Newtonian Mechanics With Examples

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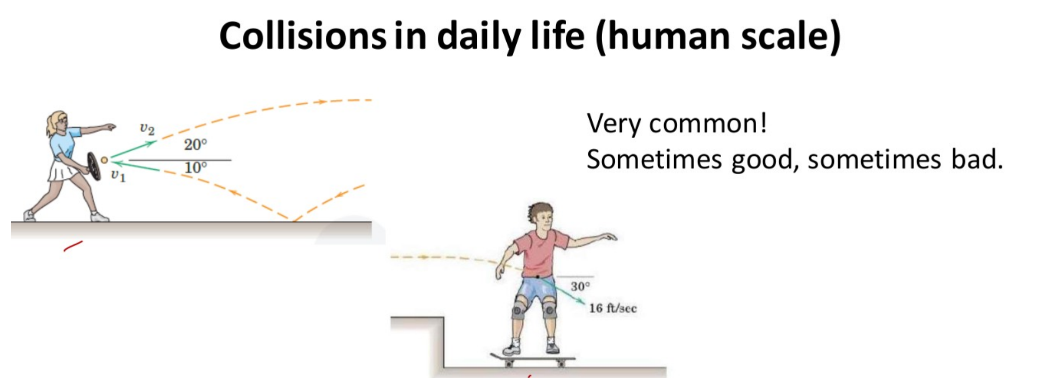
Week -06
Lecture-32

Today's lecture, we are going to discuss some examples about a very important topic in physics, which is collisions and scattering. So, we are going to look at few examples as an application of the momentum and energy principle and Newton's laws of motion that we have discussed in the earlier weeks. So, first let us start about collisions in daily life, that is at the scale of our human experience. So, when the word collision comes to our mind, usually, so the things that come to our mind is like cricket ball hitting a bat or a tennis ball hitting a tennis racket, such as in this case or some, let us say this boy who is, which is jumping on this skateboard. So, these are examples where kind of collisions are useful. And sometimes collisions are also, can be very bad, such as collision between two persons or collision between two cars, etc.



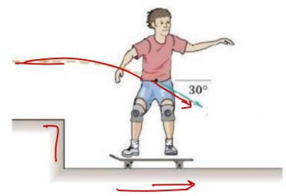
But, we all have experience with collisions and it is very common in our daily life and sometimes collision is good for us, sometimes it is bad for us. But this is something that we should know how to analyze. So, we are going to take some examples from daily life. So, our first example is jumping on a skateboard.

So, it is described in this picture. So, there is a boy, so the skateboard was initially standing still, that is stationary on the road and there is some sort of a stair. The boy was standing here and he was running on this upper surface and then he jumped on the skateboard. So, he jumped on the skateboard, like this is the trajectory of the center of mass of the boy and as a result what will happen? So, the skateboard will try to move, so it will get a jerk impact and as a result of this impact, the skateboard will try to move to the right with the boy. So, this is how he initiates the skateboard.

So, this is the example.

Example 26: jumping on a skateboard

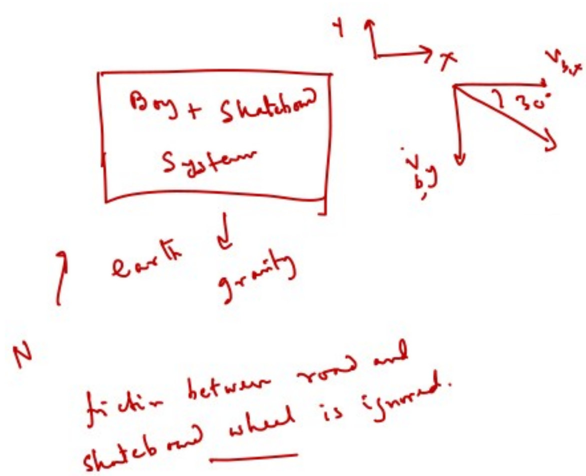
A *40-Kg* boy has taken a running jump from the upper surface and lands on his *5-Kg* skateboard with a velocity of *5 m/s* in the plane of the figure as shown. If his impact with the skateboard has a time duration of *0.05 s*, determine the final speed *v* along the horizontal surface and the total normal force *N* exerted by the surface on the skateboard wheels during the impact.



[Ans: v = 3.85 m/ s, N = 2450 N]

So, how do we analyze this kind of problem? So, let us go step by step. So, first let us define our system.

