Course Name: Newtonian Mechanics With Examples
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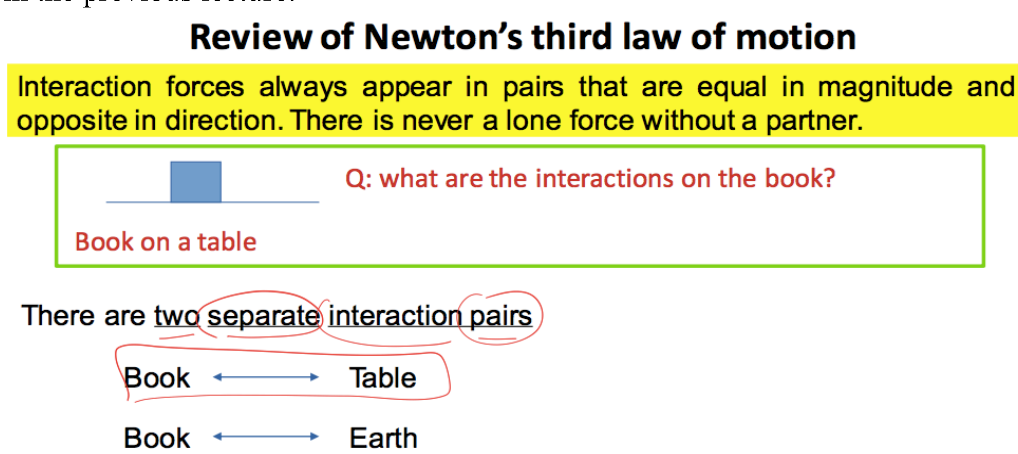
Week 02
Lecture - 06

So, let us continue our discussion about Newton's laws of motion. So, we are reviewing Newton's laws of motion. In the second week of this course on Newtonian mechanics with examples, In the previous lecture, we sort of introduced two basic mechanical model systems: the point mass and the rigid body and we sort of took some example and quickly review some very common forces in daily life that we shall encounter in different problems. In this lecture, the goal is to review Newton's laws of motion. Now, I am sure that all of you know the statements of Newton's laws of motion. So, I will sort of take a kind of unconventional route.

in the sense that I will start with the third law of motion first, then go to the second law of motion and finally, I will go to the first law of motion. Now, the reason for doing that is that we are trying to develop an analysis strategy to solve problem and from that point of view, it is easiest if we discuss the laws in this way. So, Let us get started. So, first, what does Newton's third law of motion say? So, basically, it says that the interaction forces, So here are some of the examples we took from the previous lecture.



So, it says that The interaction forces always appear in pairs. So, they always appear in pairs. So, this is the statement: first part, and these pairs are equal in magnitude and opposite in direction. So, the point is that there is never a lone force without a partner. So, by the way, I must mention that Newton's laws of motion cannot be derived from any in some sense.

So, these were originally devised based on doing experiments. So, these are kind of inferences from our everyday life experience, and the point is that this was not very easy because of the fact that we always feel the gravity and the drag force Due to the atmosphere, we are living in a frictional force. So, that sort of that makes it difficult to sort of deduce the correct laws. So, sometimes, that may be one reason why the mechanics problems are kind of you find hard conceptually. So, that is why we are trying to emphasize the concepts in this lecture and then in the coming lectures we will apply those concepts through examples.

So, let us understand this statement using this simple example of the book on a table problem. So, question is, so I have kept a book on this table, so may be something like this. So, imagine this is a table, and I have kept a book on the table. The question is: What are the interactions on the book? Now, this simple problem is again somewhat difficult to many students, it is very surprising. The point is that there are two separate interaction pairs.

Review of Newton's third law of motion

Interaction forces always appear in pairs that are equal in magnitude and opposite in direction. There is never a lone force without a partner.

