Newtonian Mechanics With Examples

Shiladitya Sengupta

Department of Physics

IIT Roorkee

Week -02

Lecture -08

So far in this course, we have covered quite a lot of ground. In the first week, we have discussed the mathematical basics of scalar vectors and tensors, which we will be continually using throughout this course. And this week, we are covering the basic theory about the Newton's laws of motion. So, let us continue our discussion of Newton's laws of motion. But before that, let us start with a quick recap of what we have covered so far. So-called theoretical background.

So, first, we have introduced two model systems. We have started by discussing what is a system. The concept of a system will be very useful in problem-solving, as we see later in the course. And here, we have given example of two ideal system, two concepts that will be repeatedly used. So these two models are one, a point particle, so we will represent our system as a point particle and sometimes a model for an extended object, which is the rigid body. Then we reviewed Newton's laws of motion, and here we took an unconventional point of view that we want to understand how to use Newton's laws of motion rather than only know the content of the statement of the law. So, we started with the third law first and the third law. The basic message is that the real forces, the real physical interactions, always appear in pairs that are equal in magnitude and opposite in direction. Then, we went to review the second law of motion. We first showed that there are three different ways to write the second law of motion, each with more general covering more types of situations than the previous one.

Then there are three messages from the second law of motion. First is that force causes acceleration and not velocity, the first thing to remember. Second to determine the acceleration, what matters is the total or the net force, not the individual force. And the third is more mass means less acceleration, so this is the property of inertia of a system. After that, we discussed the first law of motion, which tells you an experimental test, the condition under which the second law is valid, so when can you apply second law of motion? So we discussed the concept of reference frames, and we showed that the first law basically says that the second law is valid in inertial reference frames.

Then, continuing with our problem-solving point of view, we used these three laws of motion to classify mechanics problems into four different categories. So now we are gearing up gradually to solve problems, and so starting with this lecture, we are going to be more hands on and more problems oriented. To do that, we need a general thinking strategy or a method of analysis and here we introduce a very powerful tool, which is going to be our friend for rest of the course. This is called the free-body diagram. This is a very important tool and technical skill to analyze any mechanics problem.

So let us first understand. This is some go-through to take you through this concept in great detail. So my plan is to highlight the important points and also highlight the common mistakes that students usually make so that when you use the free-body diagram to solve a mechanics problem yourself, you can do it in a correct way. So, first start with the name, so there are three words free, body and diagram. Let us start with diagram. It means that when you start solving a mechanics problem, don't start just by thinking about which formula to use, just write down the formula and go on calculating some values. No, first you should get a feel for the problem, you should apply physical intuition to the situation of the problem and a very useful tool to help you get started by drawing rough sketches of the forces, and here is the diagram.

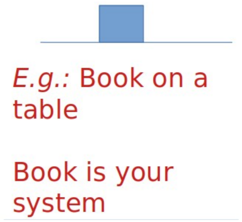
And when you draw, as I will show, we will discuss in detail in coming few slides that pay attention to the magnitude of the force the direction of the force. So you represent the force by arrows, so pay attention to what will be the length of the arrow and what will be the direction in which the arrow will point. So, the length of the arrow should be proportional to the magnitude, and the direction of the arrow should indicate the direction of the forces. Also, as we will see, it is also important to know or think about what is the line of action. This is the line extended to infinity in both directions so suppose there is a force F, so this is the magnitude of the force from tail to tip, and this is the direction. Then if you draw a line along the direction, which is extended in infinitely in both direction, this line is called the line of action. We shall see that this is important when we consider the rotation of the system. So the point is draw the sketch and draw it to the scale as much as possible.

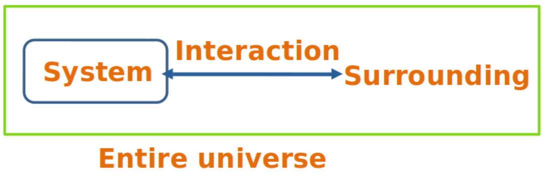
The next word is body, so diagram, so what the forces on what? The diagram should show the forces acting on one and only one body. This is very important. Do not confuse, do not draw, try to draw the forces on many different parts of your system. So first, you choose your system, very clearly define what is your system. For example, let us take this very simple introductory problem. Let us say, suppose this is a table and imagine that this is a book.

