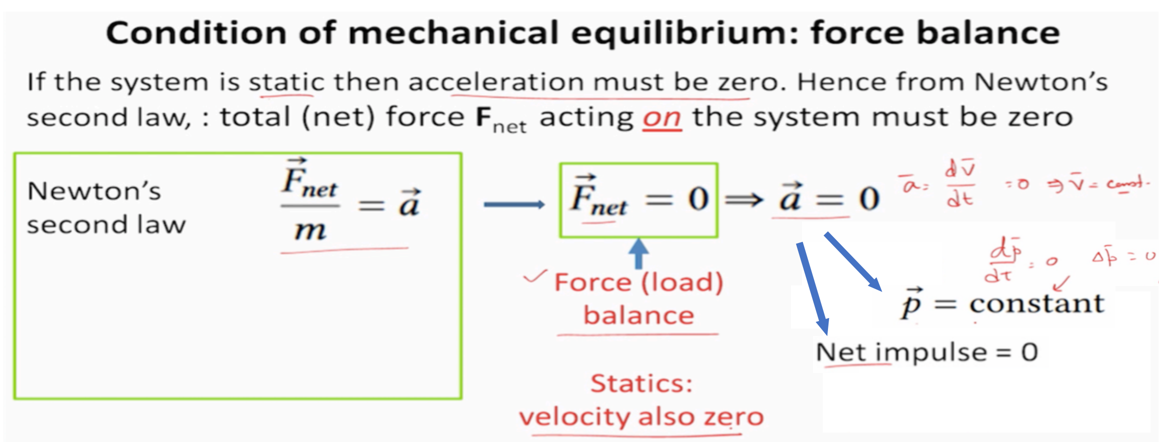
Week 03
Lecture - 09

Welcome to the third week of this course on Newtonian mechanics with examples. The topic for this week and the next week is going to be the statics or the problems we will consider. when the system is under the condition of mechanical equilibrium. So here is a quick summary of our plan for this and the next week. So first, we will start with a brief recap of what the conditions are for mechanical equilibrium. So we have discussed Newton's laws of motion in the previous week.

So we are going to use that to derive the conditions for mechanical equilibrium. Then the plan is to introduce to you different ways to solve problems under static conditions. So the first way that we are going to use is this technique of force and torque balance and using the free body diagram that we learned in the previous week. Then we will also discuss two more techniques.

one, which is called the principle of virtual work and the second is how to use potential energy diagram to classify mechanical equilibrium situations into stable, unstable, etc. So, as we said many times before in this course, our goal is to learn something about real life starting with the application of the fundamental principles of physics. So we are going to take several worked-out examples and Our emphasis will be on real-life situations so that, using he basic physics we can learn something about our day to day life. So the condition of mechanical equilibrium is obviously the force balance. So what we are having in mind is a situation in which our system is static, which means it is motionless.



So that means the first thing we need is that the acceleration must be 0. Now here I have written Newton's second law. So recall this equation. On the right-hand side, it says that the acceleration produced in a system is generated by the total force acting on the system divided by the mass of the system. So now, if the right-hand side is 0, it follows the second law that the total force acting on the system must be 0.

The net or the total force acting on the system, must be zero. So this is called the condition for force balance. In engineering literature, it is sometimes called the load balance. So load is the same as force. So if the total force is 0, then the acceleration is 0.

Now, in addition, if the acceleration is 0, then The solution is that, so this dv, dt represents the acceleration. So this implies that the velocity is in general constant. Now we are in this week and in the next week we will specialize in some particular situation where the velocity is also 0. So that the object that we are going to analyze in our system is completely motionless. Now let us look at it.