Book *b* exerts force F_T on table *T*.
Then by Newton's 3rd law, there must be a force F_b on book by table, such that $F_b = -F_T$. Electromagnetic interaction.

Earth exerts a force *W* on book *b*.
Then by Newton's 3rd law, there must be a force F_E on the earth by the book, such that $F_E = -W$.
Gravitational interaction.

Now, we should think in terms of pairs. So, what are these pairs? So, the first set of interactions is between the book and the table. So you can do this experiment yourself; put a book on your hand or on a table. So, in the force or interaction between the table and the book and there is a separate interaction, There is another set of interactions between the book and the earth. So, these two forces are really different.

So, book B exerts a force; let us denote it at Ft on the table exerts a force, Then, according to Newton's third law, It says that this is the significance of Newton's third law, that it is easy to see that book B, Because it is sitting on the table, it is going to exert a force on the table. So the book is going to exert a force on the table, but the third law really says that the table must also be putting a force on the book and that force is equal and opposite to Ft. So, this is the point of the third law. Similarly, we can easily see that the earth, the book has a weight, which means the earth is exerting a force W on the book, the weight of the book. Then Newton's third law points out that there must be a force, let us denote it by Fe on the earth by the book.

So, this is the first law, this is the weight and this force is equal and opposite to the weight of the book. So, this is the gravitational interaction. So, these two forces are really different, the first one being is electromagnetic force and the second one being a gravitational force. Now, if you are not convinced that these two forces are really different, then perhaps I can let us take a different example. So, imagine this is a ball, may be a cricket ball and so if the ball is sitting on your hand, then maybe you think that well it is not moving.

So, it must be like there is a force of gravity and force of the hand and table, They are balancing each other. But imagine that you are catching ball that is moving. You are trying to catch this ball, like catching in a cricket match. And then you can easily see that if the ball is falling with a great velocity and then it is going to hit the your hand, then the force exerted by this ball on the hand is really different from its weight. If you feel a force that is greater than the weight of this ball, So, this is a simple demonstration that the weight and the force between the book and the table are really different forces.

So, our point is that how to use Newton's third law of motion to solve problems. So, here I give you a step-by-step detailed kind of algorithm, if you may. So, how do we use Newton's third law to analyze mechanics problems? So, the first thing is that you determine which two bodies interact to produce the given force. So, suppose you identify the system. For example, let us say you are the system and earth is the surrounding.

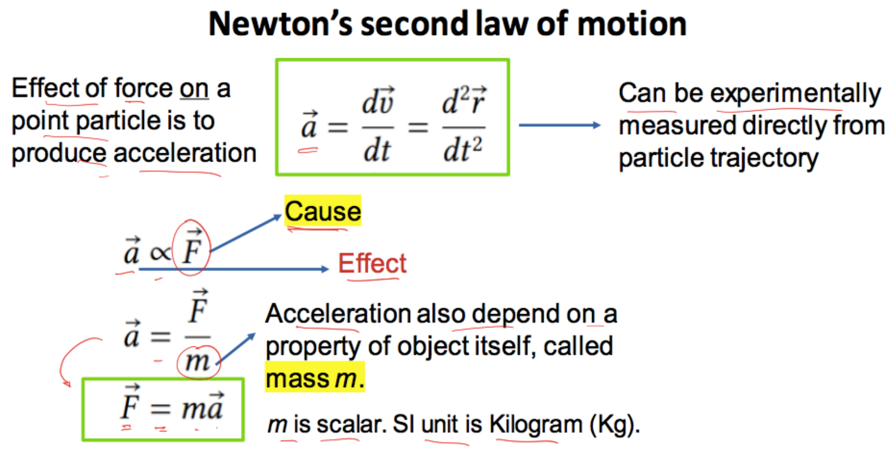
So, you and the earth are the two bodies that are interacting to produce a force. Once you do that, the next point is to describe the force on the system. So, if you remember that in the previous lecture, we said that our picture for analyzing a problem is that We are dividing the universe into two parts, this is our system and this is the surrounding and this is the universe. So, describe the force on the system. If you are the system, then the force exerted by the earth on you is the force.