So I put a book on a table, and I want to analyze under what condition the book remains at rest or it will start moving and so on. So, in this problem, first, you choose what is your system. If the book is the system, then show the forces only on the book. If a table is your system, then show the forces only on the table. Now, here I point out that as we shall take examples later in this course when I say body, it can be actually several objects.

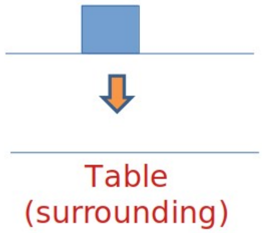
Basically, you are grouping them mentally into one composite object. So it is a mental construct. It is an imagination to define what is inside your system and what is outside your system. For example, imagine that this is actually a book, a copy. So, this is composed of several pages. So, there could be a situation where you want to think about the several pages as separate objects. But here we are sort of mentally grouping together to think about this copy as a single object.

Finally, the third word is free, which means that your body is free of contact with other bodies, which means you are actually isolating your system from the rest of the universe. Now, let us go through step by step carefully about how to draw free-body diagram. So first, as we mentioned the step 0, the 0th step is to decide on your system. For example, in this book on a table problem, let us say the book is your system.



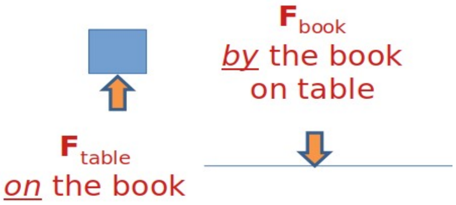


Now here, what we are doing, just to give you a perspective, this picture we introduced in one of the earlier lectures. So, what you are doing in defining the system is that you are systematically dividing the entire universe into two parts. The part that you are focusing on one part, which we are calling a system like the book in this problem, and this boundary can be a real boundary, real wall, or it can be an imaginary boundary. So this is just to define what is inside the system and what is outside the system. This system interacts, it feels, give force, and feel forces with the surrounding. Now, the surrounding can be several different objects, as we shall see. So this is the mental picture that I want you to keep in mind.



Now, step 1, so first consider the contact forces. So, these forces, as we mentioned in earlier lectures, are usually short-range. When two bodies are in contact, there are forces between them. So, from fundamental point of view, they are electromagnetic interactions by nature, but we do not know exactly what are the forces going on between the atoms of the two objects in contact. So we simply take them as unknown forces and resolve them. Usually, we resolve them into two components: normal to the contact surface and parallel to the contact surface. But we shall come to that later, but first, we need to describe these forces. So, for example, the first step is to free the body, as we said, which means mentally, you tear it from the surroundings and isolate it. For example, the book is on the table.

So this is the book and this is the table. Now, you tear the book, isolate it mentally from the table. So, now you separate them, so this is my book, and I chose it for my table and this is the table which is part of the surroundings of the book. Then, as you tear it, so, you break contacts, so you replace the contact with contact forces. So note that contact does not mean that it has to be a single point.



For example, in this case, the book covers a certain area, the contact is over certain areas. At this point, this is irrelevant, but it may be important depending on problem. So tearing means breaking contact mentally. Then, as you know, the contact forces are pairwise forces and Newton's third law, so let us say what are the forces. So, let us call the force on the book F, and I have put a subscript table to denote that this is the force by table on the book. And note that I have put an underline on the word on.

So this is a useful trick that I find to sort of remind me that on which object the force is acting. Now Newton's third law says that there is another part; the counterpart of this force is a force that is acting on the table, and this is by the book. So, this is the force that is acting on our system, and this is the force that is acting by our system. And Newton's third law says that these two forces must be equal and opposite. So, this is the meaning or application of Newton's third law.