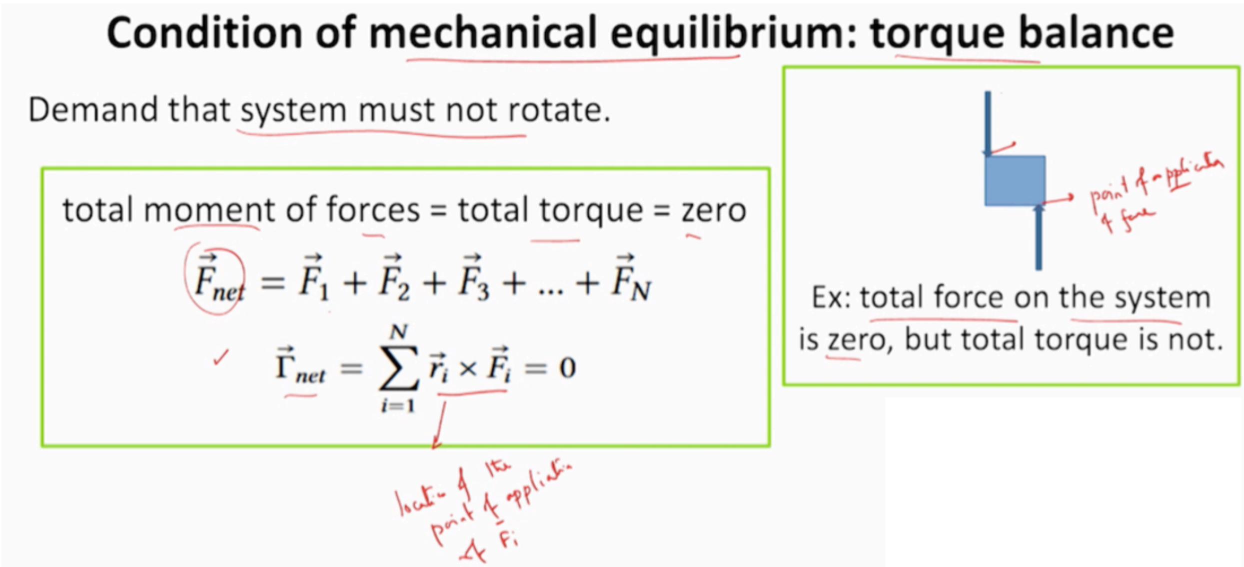
We know that in the previous week we wrote Newton's law in a slightly different way. So we say the more general version of Newton's law is that The net force represents the rate of change of momentum P with respect to time. Now again if the system is static that means that, I mean if you say that the total force is 0 then It follows that the rate of change of momentum is equal to 0. So the rate of change of momentum is equal to 0, and hence it follows that the total momentum of the system must be constant. Now again, you know that there are examples, for example if two particles are moving and colliding with each other, like a carom hitting a carom coin hitting a striker, the total momentum is constant, which does not mean that they are motionless.

So we are demanding, in addition to the fact that the acceleration should be 0 that the velocity should also be 0. So the case where the total momentum is conserved but these objects are moving, So in this case, we will consider it later in the course. Now I also want to point out that if we write down Newton's law in the third form, which is the total force which is acting on some time duration delta t, Now if you multiply the product of the total force times, this time interval represents the change of momentum. And if the total force is 0, then it follows that delta p is 0 which means that again, p is equal to a constant. And since delta p represents the net impulse of the system, so it follows that in the case of the force, if the system is in the force balance condition then the net impulse must be 0.

So I am sort of trying to give you a way of deliberately writing the same equation in multiple ways. So that you can look at the same condition in multiple ways. So this is always a useful strategy to develop physical intuition about a new situation. Now look at this example here. So this is a block, some system that is represented by a block and There is a pair of forces acting on this block, one pair from upwards and the other pair from downwards, as shown in the direction here.

Now, as we remember, recall what we said. that the force is represented by this arrow and The length of the arrow must represent the force. So even though nothing is mentioned in this diagram, we can easily see that the force is acting; the two forces are equal and opposite because the lengths are equal and the tip of the arrow represents the point of application of the forces. So looking at this diagram, even though nothing is given, no further information is given. If you draw it to this scale, then you can get a lot of information from this picture.

So this is a situation where the total force on the system evidently is 0. However, if you sort of try to, I mean, you can easily, you know from experience that this system even though this total force is 0, This system is still not motionless because it can rotate. For example, if I take this, let us say this pointer is my object and then this is situation where a force is acting from this side, another force is acting from this side, so you can easily see that it is going to rotate. So in the case of mechanical equilibrium, If we demand the system to be completely motionless, that means we demand that the system not rotate as well. So this brings us to the fact that we also demand the torque balance.