So, this is as you remember the first step in our modeling. So, we are going to take the boy plus the skateboard as our system and we will look at the what everything else in the surrounding and then we will look at the interactions. So, boy plus the skateboard, this is our system and then everything else in the surrounding. So, what are the, so this is our system. So, what else are there in the surrounding? So, they have mass.



So, earth is interacting with them. So, there is this earth's gravity. Then, the road, so the skateboard is placed on the road. So, the road will act, will provide a force on the skateboard. So, there is this normal force.

Now note that we are going to consider the system for only during the duration of the collision. So, the skateboard has just gained the velocity from 0 to v, but it has not moved really. So, this is our assumption. So, then within that condition, we are going to, so going to ignore any friction is ignored. So, this is the situation.

So, this is one important thing that you should keep in mind that before just directly plugging into formula, you should set up in this way the what are the system, what are the interactions so that you can get the right intuition about solving the problem and understand the motion. Now, in addition there are two other forces which are internal forces in the system. So, the boy due to jump collides with the skateboard, so the boy hits the skateboard at an angle 30 degree with the horizontal. So, it has a downward velocity v_y and a let us call it v_b, b for the boy, $v_{b,x}$ component. Now because of this horizontal component, so there is a downward momentum with which the boy is hitting the skateboard and if the boy after the impact the boy is and the skateboard are moving in the horizontally towards right, so only in the x direction.

So, that means the y direction the downward momentum of the boy was stopped by the skateboard. So, the boy is hitting the skateboard with some force and the skateboard is also hitting the boy with some force by Newton's third law of motion. So, there is a change in the momentum, so this is the internal force between the boy and the skateboard. So, this is the setup. So now but the point is there all these forces are kind of a priory not known.

So, in this case the useful way to analyze is to by looking at the change in momentum of the system. So, let us write it down. So, at the beginning of the interval,

$\Delta t = 0.05\,s$ of impact duration

$m_b =$ mass of boy $= 40$ kg

$m_s =$ mass of skateboard $= 5$ kg

$|\vec{v}_b| = 5$ m/s

So, at the beginning what was the momentum of the boy? So, the boy's momentum was its mass, so let us say in the x direction. The boy's momentum was $m_b v_{b,x}$. All the momentums, velocities are by the way with respect to the ground. What about the momentum of the skateboard? The skateboard was initially at rest, so its momentum was 0.

At the beginning of the interval

$\hat{x}$: $m_b v_{b,x} + 0$

At the end of the interval, the boy and skateboard are moving together. So, they have a, so let us say m b equal to mass of boy which is 40 kg and m_s is mass of skateboard which is 5 kg. Now our momentum principle, now the point is given this situation and this assumption, there is no external force in the horizontal direction. So, there is a conservation of the x component of the momentum. So, the fact we can write it by equating them.

At the end of the interval

$$(m_b + m_s)\, v$$

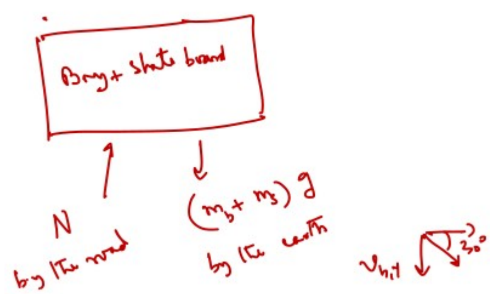
$$\rightarrow v = \frac{m_b}{(m_b + m_s)} v_{b,x}$$

$$= \frac{m_b}{m_b + m_s}\, 5\ m/s \cdot \cos 30^\circ$$

$$= \frac{40\ \cancel{kg}}{45\ \cancel{kg}} \times 5 \frac{\sqrt{3}}{2}\ m/s$$

So, this is the answer to the first part. Now to answer the second part of the question asked that what is the total normal force A in exerted by the surface on the skateboard. For that let us look at the, let us look at the what happens in the y component.

$$\Delta \vec{p} = \vec{F}^{ext} \Delta t$$



$$g = 10\ m/s^2 \qquad v_{b,y}$$

So, again the y component at the beginning of the moment of the interval, the momentum of the boy in the y direction is $M_b v_{b,y}$, skateboard was stationary, so this is, was 0. At the end of the interval, there is no y component, the no momentum of the mass or boy of the skateboard into y direction, so this is 0.