So now you place one side of this contact force pair on your system as I indicated here, and place the other side of the contact force pair by the system on your surroundings. So this is shown in this two diagram. Note one thing: that the tip of the arrow touches the object. This is to indicate that this is a contact force. Similarly, here, the tip of the arrow touches the table to indicate that this is a contact force.

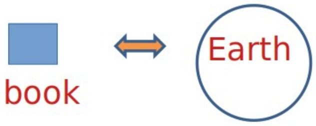
Also, note that the length of the direction of the arrow is vertical. So I am sort of assuming, thinking in a situation where the book is at rest on the table and this implicitly I am assuming that is the example. So, in this case, the contact force between the book and the table is vertical and the length of the arrow that is from here to here and the length of the arrow of the force by the book and force on the book are equal. So this again reflects that these two forces are equal in magnitude and opposite in sign.

$$\boxed{\mathbf{F}_{\text{table}} = -\,\mathbf{F}_{\text{book}}}$$

So now that we have described the contact force in this particular example, similarly, in this particular example, there are no other contact forces. In other cases, there could be more contact forces or short-range forces.

Once we have found out all the short-range contact interactions, then go to the long-range interaction. So, what do you mean by long-range interaction? So, simply speaking these are interactions that can exist without contact. How is it possible? So take, for example, the gravitational attraction due to the earth. So we know the centre of the earth is pulling this book with a certain force, which is called the weight of the book.

And here, you do not have to be in contact with the earth. The book does not have to be in contact with the earth. Another such long-range interaction, a very common example, could be an electromagnetic force. For example, in the lab, suppose you have an electromagnet or a bar magnet, and they create a magnetic force which let us say in magnetic material such as an iron or another magnet, a permanent magnet can fill if you bring them near to the first magnet but without you do not have to put them in contact. So in this example, there is no magnetic force. The only long-range force is the gravitational force, the weight of the object.

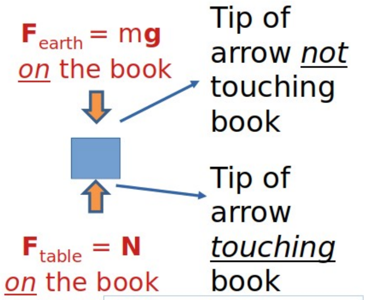


So book and the earth, so the book is our system, and the earth is in the surroundings. So, there is a second interaction between the book and the surroundings, which is the long-range gravitational interaction. Now we want to find the long-range forces, and in this case, one question that we want to ask is, in the given problem, can you represent the system by a point particle? If we can do that, then often, it simplifies the analysis hugely. So in this problem, actually, under the situation the book is at rest, you can sort of denote. You do not have to describe the book by a block. You can also denote by a point mass.

And similarly, the earth you can denote by a point mass. Otherwise, if it is not possible if you want to consider the rotation of the object, then often it is not a good idea. It is a good idea to describe the system as an extended object. In that case, ask yourself, where is the center of mass? So we shall come to the concept of the center of mass and how to find it in great detail in the later part of the course, but at the moment, I am sort of assuming that all of you have some basic idea about what a center of mass is. It is simply speaking a representative point, so if you want to represent an extended object by a point mass, then you want to put that point mass somewhere with respect to the extended object, and that special point is the center of mass. For example, think of a cricket ball. Let us say a bowler is delivering a cricket ball.

As you know, the ball is doing a lot of stuff. It can swing, it can rotate around its own axis spinning and turning or swinging, etc. But you can also see that something in the ball is sort of following a kind of parabolic trajectory. So, that point is the centre of mass. So now when you draw, coming back to our analysis, when you draw the force, the long-range forces such as, in this case, gravity, the line of action must pass through the centre of mass because each point in the book is attracted individually by the gravity and the resultant force is passing through a representative point which is the centre of mass. So once you keep in mind then as before with the contact force now, you can draw and describe the long-range forces.

