Newtonian Mechanics With Examples

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Week -02

Lecture -07

Welcome to the second week of our course, Newtonian Mechanics, with examples. We are discussing Newton's laws of motion. As you know, in the last lecture, we covered our review of Newton's third law of motion and Newton's second law of motion. So we are following a path which is unconventional. So, we are going backwards. So first, we discuss the third law, then the second law, and then we will go to the first law.

Newton's laws are very simple to state, but there are lot of subtleties involved, so, there are a lot of conceptual misunderstandings among students. Hence, I quickly summarize and recap of what we discussed in the last lecture about the message of Newton's third law and second law. So, the learning from Newton's third law of motion is that it says what is a real physical interaction. The hallmark of a real physical interaction is that they always appear in pairs.

The real force, which you can contrast with something called fictitious force. Some of you may have heard this term. This arises if you are measuring the forces and acceleration using a system of reference which is itself accelerating. We will not consider those kinds of examples in this course, but these non-inertial frames are there. There is a difference between those situations and the real physical force.

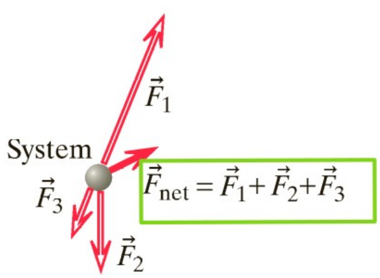
The real physical force always appears in pairs equal in magnitude and opposite in direction. Now I point out here that when we analyze the mechanics problem, there is often a common misconception to consider both parts of this interaction in analyzing the mechanics problem. This is usually incorrect. So, you should usually focus on your system, and if you define your system clearly, then you should consider only one side of this interaction as the force acting on the system. We will discuss this point more concretely when we take examples.

But let us now go to Newton's second law. The message from Newton's second law; the first message is that force is the agent that causes motion. It causes acceleration and not velocity. So, this is a great misconception, and I will explain in a few minutes why this misconception is so common. But this is what took the genius of Newton to realize that the effect of force is not to change the position but to change velocity.

So, force causes a change in velocity, which is acceleration. Now here, there is one point that I want to highlight, a common misconception among students is that acceleration is a change in velocity, so a change in velocity is a vector. Vector has both magnitude and direction, which means it can change in two ways. The first way you can think of is that the velocity changes in magnitude, keeping the direction fixed. For example, a body moving in a straight line speeds up or slows down.

So, for example, if you take something and drop it from the air, it follows a straight line in this case, but its speed increases if it goes downwards. Similarly, if you throw something vertically upwards, its speed will decrease while climbing upwards without changing direction. This is a common situation, and it is easy to understand that the acceleration is due to the change in the magnitude of velocity. In another scenario, the magnitude of the velocity can remain the same, but its direction can change, and if the direction changes, there must be an acceleration. So a body moving in a circle, for example, even at a constant speed, has a non-zero acceleration because, if the body is moving in a circle, then its velocity is along the tangent, and the tangent direction is continuously changing, so there must be an acceleration.

The second message from Newton's second law is that what matters is the net force. i.e., what causes the acceleration is not the individual forces acting on the system but the vector sum of all the forces. For example, in this picture, which we discussed in the last lecture, so, this is our system. Let us say some particles on which there are three forces acting, but the particle will move in the direction of neither F1 nor F2 nor F3 but in the direction of the net or the total force, which is the vector sum of F1+F2+F3.



Net Impulse = $\Delta \vec{p} = \vec{F}_{net} \Delta t$

So this was stated in the momentum, the impulse version of Newton's second law, that the change in momentum is equal to the total force times the Δt, which is the net impulse on the system.