At the beginning of interval	At the end of interval
$m_b v_{b,y} + 0$	0

Now note that we cannot equate them because there are forces in the external direction, so we have to remember our principle that the change in momentum in a time interval Δt is equal to the net force acting on a system and this is our system. So, what are the forces? So the force, there are two forces, one force by the road and the other force is gravity which is by the earth. As we have discussed many times that these two forces are very different forces, they have different physical origin. So the N is positive. Now this gives us the N.

$$-m_b v_{b,y} = \left[N - (m_b + m_s)g\right]\Delta t$$

$$N = \underline{(m_b + m_s)g} - \frac{m_b v_{b,y}}{\Delta t}$$

$$= (40+5) \times 10 \ \text{kg}\frac{\text{m}}{\text{s}^2} - \frac{40\text{kg} \times (-5 \text{m/s} \sin 30^\circ)}{0.05 \text{s}}$$

$$= \underline{2450 \ \text{N}}.$$

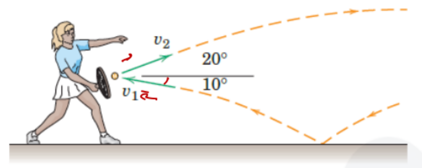
Cannot ignore weight !

So, I will let you do the simplification and the, so show that this answer is about 2450 Newton.

Now note one thing that sometimes we often say that during a collision the forces due to impact are so large that you can ignore the other forces that are continuous. In this case this weight is a continuous force that is acting continuously and this part is the impact or impulsive force due to the collision. But note that if you compare the magnitude of these two forces then the impulsive force is large but not too large compared to the weight, so you cannot ignore weight in this case. So, this is how we analyze this sort of problems.

So, let us take another example of collision from daily life. So, this is a striking a tennis ball. So, here is a tennis player and there is a ball which has dropped. So, this is the trajectory of the ball shown in the orange. So, it hit the ground and then was on the rise and then she hits the ball and returns it.

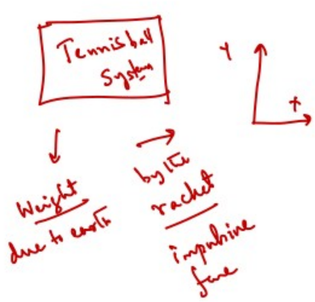
A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is v_1 = *15 m/s* and after impact its speed is v_2 = *22 m/s*, with directions as shown in the figure. If the *60-g* ball is in contact with the racket for *0.05 s*, determine the magnitude of the average force ***R*** exerted by the racket on the ball. Find the angle *b* made by ***R*** with the horizontal.

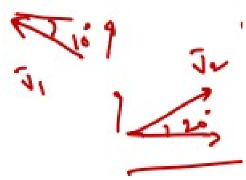


[Ans: Rx = 42.5 N, Ry = 6.49 N, *b* = 8.7 degree]

So, this is the question. Again as before we first start by defining our system.

In this case, it is simplest to consider the tennis ball as our system because the problem ask about what are the force acts on the tennis ball, so this is our system. So, then the racket and is in the surrounding, so in the surrounding so what are the forces acting on this tennis ball. So, the one force is its weight which is due to the earth, so these are the interactions and the other force in some direction is by the racket. So this force is kind of an impulsive force due to collision and this is a continuous force. So, now our goal is to analyze the problem.





Again, we will follow the approach that we will look at the time interval Δt during which so the Δt the impact time interval. So, at the beginning of this time interval and we look at the momentum of the system at the end of this time interval. So it is given in this picture that this is the v_1 and it makes an angle 10 degree. Now we define as by so these are the positive directions of x and y axis. So, the ball the momentum of the ball was v_1 and then let us say x and at the end of the time interval the momentum of the ball is v_2 so it goes with a angle 20 degree.

So, if we look at the x direction so what happens the x component came towards left gets hit and then moves towards switches reverses and goes to the right with a bigger magnitude. Then if we look at a difference this difference must be equal to the external forces acting on the system. Now in this case the only external force in the horizontal direction. So, the gravity doesn't have a component in the horizontal direction. So, the only external force must be due to the racket.