Now, the next two points are often not required to solve a problem, but you should complete the thinking because it means to describe the third-law counterpart as the other side of the interaction which means that if you are the system and you are filling the force by earth, The third law says that the earth is also being filled with force by you. So, this is what I mean by third-law counterpart and to appreciate the true meaning of the third law. So, you should remind yourself that how strong this counterpart force is and in what direction. For example, in this case, you are pulling the earth upward by a force of magnitude equal to your weight. Let us now go to Newton's second law of motion.

Now, in this course we will emphasize on problem solving skill, developing your skill to solve not only the known problem but also any unknown mechanics problem. So, this is going to be one of the core concepts that we will be using again and again in this course. So, let us go through it slowly. Now, we want to cover all sorts of mechanics problems. So, what I am going to discuss is to give you three different versions of Newton's law, the same law written in three different ways.

Each version is more general compared to the previous one. So, let us start with the first one. So, what does Newton's second law of motion say? It says the effect of force, so you all have an intuitive understanding from our experience of what a force is. So, effect of force on a point particle is to produce acceleration. So, this is the statement of Newton's second law of motion.

Now, note that acceleration is a vector which measures the change of velocity, The rate of change of velocity with respect to time, can be written as the second derivative of the change of position with respect to time. So, the point is that acceleration is something that You can directly experimentally measure the position and velocity of the particle as a function of time. So, to know acceleration or to define acceleration, you do not need to know anything about Newton's laws of motion. So, then what does this law says? It says that what is the cause of this acceleration, What causes this acceleration? The answer is that it is the force that causes the acceleration. In fact, the acceleration produced is directly proportional to the force applied on a point particle.



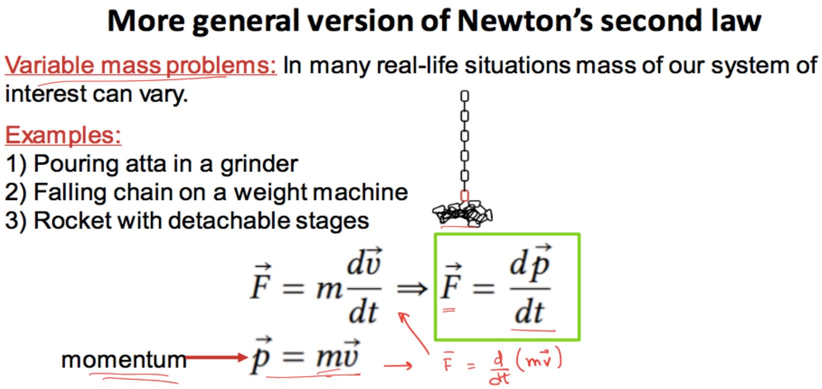
So, force is the cause, and acceleration is the effect. Now, if you replace this sign of proportionality with the equality sign, then the constant of proportionality is related to the mass. So, this mass is a property of the object itself. It is not a property of the force that is acting on the particle. but the property of the object itself.

So, acceleration also depend on a property of the object itself and this property, as we know, is called inertia, or the mass of the particles. Now, mass is a scalar quantity. So, you may recall our discussion from the previous week. What is a scalar, and what is a vector? So, mass is an example of a scalar and its SI unit is the kilogram. So, we can rewrite this equation in a more familiar form that the force is equal to the mass times the acceleration.

Now, there are some situations in real life where the mass of the system that we are studying can vary and is not constant. Usually, in mechanics problems, we assume that the mass of the system remains constant, but this is not always the case. For example, if you take a packet of wheat to your nearest shop to grind it and get some atta or flour out of it, So, the shopkeeper will pour this wheat grains into some hopper, a machine which then runs and grinds it and makes a fine powder out of it. So, in this case, if you notice that The amount of wheat that is going into the machine is changing. So, its mass is changing with time.