The third message from Newton's the second law is that more mass means less acceleration, which means that the effect of this net force on an object is inversely proportional to its mass. This is evident from this equation.

$$\vec{a} = \frac{\vec{F}}{m}$$

If we write the acceleration as force divided by mass, you can see if you keep the same amount of force, but then, if you increase the mass, the acceleration caused will be less. Now if we combine all these three messages in a single picture, the first part is that this acceleration is the effect caused by the total force, and now, if you replace this proportional sign with an equality sign, now this equality sign itself is important because this means that the right-hand side and the left-hand side of this equation can be independently measured.

$$\vec{a} \propto \vec{F}$$

$$\vec{a} = \frac{\vec{F}}{m}$$

$$\vec{F} = m\vec{a}$$

And then this equality says that if you can determine the independently, F, the left-hand side, and the acceleration and its mass on the right-hand side, then they will be equal, where F represents the total force acting on the system. And this equality in both magnitude and direction. So, a consequence of this fact is that the acceleration of an object is always in the direction of the net force or the total force acting on the system. So this equality is important because you can independently measure the left-hand side and the right-hand side and it is Newton's second law which shows that they must be equal.

Now we go to Newton's first law of motion. So here this is the least intuitive out of all the Newton's laws of motion. Why? Let's consider the following example. Imagine that you are sitting on a rotating merry-go-round. So merry-go-round is this toy horse which moves in a circle, and it is a favorite toy for kids usually you encounter in a fair. So if you all know that you will feel a force that is trying to push you away radially outward direction.

However, with respect to yourself, you are clearly at rest, you are not moving, that is your acceleration is 0. So apparently this is a situation where you can feel a force, but it does not cause acceleration. So, does it contradict Newton's second law? This is the question. To answer this question, we need another independent law, which is the Newton's first law of motion. So the question is, under what condition can we apply Newton's second law? And to answer this question, you need an experimental test, and this experimental test is Newton's first law of motion.

So, let me describe the steps in the test. So the step one is you prepare an isolated test body on which no force acts. So this is the special situation. No force is acting on an object on your system. Then the second part is to observe its motion. Now the test says that if such a body continues to move at constant velocity when no force is acting on the body, then that is in a straight line at a constant speed, which is the same as saying constant velocity.

So if you observe this, then this is the condition in which you can apply Newton's second law of motion. So, it may sound a little bit confusing and difficult to digest. So, let us go through it slowly. First is an isolated test body. Now, this part is very difficult, and this is the reason why Newton's first law is very counterintuitive to our daily experience.

Why? Because we, the humans on this Earth, are immersed in an environment where, there are always two forces present. The first force is the gravitational force our weight due to the attraction by the Earth, and the second is that there is always some sort of friction. So for example, we are immersed in the air surrounded by the atmosphere. So we are continuous. When we move through the air, we feel the drag force by the air. Sometimes we are in the water, so when we move or swim in the water, we feel the drag force due to the water.

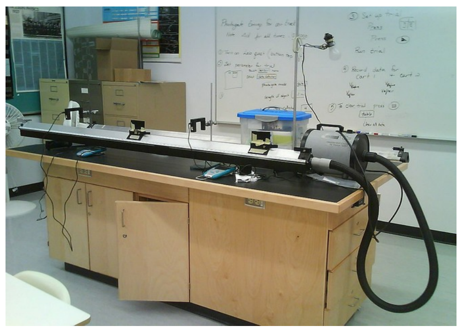
We walk on the ground, and then we come in contact with some surface where we experience friction. So, these two forces are always present in our environment, and our daily experiences and physical intuitions are heavily biased as a result. So the isolated test body means to remove all the forces. So, that means we have to consider a situation where you must remove the gravitational force as well as friction. Now, how to do that? Now here is a beautiful one, the first one is actually removing gravity or switching of gravity is actually easy.

This is a beautiful example illustration of the concept that force is a vector. So, all you need to do to remove the effect of gravity is to consider the motion of the test body in a direction which is perpendicular to gravity. So, as you know that, gravity is the direction that says the vertical direction because it is towards the centre of the Earth and this is the direction we usually call the vertical direction. So if you consider a direction which is perpendicular to the vertical direction, that is, the horizontal direction, then gravity has no component in the horizontal direction. So, there is no force of gravity if the test body moves in a horizontal direction.