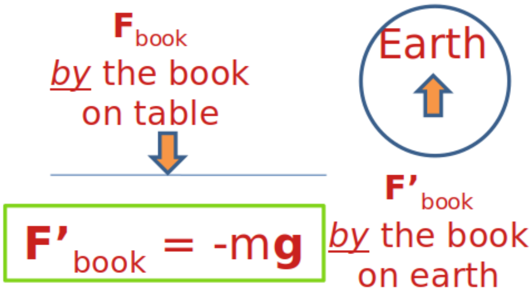
So again, the rule is the same. So this is our system, the book. And this force is the force F, which is acting on the book. So, in the common notation, usually, we denote it as mg or the weight of the book. So this is a force on the book. So note that I have also shown for comparison with the contact force that we have described in just before.



Note the difference that deliberately, in the case of contact force, I have the tip of the arrow is touching the book to denote that this is a contact force. Whereas, when I draw the weight which is going vertically downward, I have deliberately put the tip of the arrow away from the book denoting that this is a force without contact between the center of the earth and the book. So again, just to say in words, there is a pair of interaction, which is the earth and the book. Place one side of this long-range force pair on your system, which is shown by this arrow, and place the other side of the long-range force pair on your system on the surrounding. This we have not shown here, so it is shown in this particular case.

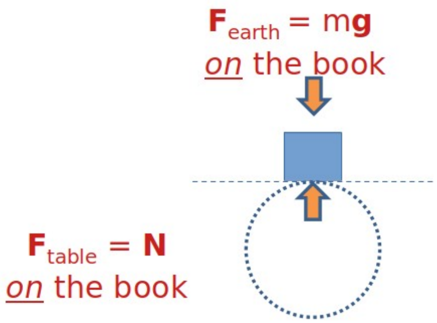
So here we have shown it. So this is the earth, and this arrow is the force by the book on earth. So this is a crucial point. Let us come back to emphasize this point. It is a common misconception to confuse the force, the weight of the interaction between the book and the earth which is the weight and the interaction between the book and the table which is a contact force. Do not confuse between them. So, as you see in this picture, these are two separate interactions, two completely different forces.

So here I am showing the forces by the book on the surroundings to emphasize. So, book is exerting a force on the table, which is shown by this arrow. Whereas, the book is exerting another force on the centre of the earth, which is the other side of the pair, which is given by the weight of the book. This is equal and opposite to the weight of the book by Newton's third law. This force is different from the force that the book exerts on the table.



And in general, these two forces are different in magnitude also. In this particular example, I am implicitly assuming that the book is at rest on the table. So they must cancel each other. So, in this particular situation, as we shall see later, they must cancel each other so that the book can be at rest on the table. But in general, the magnitudes can be different. So, the force by the book on the table and the force by the book on earth can have different magnitudes in general. In this particular example, they are the same in magnitude.

And finally, there is step 3 which is usually optional in the sense that in simple cases, this is not required too much. But in complicated situations where drawing the free-body diagram can be tricky, it may be helpful to sketch the surrounding objects interacting with the system to remind us of the physical origin of the interactions. So, for example, when you draw the free body diagram of the book, draw the book to denote this is your system, and then you draw the forces, the force by the weight of the book, which is the force by the earth on the book and the contact force by the table on the book which is denoted here.

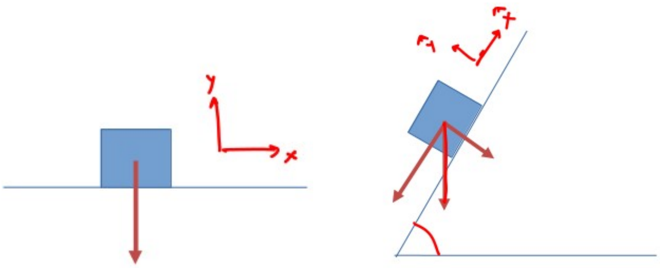


Also, note that here, I have sort of drawn the table as a kind of dashed line and the earth as a dashed line to remind ourselves that these two are really different forces coming from different physical origins. So, this particular simple situation it is not mandatory, but complex situation may be useful. So, the next aspect before we consider workout problems, so there are three aspects of the forces that I want to point out. So, the point is that real-life situations can vary from simple to complex. So there can be a simple mechanical problem such as this book on the table where the free body diagram is very simple, but it can be also very difficult.