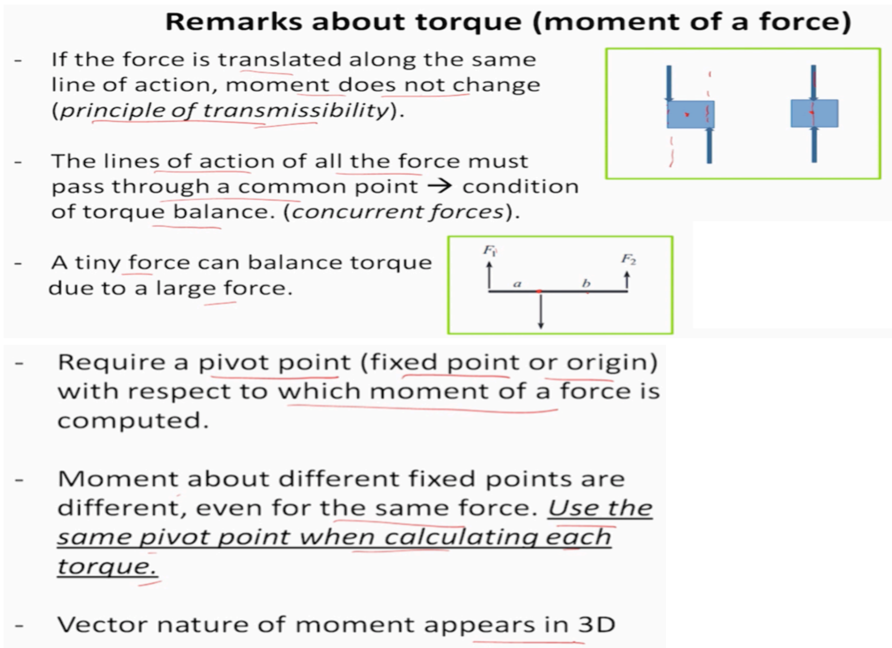


So the total moment of force, which is also the same as the total torque, must be 0. So in the second line, it represents the total moment of force. Suppose there are several forces acting on these objects, such as the two forces acting on them in this picture. So then the F1, F2, F3, and so on till Fn, so N forces are acting. Then the resultant force is given by this F net and the resultant is now each force you can generate at moment.

which is represented by this term, ri cross Fi, where ri is the location of the point of application of Fi. Then this represents the torque generated by the force Fi and if you add up all the torques or all the moments, then this resultant moment must be 0. So this is the second conditions to require for mechanical equilibrium. Now I am going to make several comments about torque because these are the things I find that students sometimes find it confusing to understand the condition of torque balance. You can learn from experience perhaps it is easy for to understand the condition of force balance.

But it may be difficult to understand the condition of the torque balance. So the first point is that in order to compute torque or moment of a force, first thing you require is a pivot point. A point about which there is a fixed point or you may call that the origin of your coordinate system, or reference frame from which you are going to compute the moment. So recall that this is ri cross Fi is the moment, so for to calculate ri cross Fi you need. It is not sufficient to know only the force; you also need to know the location of the point of application.

Which means that you need to and this location is arbitrary. You have to specify with respect to which point you are defining this vector, ri. The second point is that it follows from this first point that if you take different fixed points as your location for calculating moment, the origin for calculating the moment or the pivot point, you will get different values of the moment even for the same force. So this means that in a problem, if there are several forces, then you must use the same pivot point. When calculating each torque, this is very important.



The third point is that the moment is a vector and this vector nature is not so clear if you consider a planar force; all the force acting on the same plane, but it is really evident in three dimension. Then there is some useful property of the force that if the force is translated along the same line of action, The moment around the same fixed point does not change. So this is something that will be clear as we work through the example. This is called the principle of transmissibility, and this is a very useful trick we can use to solve problems. Now, I mention one thing: the fact that if in the mechanical equilibrium condition, that is the total torque is 0, the condition of torque balance will follow.