This is the easy part, and this is a beautiful illustration of the concept of dot product. So we have discussed this in the first week of this course. So, if you know the force, then a force does not have any component in its perpendicular direction. So now we have removed gravity.

So this is an easy part. What is difficult and challenging is to remove the friction and all sorts of contact forces. So, to do that, you have to make sure that the test body does not touch any other body. So, no contact. This is a bit difficult to achieve in an experiment.

But suppose you can do that. So, the closest approximation of this kind of system achieved in a laboratory is usually demonstrated by something called a linear air track. So here is the picture of such a system. So note that there is a horizontal track, which is the silver-white path. So this is a track on which there are pinpoints. So, there is a jet of air coming through this pin nozzle, and this is the slider. Because of this air jets coming up it is not in contact with the track itself, but it is floating in the air, and then it can move along the track in the horizontal direction. So this is a simple system in which you can approximately turn off all the friction, the gravity and so you can generate a condition where no force is acting on your test body.



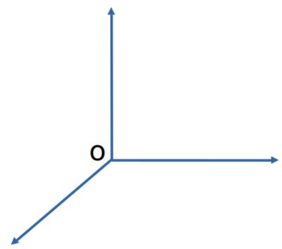
Linear air track : IIT Kanpur https://www.youtube.com/watch?v=IQhooi4abE0

Verification of Newton's second law of motion using air track: IISER Kolkata https://www.youtube.com/watch?v=FLdfXBnUToc

So I have put a couple of YouTube links where a video demonstration of the motion of the slider on the linear air track is shown.

The first one is taken from a demonstration in IIT Kanpur, and the second one is a more detailed version explaining what this linear air track is and how to use that to experimentally verify Newton's second law is from IISER-Kolkata. Now the second point is that you have a concrete example of a test system now you have to observe its motion.

Now, what do you mean by observing the motion? It means you need to use a reference frame to measure the motion, position velocity, acceleration, etc. Now, what is a reference frame? A reference frame is nothing but an origin and a set of coordinate axes drawn from that origin and a clock. So, for example, in this picture, this is the origin from which we have taken a set of coordinate systems. So, these are Cartesian coordinate systems. At this point, it does not matter with which one is x, y, z, and you must have a clock on this.



So this is a clock to measure the time. Now that you have this setup, you can observe the world and what is happening the motion of this test body. So, you can measure the position vector, velocity, momentum, acceleration, force, torque, and everything. This is called a reference frame. So this concept, sometimes, is not used explicitly, but whenever we analyze any mechanics problem, we either explicitly or implicitly always working from a particular reference frame.

So here are some examples. For example, if we do not mention anything, that means usually you are working from the frame of the Earth, meaning the frame in which the Earth is at rest. So now if you observe Earth from a satellite in space, as you see in some of the space-related movies that Earth is actually moving with around the sun and also spinning around its own axis. That means if you attach such a coordinate system with the Earth, then that is also moving with the Earth around the sun and rotating with the Earth such that the frame is at rest with respect to the Earth.

So this is the earth frame. Another common example is a frame of a train. So imagine you are an observer sitting in a train which is moving at a constant speed on a straight railway track. Another common example could be the frame of a lift which is falling freely toward the Earth. Another example that we discussed just a while back is the frame fixed to a rotating merry-go-round. In which the observer sitting on the merry-go-round is at rest.

So these are the examples of reference possible reference frames. Now the question is in which of these frames you can use Newton's second law of motion to analyze problems. So this is the test. In the candidate reference frame observe which means actually 'measure' how the isolated test body such as this linear air track moves. the isolated test body means no force acting on it. Under such conditions, if it moves with a constant velocity, then we can apply Newton's second law of motion in that frame.

So, this is the content of the first law of motion. Such a frame is called an inertial reference frame. So Newton's first law basically says that inertial frames exist in nature. So this is the physical meaning of the first law and sometimes you encounter in some textbooks that you can derive Newton's first law from Newton's second law by putting the F is 0 you get the change in momentum is 0. So it moves with a constant momentum or constant velocity, but that is not the point of the first law.