For example, if you are studying civil engineering for example, and you want to analyze the stability of a complicated structure such as the Eiffel Tower, then you can imagine that the free-body diagram will be very complex. So, to correctly draw the free body diagram, you must remember the following message from Newton's laws. So force causes acceleration, that is it changes velocity. Now velocity is a vector, which means either the magnitude or direction or both can change. That means the system can either move in a straight line as the translational motion, or it can rotate, or it can do a complicated motion, which involves both translation and rotation together, such as the motion of a bicycle on a road.

Now, this rotation is actually an effect of force, but this is caused by not the force itself but by the moment of force which is also called the torque. So, to include rotation, so, there are three important points about force that you should remember. First, as you know, this is a vector, so it has a magnitude, and it has a direction but also, to include the rotation, you should pay attention to the line of action of the force. As we shall see the line of action determines the moment, which in turn determines the rotation. So in practical terms, when you draw a free-body diagram to represent a bigger force, draw longer arrows, the direction of the arrow should describe the direction of the force, and pay attention to the line of action and ask yourself this question, Does it pass through centre of mass? Now, if the forces and moments are vectors, that means that you can resolve the forces into components.

So for example, let us say I consider a force on a plane, that has only two components, a component along the x direction and one along the y direction, then let us say this is a simple force and then you choose this is your x-axis and this is your y-axis, then as we discussed in the first week when we defined the dot products, this is the force, the x component of the force and if you draw a perpendicular then you get the y component of the force.



So you can resolve the force into components, and we shall see that this is a very important and handy trick to solve the problem, but I must highlight one thing: components are not unique. You can choose your coordinate axis according to the problem. So here is an example, for example, suppose this is a picture representing a book on a horizontal table, so, in this case, a natural choice of a coordinate system is maybe you take the direction parallel to the table as your x-axis and the vertical direction as your y-axis. But if you rotate this picture by some angle, then it will become your favorite block on an inclined plane problem. So, in this case, consider the force mg, that is, the weight of the object.

So you see that in this case, the natural way to resolve the force into components is to draw this not as the earlier direction of your x and y axis, but in this case, as a direction which is parallel to this inclined plane as your x-axis and perpendicular to the inclined plane as your y-axis. As before the little hat over x represents the unit vector, but we shall analyze this problem later, your inclined plane need not be always an angle which is less than 90 degree, it can also be 90 degree or even bigger than 90 degree such as in this particular situation. So you see that in this particular situation, the component of the force has a different direction compared to this situation, where the angle of the inclined plane is less than 90 degrees. So the take-home message is that resolve the force into components to suit the problem. The opposite part of resolving the force into the component is adding the force, adding the components vectorially to get back the resultant force.

For example, if you have two forces acting on the system, then F1 and F2, and you can resolve each of them into Cartesian components along the x-axis and y-axis, then the component of the resulting force will follow the vector, which you can find by the vector law of addition as shown in this particular formula.

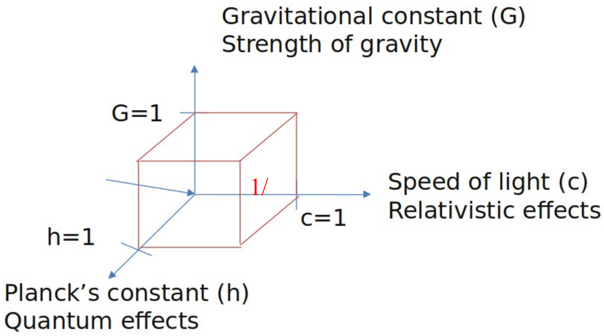
$$\vec{R} = \vec{F}_1 + \vec{F}_2 = (F_{1,x} + F_{2,x})\,\hat{x} + (F_{1,y} + F_{2,y})\,\hat{y}$$