Another example that is shown on the slide is if you have a weight machine with a weight balance like the one you may see in large shops. And suppose you have a chain and this chain was initially hanging fully from the air without any contact with the weight machine and then you touch it, gradually lower the chain and touch the weight, the balance, and the floor of the balance and then you release it. Then the part of the chain that is in contact with the floor of the weight machine is gradually changing with time. So, initially, there was no part that was touching, after some time All the chains will be falling on the weight machine. But during this time, the mass of the chain that is in contact with the weight machine is going to vary.

So, the weight machine and the balance will register a variable mass or variable weight. Another very common example is a rocket with various stages. Like the one that we use to send people to the moon or on Mars missions, etcetera. So, in this case, when the rocket is lifted from the earth, it has a lot of fuel and as it progresses, more higher and further and further away from the earth, then the fuel is consumed and some of the part gets detached from the rocket. Which gives a boost to the velocity of the remaining part of the rocket.



So, these are examples of what are called variable mass problems. So, in this case, a slightly more general version of Newton's law should be used, using the definition of momentum. So, as you know, that momentum of a particle of mass m is the mass times velocity. So, P is equal to m times v. Then Newton's second law says that the force, which is the cause for change in momentum and the force is in fact equal to the rate of change of momentum So, F is equal to dP dt.

Obviously, if you plug in the expression for mass times velocity, then the force is equal to the momentum if the mass is constant, then it reduces back to the familiar expression of force. which is equal to mass times acceleration, as we discussed in the last slide. Now here, I want to sort of make a some remark on momentum. So, I sort of want to connect this course with some other courses, such as quantum mechanics or electromagnetic theory that some of you may have are taking simultaneously in your curriculum. So, in Newtonian mechanics such as the course that we are studying, so we know the momentum of a particle with mass m is defined as mass times velocity.

Now, I must mention that here is a hidden assumption In this formula, the speed of the particle should be much smaller than the speed of light. If the speed of the particle is comparable to the speed of light, so c represents the of light. So, we need to modify this definition little bit which is subject of a course in special theory of relativity. However, so some, I want to make this point that you are in a course of quantum mechanics. For example, you have also come across another type of particle which are for example, the photons or the particles of light.

And there, you may have come across the De Broglie hypothesis that the particles of light also carries momentum and this is given by a completely different formula. So, the momentum, So the photon particle of a quantum particle of light is a massless particle. So, its momentum is given by this expression where k represents the wave vector or wave number, which is inversely proportional to its wave length times this universal constant called the Planck's constant. Some of you may also come across this fact about massless particles carrying momentum in a slightly different version in a course in electricity and magnetism or optics. So, there you may have studied about electromagnetic radiation.

Some remarks on momentum

In Newtonian mechanics course:
Momentum of a particle with mass *m*: $\vec{p} = m\vec{v}$ $\quad v \ll c$ → speed of light

In quantum mechanics / modern physics course:
light particles - *massless* - carry momentum $\vec{p} = \hbar\vec{k}$ = $\frac{2\pi}{\lambda}\hat{k}$

Electricity and magnetism / optics course:
electromagnetic radiation carry momentum.

Concept of momentum applies beyond mechanics.
Q: Why?
A: Due to the momentum conservation principle

So, this is very important example. in communications, like different kinds of radiation such as microwave and other light, x-rays, etc. So, these are very much part and parcel of our modern life and technology, So this electromagnetic radiation also carries momentum, which is related to its energy. So, the point I am trying to make is that the concept of momentum. As we see in these examples, this applies beyond the scope of Newtonian mechanics. Now, you may wonder why momentum is so important.

So, this is actually a very important concept and it is actually a sign that momentum is really a very useful concept. So that people tries to apply the same contact in different and more and more different situations and try to generalize its definition. And where is this importance? The answer lies in the conservation of momentum. So, the law of conservation of momentum, we shall discuss it in the later part of this course. So, that is why people find the concept of momentum is so important.