So, this equation let us call it equation 1 will give us the x component of the force the force R. Similarly for the y direction so for the y direction the if you look at the y direction of the velocity it was going like this and after the collision so y component it was rising the ball was rising so the y component of the velocity is in the positive direction. After the collision ball is still rising so look at the picture so after the collision the ball is still rising with a different speed in the y component. So ball is still rising there is a difference so there is a change in momentum in the y direction as well so before the collision at the beginning of the time interval the momentum in the y direction is this much and at the end of the time interval at the end of the time interval the momentum is this. So then the change in momentum is and then by applying our momentum principle the change in the momentum of the system must be due to the external force acting on the system times the duration of the interval. So, there are now two external forces one is R_y which is acting in the upward direction the other is the weight of the ball which is acting in the downward direction so according our sign convention R_y is positive and the weight is negative times the time interval.

At the beginning of this time interval

$m_b v_{1,x}$

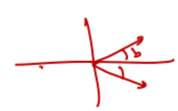
$m_b v_{1,y}$

At the end of time interval

$m_b v_{2,x}$

$m_b v_{2,y}$

$$\boxed{m_b (v_{2,x} - v_{1,x}) = R_x \Delta t}$$



$$m_b (v_{2,y} - v_{1,y}) = (R_y - m_b g)\Delta t$$

$$R_y = \frac{m_b (v_{2,y} - v_{1,y})}{\Delta t} + m_b g$$

$$R_x = \frac{m_b (v_{2,x} - v_{1,x})}{\Delta t}$$

$$\boxed{\begin{aligned} \cos b &= \frac{R_x}{R} \Rightarrow b = \cos^{-1}\frac{R_x}{R} \\ \sin b &= \frac{R_y}{R} \Rightarrow b = \sin^{-1}\frac{R_y}{R} \end{aligned}}$$

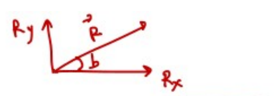
$$\tan b = \frac{R_y}{R_x} \Rightarrow b = \tan^{-1}\frac{R_y}{R_x}$$

So, then from this equation we can rearrange the terms to get the R_y. So, our rule is multiplied by Δt so the change in the x component of the momentum should be the $R_x\Delta t$. So then from here we can write R_x.

$$v_{1,x} = -15\,m/s \cos 10^\circ \qquad m_b = 0.06\,kg.$$
$$v_{1,y} = 15\,m/s \sin 10^\circ \qquad g = 9.81\,m/s^2$$
$$v_{2,x} = 22\,m/s \cos 20^\circ$$
$$v_{2,y} = 22\,m/s \sin 20^\circ$$
$$\underline{0 < b < 90^\circ}$$

Now Δt as I mentioned is 0.05 second. So then I invite you to plug in these numbers and compute what is R_x and R_y. Now once you get this so the second part of the problem ask that what is the direction of R_x and R_y.

So, here in this case how do I get the direction so this is R_x this is R_y so then the by vector law of addition the resultant of this is the the actual force acting in this direction and this is the angle b. Now one thing that so from once you compute Rx and Ry we can get the magnitude which is R as simply from the components and then we can get and remember from this picture it is clear that



$$|\vec{R}| = R = \sqrt{R_x^2 + R_y^2}$$
$$\left.\begin{aligned} R_x &= R \cos b \\ R_y &= R \sin b \end{aligned}\right]$$

Now here one thing I want to mention that in this situation is clear that b is a angle in the first quadrant. But in general, we do not know what is a b. So it can be in any quadrant So, this is a generic problem in computation. So, you have determined the components and now we want to find out the direction. So, in this case you always get two solutions because if you have an angle so if you have an angle like this as acute acute angle like this so there are this angle will also have the same cosine components with different sign components. So, in general if you have a trigonometric equation of this form. So, it is always safe to compute b as an inverse of both cos and the sin and then determine. So then you get an unique answer because when you check both of them, the usually textbook what they mention is to calculate tan b which is

$$\tan b = \frac{R_y}{R_x} \Rightarrow b = \tan^{-1} \frac{R_y}{R_x}$$

So, this I would not recommend because in this has an ambiguity because there are two angles with the same tangent by as you know from the trigonometric property of angles. So, it is better to look at the simultaneous solution of inverse cos and inverse sin to determine the angle. So after this our next example will be something that is not so familiar from our everyday life but now we are going to go towards smaller length scales. Even at smaller length scales the collisions play a very significant role. So, that is going to be our next example. Thank you.