The first law is actually a statement that when you have permission to use Newton's second law. This is why this is an independent law of motion. Now note one thing there is no unique inertial frame. If you can identify one inertial frame, let us call that a base reference inertial frame, then any reference frame moving with a constant velocity with respect to your chosen inertial reference frame is also an inertial reference frame, so this is called the principle of relativity. So now, after we introduce and discuss the physical meaning and content of Newton's laws of motion.

So now, as we said, the goal of this course is to develop a problem-solving skill, the mechanics' problem-solving skill. Now, of course, if you want to develop problem-solving skill in mechanics, the only way to do this is to solve mechanics problems, to struggle with mechanics problems, and come up with a solution. Only then you have the confidence that you can solve mechanics problems. There is no other shortcut. However, the point is that the number of possible problems in mechanics is basically infinite. You cannot definitely solve an infinite number of problems.

So you have to develop your skills in a smart way, and here I suggest some strategies for that. So, the strategy is to identify the different types of problems that is you typically encounter either in real life or in your mechanics textbook. Now, I have given you a sort of classification. So the picture of the system that I have in mind is the system which can be modeled as a point particle.

Now we will encounter this in real life and in this course situations where the system cannot be modeled by a point particle, you have to assume it is an extended object. In those cases these are more complicated scenarios. These cases, we can have other types of problems, but for point particles, we can think of four kinds of situations. All of these four situations are, of course, valid for extended objects.

So here I write down to recall the Newton's second law, so the version in which the total force divided by m gives you the acceleration of the object. So the first situation is when the net force is 0, then Newton's second law shows that the acceleration must be 0, which means the system moves with constant velocity which is Newton's first law. A special case of interest is when the system is not moving at all. So this is the so-called statics example. So, in this case we say that the system is in mechanical equilibrium and we are interested to know what are the conditions of mechanical equilibrium.

Now, typically, the problems, what kind of problems we encountered here, so this gives you an equation, so it says that if you add the total force, you get 0. Now typically one force is unknown, so you can solve this equation so F total is equal to 0. This is called the force balance. We shall see more in detail in the coming few lectures that you need an additional condition that the total torque is also equal to 0 in this case. This gives you two equations if you add the torque condition, this is the torque balance. So you have two equations, so at most, you can have two unknowns in unknown forces. Now you get two equations; solve this equation to find the unknown, so this is the typical problem.

Now here is a side remark again: how can a force be unknown? So I want to draw your attention to two categories of common forces that you encounter in everyday life. So the first category and typical example is the weight that is the gravitational force due to the Earth. Let us say you take a point particle of mass m; then, its weight is always m times g, where g is the acceleration due to gravity and always towards the centre of the Earth. Meaning that this force is always known both in magnitude as well as in direction, even without solving any problem.

Now, contrast this example with another situation, for example, the contact forces between two surfaces, so remember a typical block on a plane problem. So, for example, if I take this block on the surface of the notebook, then the contact force between the block and the notebook, now, in general, this contact force is unknown. So, as you know, typically, what we do is that we resolve this into two components, one which is perpendicular to the surface and one which is parallel to the surface. Parallel to the surface is usually called friction, but again, this force is self-adjusting and unknown in general. You cannot write it down; you can write it down symbolically as, let us say, N and F, N for the normal component and F for the friction, but you do not know the magnitude of N and the magnitude of friction.

Usually, you solve Newton's second law or the force balance or torque balance to determine the contact forces. These forces are called self-adjusting and are usually unknown, both in magnitude and direction, or in general in both. So there are two kinds of forces, and interestingly, this kind of contact force usually has a short range of interaction; that is, they always appear when you have two objects which are either in contact or very close contact.

Now the second situation is a slightly more general version. That the total force is now non-zero, but the total force is given, that is, each force acting on the particle is given. So in this case our goal is to determine or compute the trajectory of the particle, how is the particle moving. Consider the example of a projectile motion.