Still does not cover everything!

Click to play

Collision problems: In many real-life situations a large magnitude of force acts for a very small duration so that acceleration tends to infinity.

$$\text{Impulse} = \vec{F}\Delta t$$

Δt is a (small) time interval

Impulse:
In such cases, impulse of force is more useful.
It is a **vector**.
In SI, unit is *N-s* (Newton-seconds).

Note: impulse can be computed for *any* force using the above formula.

Now, coming back to our discussion on Newton's second law, So, we have covered situations where the mass of the system or particle is constant and we have also covered in a situation where the mass of the system can vary, and still it does not cover everything. For example, So there is an important class of examples. For example, the collision problems in which a large magnitude force can act for a very small duration. So, this is on the left-hand side as an example. So, this is a famous experiment or toy called Newton's cradle.

So, this ball is hitting ball from the outside on the extreme end; it is hitting the ball in the one of the middle balls and as a result, one of the balls is moving out of the other side. So, this is an example of a collision. Now, in this case, force may not be very well defined mathematically; that is the issue. Because if you take the duration, so the duration or by which the force is acting during the collision is very small, the time window is very small and the force, so if you reduce the time window, the force becomes very large. So, this is a situation where force is not mathematically very convenient to handle.

Most general version of Newton's second law

We can re-write the Newton's second law in terms of impulse of the force.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \Rightarrow \text{(re-write)}$$

$$\Delta\vec{p} = \vec{F}_{net}\Delta t \text{ = Net Impulse}$$

The change of momentum of a system is equal to the *net* impulse applied to the system.

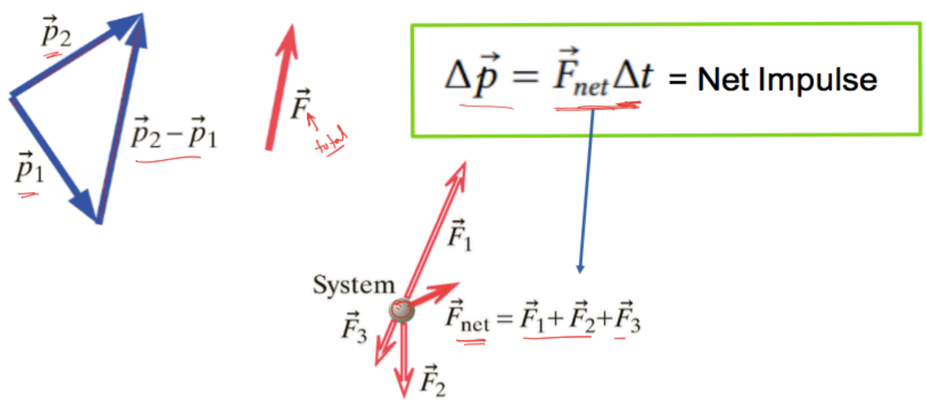
So, we define another quantity called impulse, which is nothing but the force times the duration of the time interval. So, it is simply force times delta t. So, impulse is a vector Because force is a vector, it is multiplied by a scalar over the time interval. So, an impulse must be a vector. So, note that I put it in a bold face to indicate that the left-hand side is also a vector.

So, it is a unit in the SI system; its unit is Newton seconds. So, the unit of force is multiplied by the unit of time. Now, I make a point to highlight here that this definition can be applied to any force. that is, even though this is mostly when you think about impulse, It is very useful in situations where a large force is acting for a short duration. This is the typical situation of collision where this formula is used most often, but there is nothing in this formula that says that force has to be very large or delta t has to be very small.