If we demand that the total torque be 0, then the lines of action of all the forces must pass through a common point. So this is a kind of illustration, and I am going to illustrate it through this picture. So again, if you take the same picture, you see that in the left-hand side picture, So this torque generated by each of the force, suppose I take the center of this box as my pivot point and I can compute the torque generated by each of the force and in the left-hand side picture, clearly the total torque is not 0 and on the right-hand side picture, so only difference between the left-hand side and the right-hand side picture is that the lines of action of the forces, the two
Force do not pass through a same line, they are not, the line of action are not parallel, they are not intersecting each other and hence the total torque is not 0. In this case, on the right-hand side of the picture, the both the lines of action passes through the center of the object and it is clear from the experience that, in this case, the total torque produced by the top force and the bottom force must be equal and opposite, so they will cancel each other out. So, this is a geometrical property of the definition of torque.

So I am not going to derive it, but it is very clear from experience, so I just want to highlight this. Another point is that because the definition of torque requires the not only the value of force, But also the distance of the line of action from the pivot point, So we can change torque not only by changing the magnitude of force but also by changing the distance. So this is what we sort of use in day to day life when we close a door, So you know from experience that if you push the door, suppose this is an object and you are computing the torque about this point and this is a fixed point, Now if you apply a force at this point. then the line sub, the distance between the line of action and this fixed point is long big and If you apply the force here, you are reducing the distance. So, in order to move the object by the same amount, you need to apply more force.

So this is also illustrated in this picture. So, the consequence is that, in order to balance torque, You do not; it is a completely separate condition from the balancing force. Even if you have a small force versus a large force, you cannot achieve a force balance. but you can achieve torque balance. So, the torque balance and the force balance are different conditions.

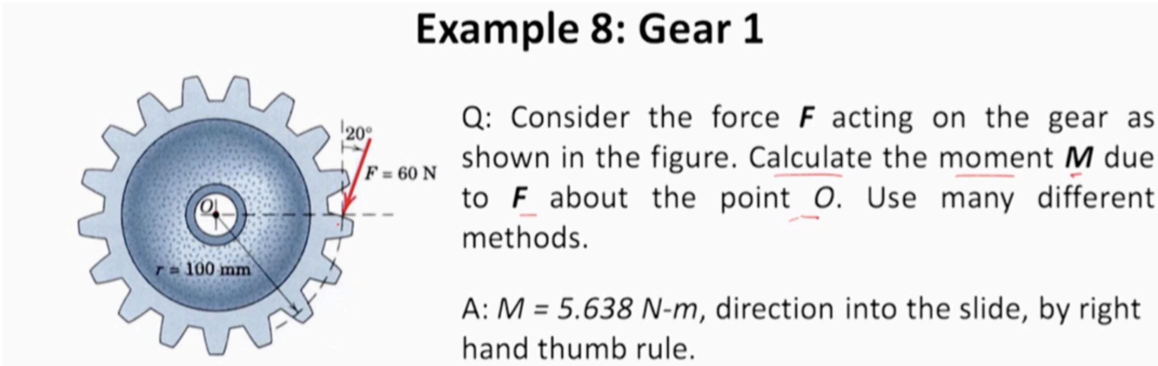
So, for example, in this picture, this is my pivot point and then we are applying two forces. Now, these two forces can be assumed if I calculate the torque. So we can see that the magnitude of F1 and F2 need not be same and in fact, by adjusting the value of A and B, We can have different amounts of force. Even a tiny force can balance the torque due to a large force. So now, we are going to illustrate these points through examples worked out.

Now, before I go to the examples, let me state the strategy that we are going to use. So, we are going to use what I am going to call a box model for any mechanics problem. So, it is not required to go through the details; it will be clear through the examples. But the first step is that we choose the system. The next step is that we represent the system by a box or schematically by a usually something-by-box and then everything that is not part of the system.

We are going to call it surrounding. Now, system and surroundings exert force on each other, So we identify these interactions, and then we draw a free body diagram and solve the problem. So this is going to be our overall strategy for solving problems using the free body diagram approach and the force band torque balance approach. But first, before going to take examples of situations where we have force and torque balance, We first take a few kinds of simple problems about how to calculate moments. So, this is our first type of problems we are going to solve to calculate moments and to sort of illustrate the points that I made.