$$R_x = F_{1,x} + F_{2,x} \qquad R_y = F_{1,y} + F_{2,y}$$

So these are kind of handy tools that will be very useful when it comes to problem-solving. So far, we have covered somewhat technical and mathematical grounds to develop a method of analysis. So, let us end this lecture on a somewhat lighter note. So, as you know that in physics, the grand aim of physics is to answer the question, what is this universe made of? And in particular this particular question can be broken into three parts.

What is the structure of the matter that we see around us? This is about the molecules and atoms and nucleus and so on. And also, what is the nature of the light, or more precisely, the electromagnetic radiation that we see, like sunlight and heat and all the radio waves, microwaves, etc., which is also what we see around us? And third part is how matter and radiation interact with each other. These are the three grand questions, and a lot of work has been done to find answers to this question. Now, as you know, Newtonian mechanics was invented by Newton almost 400 years ago.

We can take a look back over the progress of physics since the time of Newton to appreciate the position of Newtonian mechanics in the current understanding of physics. In this picture, I have drawn three axes. Along each axis, I have drawn one particular fundamental universal constant. So, along this axis, it does not matter which one is your x, y, z.



So this axis represents c, which is the speed of light. This axis represents Planck's constant h, and along this axis, I have shown the gravitational constant capital G, not the small g. The c-axis represents the relativistic effect. The h-axis represents the quantum effect, and the G-axis represents the strength of gravity. What do I mean by that? In some units, you can choose all of them to be, let us say, 1. So if c is 0, this means that the speed of the system that we are studying is very less compared to c.

I am sorry, this should be one by c. If (1/c = 0), it implies that we can take the speed of light to be practically infinity, which means that the speed of the objects that we are studying as our system is moving very slowly compared to the speed of light so that we can ignore the relativistic effect. And if 1 by c is 1, we cannot ignore the relativistic effect. We have to include the relativistic corrections explicitly. Similarly, if I take 'h' to be zero, that means I ignore quantum effects, and 'h' is equal to 1 represents the fact that in our system or the natural phenomenon we are studying, quantum effects are very significant, and you cannot ignore them. Similarly, G is equal to zero, representing the case where the gravitational force is weak. So, that weak means the gravitational force can be described with reasonable accuracy by Newton's laws of universal gravitation.

Whereas G equals to 1 represents the fact that the gravitational force is very strong, for example, between a black hole and a star or between two massive stars. In this case, the gravitational attraction cannot accurately be described by Newton's law. You have to go to Einstein's general theory of relativity. So then Newtonian mechanics describes a situation where the following things are true, that the objects are moving at a low speed, so we ignore the relativistic effect. There is no quantum effect in the situation, so we ignore the quantum nature of the objects. And third, the gravitational force is accurately described by Newton's law, so the gravity is weak.

That means we are sitting at the origin. So, this cube, each vertex of this cube represents a different combination of these three effects, and if you consider all these vertices together, that covers all possible scenarios that theoretical physicists people who study theoretical physics think about. So, we can call this a kind of physics cube. So this cube covers all of physics and what we are going to study. The kind of problems that we are going to study in this course will be something where we are sitting at the origin, which means ignoring the relativistic correction, ignoring the quantum nature of matter and radiation, and describing the gravitational force by Newton's law of gravitation. So in the next week, we are going to start solving problems, given this background theory that we have reviewed this week and the mathematical preliminaries that we covered in the last week, we are now ready to solve problems.

So from next week, we shall follow our journey. Remember the classification of problems that we did in the earlier lecture. So, we shall consider the first category of problem, which are the problems where the objects are not moving at all. So, this is the so-called statistics problem, and we shall work out several examples step by step to help you gain knowledge and practice in solving mechanics problems. See you next week. Thank you.