So, this formula impulse can be computed for any force. Now, using the concept of impulse we can now rewrite the Newton second law as the most general version of the Newton second law which covers all sorts of mechanics problem situations that I know of. So, how do I write it? So, in the previous version, we said that f is the rate of change of momentum. So, if you multiply both sides by delta t, So, we rewrite this formula as saying that the net total change of momentum is equal to the force the total force or the net force times the duration of the time window over which the force is acting. So, let me repeat the change of momentum of a system. Let us say a point particle is equal to the net, that is the total impulse applied to the system.

Now, this is probably not very familiar. This version of Newton's second law may probably not be very familiar to some of you. But we shall see that in some situations, this is very handy to analyze the problems. Especially in collision problems and variable mass problems. So, now you know all three versions of the Newton second law that cover all possible situations.

Most general version of Newton's second law



So, now you know how to write Newton's second law. So, let me illustrate this third version graphically. So, it basically means that suppose P1 is the momentum of a particle at the beginning of the interval and P2 is the final momentum at the end of an interval delta t. So, then this is P1 so this arrow is in the magnitude is P1 and direction represents the vector P1, and this arrow represents the vector P2. Then, from the law of vector addition, we know that this represents the total change of momentum of the particle during the interval delta t. Then Newton's second law says that this total change is proportional to the total force.

F must be the total force acting on the particle. So, note that the direction of this F and The direction of the change in momentum (P2 minus P1) is parallel; that is, they have the same direction. Because that is the first message from this equation that the direction If you look at this equation, you must see that the direction of the delta P is in the direction of the total force. Now, this is an important point that is so I illustrate it with this picture. Suppose this is our object our system and in most of the situations that we encounter in mechanics problems as well as real life. There are multiple forces acting on a single particle or a single system, such as 1 F1 F2 F3.

So, there are three forces acting on this system. The momentum, or the net change of momentum, is not the impulse of an individual force but the total force. So first, we have to calculate the total force acting on the particle. Which is the the vector addition vector sum of all individual forces, and then The net change of momentum will be in the direction of the total force. So, see that the net particle will move neither in the direction of F1 nor in the direction of F2 or in the direction of F3, but in a completely new direction, which represents the direction of the total force. This is very important and of course, the magnitude of the change in momentum and magnitude of the total force are not same because You see that there is a factor of the time interval.

They are not of the same dimension of course, and Because of this time interval, the magnitudes are also different. The numerical values are also different. So, we shall come back to these impulse in a later part, But now let us quickly illustrate the concept of impulse using a couple of examples.

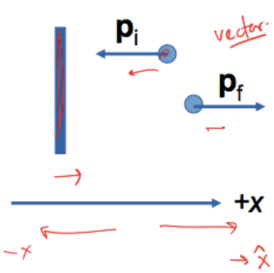
So, in this example, there is a wall. So, this is a wall and there is a tennis ball Which is traveling in the negative x direction.

So, this is the positive x direction. So, this is the negative x direction. So, the tennis ball is traveling in this direction and it hits the wall and then bounces back. So, the question asked that the initial momentum is 2.8 kg per meter per second in along this direction. After the collision the magnitude of the balls momentum is nearly same.

So, Pf and Py are equal in magnitude, but the ball is now traveling in the plus x direction, which means the direction is different. What is the impulse applied to the ball by the wall? So, before I go to the calculation Let me note two things. The first is that momentum is a vector. So, this is a vector. So, even though the ball has same magnitude the momentum before and after collision, But their direction is different.

Hence, there is a change in momentum. So, delta P, which is a vector, which is Pf the final value minus the initial value; this is not 0. So, we have to consider the direction. Also, when you calculate the change in momentum, the second point is that, so how do we calculate this change in momentum? So, note that in this case, we do not know anything about the force that is acting between the ball and the wall. We also do not know that for how long the ball was in contact with the wall; that is the duration of the collision. But the beauty of Newton's second law is that, even though you do not know this, but it says that because there was, there must be a force between, because we observe that there is a change in momentum which means that there must be a force acting on the ball, and the cause of this change in momentum is the force exerted by the wall during the collision.