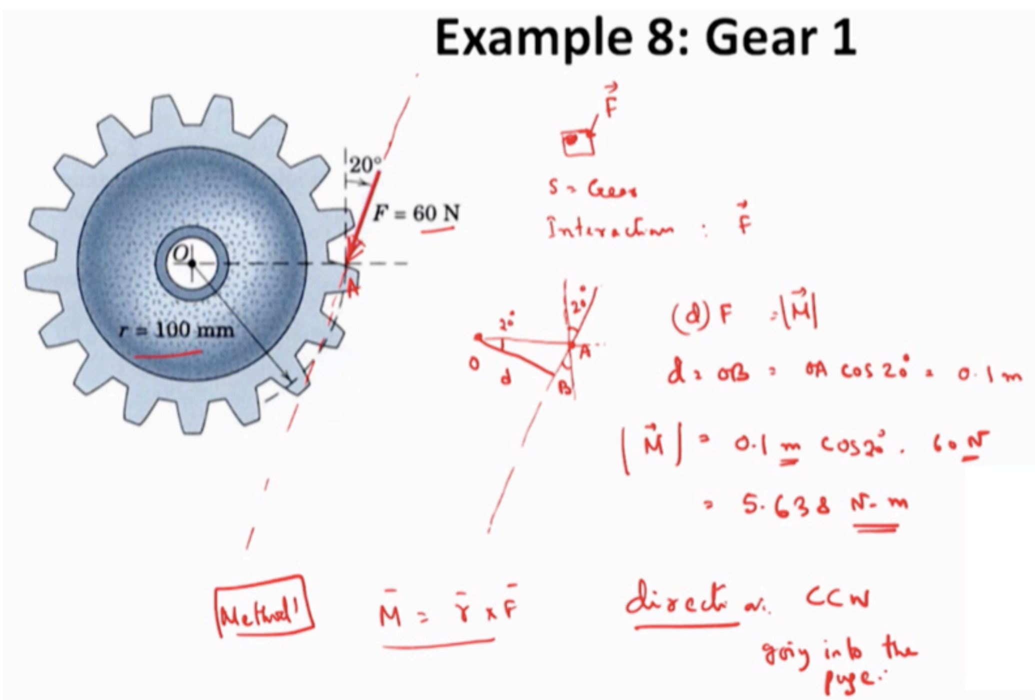
So, this is our first example. So, this is a very well-known device; it is a gear. So, this gear is something that is found in all sorts of engineering machines. So, this is a wheel with some teeth and so in the useful machines then there are several gears which are connected to each other and they can rotate and The rotation can be controlled through these teeth and you can have different applications. But in this problem, what is the problem? Consider the force F acting on the gear as shown in this figure. So, the problem is to calculate the moment M due to F at the point O.

So, the pivot point is mentioned, so take the center of the gear as the pivot point. The force is given; calculate the moment. So, you are going to use several different ways to calculate it. So, first, let us take a simple geometric way. So, it is important to sort of draw a diagram to remind ourselves to do it and analyze it systematically.



So, this is our box, so this is our system. We are going to choose the gear as our system and then everything else is surrounding and in the surrounding it sort of exerts a force which is F. Now that force is a vector, it is noted down its line of action, the magnitude. The magnitude is 60 Newtons, and its direction is pointed downwards, as shown in this figure and this is the line of action of the force. Now, let us say this is the case, So these are the interactions, so we have noted down all the interactions; the only interactions with the surrounding is F.

Now we are going to analyze it. So, the first thing is that we simply need to calculate the moment. So, let us say our first way to calculate the moment. So this is my pivot point O and this is my line of action of the force. Then the perpendicular distance from this line of action, let us call it D, times the magnitude of this force, which gives the magnitude of the moment. So, we need to calculate: So the F is given; we need to calculate D, which we can calculate from the geometry.