In this case, the force due to gravity is known. It may also have a drag force due to the air through which it is moving. So that is also given. In this case, we want to know what is the path the trajectory the particle is taking and how the position and velocity change with time. Now this is the most straightforward application of Newton's laws, so this is usually done in two steps. So the first step is to apply the second law in whatever form, for example, let us say we use this first form in which we know the total force acting on the particle and then apply Newton's law to compute its acceleration.

$$\frac{\vec{F}_{net}}{m} = \vec{a}$$

known unknown

So this is known, and this is unknown. Now, once we calculate the acceleration, you can relate to velocity using the definition that acceleration is the rate of change of velocity, and you can relate to the position with the definition that acceleration is the second derivative of position with respect to time.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$$

Note that these are differential equations. So, this situation is now a mathematical problem of solving a second-order differential equation. Now, as you know, if you want to get a unique solution out of a second-order differential equation, you need to supply two constants of integration. So these are, for example, can be thought of as the initial position and initial velocity at t equal to zero. So, if you know where you are starting to throw the projectile, what its initial velocity magnitude is, and the direction of its initial velocity, then you can determine the subsequent trajectory of the projectile. So, this is a mathematical problem now.

Now, there are two special cases of this situation. So, this case one in which this force is only a function of time. For example, some of you may have studied the oscillators and the motion of oscillators. So, let us say there is an oscillator, so this is a common example in mechanics, electrical engineering, and electronics communication engineering. So, suppose this oscillator is driven by a periodic force, which can be an AC signal, or it could be a mechanical driving force. In this case, this force can be represented as some F, not sine omega t, where omega is the frequency of the force in some direction. So, in this case, the force is only a function of time.

And the other scenario in which the force is only a function of position. For example, the gravitational force, as you know, Newton's law of gravitation shows that the force doesn't depend on time but it depends on the distance between the two objects. Similarly, the spring force which says that the spring force is proportional to the extension of the stretch of the spring. So it depends on the position of the endpoint of the spring.

These two examples are special cases I mention because the mathematical tricks and tools to solve the differential equation are slightly different in these two special cases. Now, situation 3 is the opposite, meaning the trajectory is given. You know the position, velocity, and acceleration. So acceleration is known. Now, you determine the forces acting on the particle. The force is now unknown. So the most famous example is, given the Earth's orbit around the sun, deduce the force between the sun and the Earth. So, to deduce that this force is inverse squared, proportional to the inverse square of the distance between Earth and the sun.

That is, it follows Newton's laws of gravitation. Now, this example is becoming more and more relevant in the 21st century because nowadays, we launch many satellites in orbits near Earth and far away from Earth. We have access to a lot of real-time data about the position velocities of these satellites, and using these satellites or trajectories of these satellites, we can easily determine what the forces acting on the satellites, for example, by Earth or by sun, etc.

The fourth situation is probably the least common, but I mention this for the sake of completeness. So this is a situation in which both the trajectory, the position, velocity, and acceleration, as well as the total force are given. Now, you want to determine some properties of the particle, some properties of the system you study.

One example that I can think of is electromagnetic theory. In this case, suppose there is an electron which is moving in an electromagnetic field, so it is experiencing the Lorentz force. So in this field, under certain conditions, you get a trajectory of the electron, which is a circular trajectory. So, you can design different kinds of trajectories in which the electron will move by playing with the electric and magnetic field that you apply on the electron. So this trajectories are known, the applied electric field magnetic fields are known. From this, you can determine the charge-to-mass ratio (e/m), which is a fundamental physical constant of the electron. It gives you a very fundamental property. This is also a standard first-year B. Tech experiment in many places.

So now, the reason I mention this is that now we have applied the first principle, the basic Newton's law; just by looking at Newton's law, we can think of these four types of situations that can arise. So that now we have a handle some idea about what kind of problems we may encounter in mechanics. So, our next goal is to develop an analytical tool or thinking strategy to systematically attack the problems. Our next goal is to develop how to use the free-body diagram, which is our first technique we shall use in this course to solve the mechanics problem. This will be the subject of the next lecture. Thank you.