Example 6: impulse in a collision



A tennis ball traveling in the −*x* direction with magnitude of momentum *2.8 kg m/s* hits a wall and rebounds. After the collision the magnitude of the ball's momentum is nearly the same, but the ball is traveling in the +*x* direction. What was the impulse applied to the ball by the wall?

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i \neq 0$$

$$\vec{p}_i = -2.8 \text{ kg m/s } \hat{x}$$

$$\vec{p}_f = +2.8 \text{ kg m/s } \hat{x}$$

$$\vec{p}_f - \vec{p}_i = \vec{I}_{mpulse} = [2.8 - (-2.8)] \text{ kg m/s } \hat{x}$$

$$= 5.6 \text{ kg m/s } \hat{x}$$

So, we can compute the net effect without knowing either the force or the duration of the collision. So, how to calculate? So, let us first write. We have to be careful about the vector. So, we say that P i is, so the magnitude is 2.8 kg meter per second and it is suppose we call this, we define an unit vector in the plus x direction as x hat, then its direction is in the negative, so I put a minus sign and Pf, so these are given in the problem as 2.8 kg/meter/second. So, then Pf minus Py which is the required impulse, is minus 2.8 kg/meter/second. Which is equal to 5.6 kg/meter/second in the positive x direction.

Example 7: impulse due to gravity

A tennis ball of mass m = *100 gm* is freely falling due to gravity. What is the impulse due to gravity in a time interval of *1 s*? [Assume $g=10\ m/s^2$]

$$\Delta\vec{p} = \vec{F}_{net}\Delta t$$

$\vec{F} = mg \downarrow$

$\Delta t = 1s.$

Impulse $= \Delta\vec{p} = \vec{F}\Delta t$

$= 0.1\,kg \times 10\,m/s^2 \times 1s$

$= 1\,kg\,m/s.$

Note: impulse can be computed for *any* force.
Type (1): Large force in a short time interval.
As Δt → 0, impulse remains finite.
Type (2): As Δt → 0, impulse tends to zero. → continuous force

So, the direction of the force by the wall on the ball is in this direction. So, in the next example, we will calculate the impulse due to gravity. So, a tennis ball of mass m, which is equal to 100 grams, is freely falling due to gravity. What is the impulse due to gravity in a time interval of 1 second? So, assume that the acceleration due to gravity is 10 meter per second square. So, again In this case we, so this is a situation in which we know the force acting on the ball which is its weight m times g in the downward direction, and the time interval is 1 second. So, then using Newton's second law, so in this case we know the right hand side of the equation is given, The left-hand side is unknown, so this is the opposite of the previous problem.

So, then we can calculate the impulse or the change in momentum, the f times, so in this case there is just one force acting on the particles. So, which is the mass, I am going to convert everything to the SI unit, So that I do not make mistakes with units. So, 100 gram is 0.1 kg times the acceleration due to gravity times 1 second. So, I am deliberately writing the units, so that you can see that they also play a role in the final calculation.

So, what you get is 1 kg, so this meter second square in the denominator and the second in the numerator, so one of them get cancelled, so this is and 0.1 times 10 is 1 into 1 is 1, so kg meter per second, so the units checks out. So, the point, the message here is that Again, as I repeat, impulse can be computed for any force; it is defined for any force and There are typically two situations we encounter: one in which a large force is acting for a short time interval. In this case, the hallmark of this scenario is that if you reduce the time interval towards 0, then The impulse remains finite.

So, this is the situation where impulse is the natural way to analyze the problem. The other situation such as in this example, This force due to gravity is not large, so in this case, in fact, it is constant. So, in this case, if delta t tends to 0, The impulse also tends toward 0, so this is a hallmark of a situation where the force is continuously changing. So, in the next lecture, we shall continue with our review of Newton's laws of motion, and we will go into Newton's first law of motion.