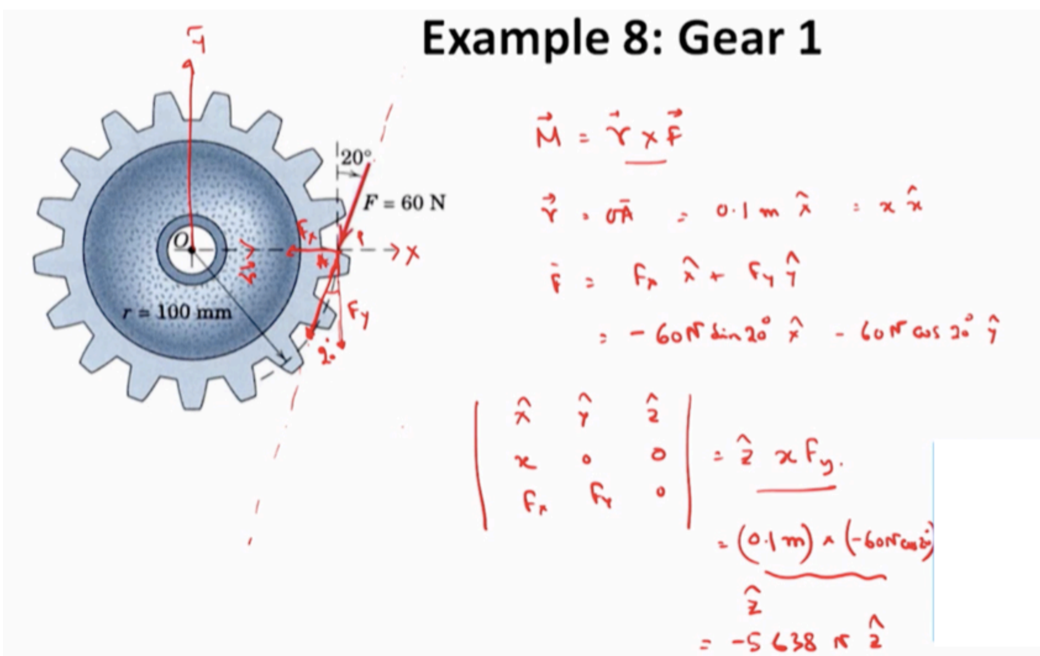


Let us say this point is A, this is the point of application of the force and this is a perpendicular on this line of action; let us call it B. So, we need to calculate A, and B. Now given that this angle is 20 degree, which means this angle is same and from the geometry, it follows that this angle must be 20 degrees. You can check it out for yourself. So, then D, which is OB in this triangle, is equal to OA times cos 20 degrees.

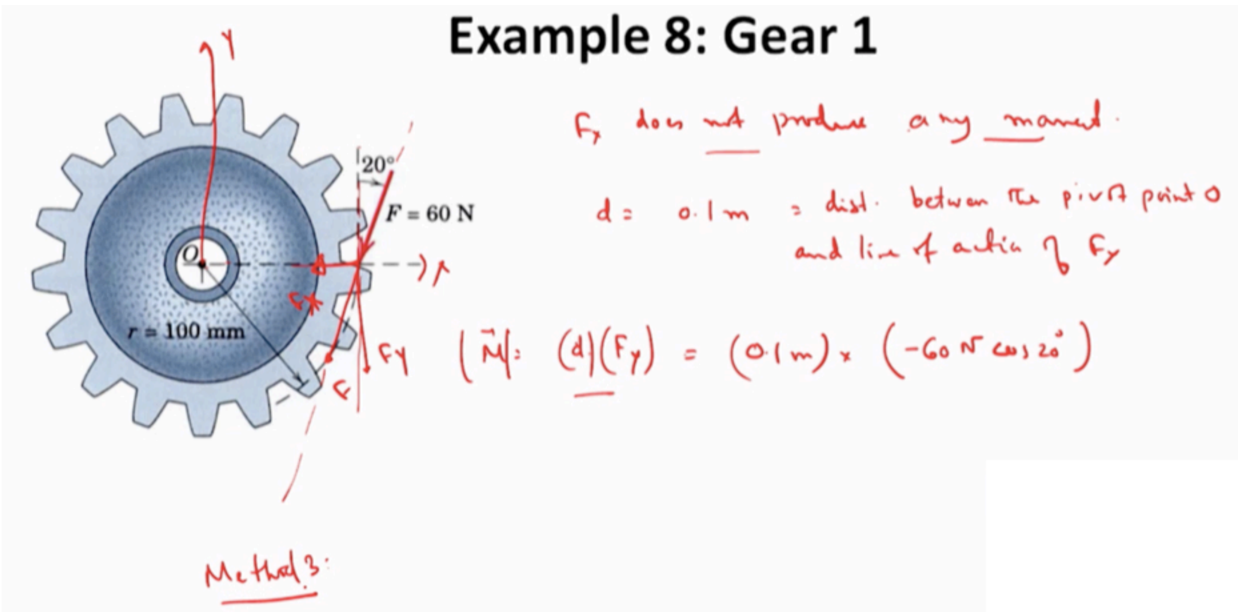
So, then our magnitude of torque is equal to, so this is equal to, So OA is given, which is the radius of this gear, which is 100 millimeters, So I am going to use the SI unit, so I am going to write it in meters. So, then, my force magnitude is 0.1 meter into cos 20 degree times 60 Newton. So which is equal to, so you can plug in the numerical values, and the answer is 5.638 Newton meter. So remember, 1 meter Newton coming from here, So the unit of torque is different from the unit of force, and it is a combination of the units of force and distance.

Now, what is the direction? So, the direction you can find by applying the right hand thumb rule, So this force is going this way, and this is a cross product. So M is a cross product, so you can apply the right-hand thumb rule. So, the direction must be going vertical to this plane of the screen and going inside the screen. So, this is our method 1, a sort of geometric method to calculate, so the direction is counterclockwise and going into the page.

Now, I am going to use it in a different way to calculate. The second way to calculate the torque is to get a feel for the moment. So, we are going to apply this definition that torque is given by R cross F. Now we choose our coordinate system such that this is our x axis and this is our y axis. Now if we, instead, redraw this force by moving it little bit further, So that the tail of the arrow coincides with the x axis.



So, we are shifting the force along the line of action and this does not change the cross-product value, so it does So it does not change the torque. So, this is what we mean by the principle of transmissibility. Then we can see that if we resolve the force along x and y direction, then this is going to be the y component and This is going to be the Fx component of the force. So, from this choice of coordinate system, the position of the point of application So, this is my R, this point A, so this is OA, so this is equal to 0.1 meter x hat and There is no y component. Let us call that for the moment x times x hat to keep the algebra general. Then the F is Fx plus Fy and So this angle is 20 degree, so this must be minus 60 Newton sin 20 degrees, and This must be minus 60 Newtons multiplied by 20 degrees y hat. So, this minus represents the direction. So, then this R cross F is calculated by using this, So you can see that since it has only a z component, other components are 0, So the z component is given by the x component of R and the y component of force, So this is 0.1 meter times minus 60 Newtons cos 20 degrees. So, this is the magnitude and the direction is perpendicular. You can very easily see that this is the same expression as before, so you get the same answer. And here we do not have to apply the thumb rule; it is already taken care of. Now, we can solve the same problem in another way to point out an important aspect. So, in this case, let us say we draw this picture again, so this is my F.



I have shifted it along the line of action, Now note that the component of Fx, the x component of the force, passes through the line of action of the x component of the force on this x axis, so it passes through the pivot point. So Fx does not produce any moment, so all the moment is actually coming from the y component and the distance between the line of action of the y component and the pivot point O is very simple. This is just the radius of this wheel, so that distance is just 0.1 meter. Which is the distance between the pivot point O and the line of action of Fy.

So, then our torque will be simply the D times Fy, this is D magnitude and this is going to given by again. So we have just calculated this 60 Newtons times cos 20 degree and you get the same answer as before. And the direction is of course, this will not give you the direction, but I mean the direction you can get from the right-hand thumb rule and it will be vertical and going inside the screen. So, this shows, this is a third method of computing the force. So we introduce three methods and each of them shows a slightly different aspect of moment calculation.

So I hope now have better intuition about calculation of moment. So, we will consider more examples of moment calculation in the next lecture. Thank you.