Newtonian Mechanics With Examples

Shiladitya Sengupta
Department of Physics
IIT Roorkee
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Welcome everyone. So, we were discussing projectile motion. And in the last lecture, we sort of reviewed the difference between the projectiles in real life versus your idealized version in the typical textbook. Now here, when I say projectile, what I have in mind, so that you should not think that projectile is simply just a ball that you throw in through air, but it is also a bird which flies through air or a plane that flies through air or a rocket. So, we cover everything that can move through. It is a broad definition.

And what we discussed in the last lecture is that apart from gravity, there are several other forces that are present and the effect of the air, the fluid through which the projectile moves is very, very significant on the how the projectile can move. And in the last lecture, we mentioned three forces, the drag force, the lift force and the thrust force. So, we discussed drag force in the context of friction. So, today, our first focus is to look at the effect of lift force.

So, what is a lift force? So, we will try to understand. This is an another effect of the air on the projectile. So, this is what is responsible for, so this force is present if there is a fluid flow surpassed the projectile and this force is responsible for bird, flight of the birds and planes. So, we will take a simple example in which we will look at the something called hovering. So, this is a picture of some bird, it is a very common bird in India.

Very small tiny bird like this size. Its wingspan is about 10 centimeter, and it is also very light, about let us say 10 grams. So, this is called a sunbird. And you have seen this bird, they come to the flowers and take the nectars. So, they are very important for pollination of flowers.

And you have seen that when they are in the taking the, sucking the nectar from the flower, they are not at all moving, they are not at all flying, they are standing still at one place in air. So, this motion whereby, so what they are doing, they are rapidly flapping

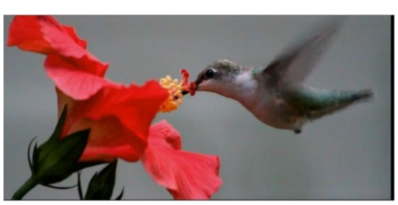


Image ref: sunbirds of Kanha National Park
https://www.youtube.com/watch?v=osp5tGS0U1Y

Ex 24: Hovering of a sunbird

Q: How much power does a sunbird required to hover? Assume wingspan, $L=10\ cm$, mass of a bird, $m= 10\ gm$, density of air $\rho_{air}=1\ Kg/m^3$.

their wing and due to that they can stand still at one position in air and that is called hovering. So, the goal of this example is to understand how is it, how the bird do that. So, to make things concrete, so here is a question that how much power does sunbird required to hover? So, we assume certain parameters about the bird. So, I must mention that these parameters are little bit guesswork.

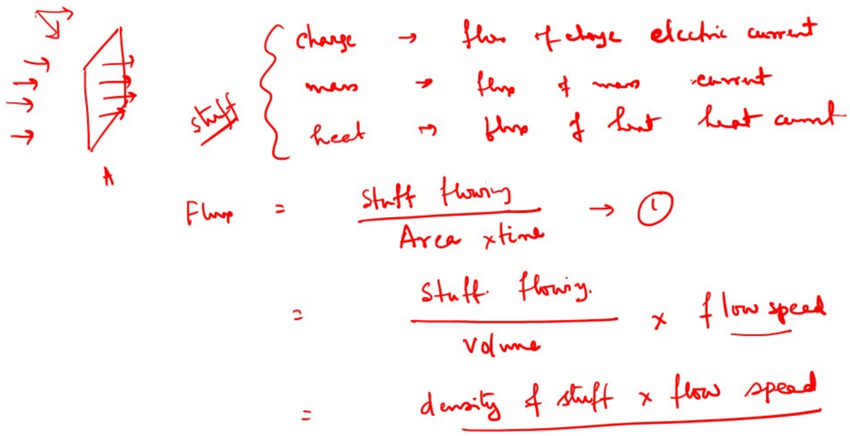
So, they are, I mean they are not very accurate, they can be off by a factor of 1 or 2. So, this our goal is to sort of get a quantitative estimate to get a feel about the numerical feel about the problem. So, how does the bird stay still or hover at one place? You know what, now here again as I said that we can, we shall take two different point of views to analyze this problem. So, first point of view is something that is already familiar to us which is like drawing a free body diagram. So, we are going to take the bird as our system and then we will, everything else in the surrounding and there are two important forces acting on the bird.

So, surrounding will consist of the earth and the air and we will look at the interaction between the bird and the air, bird and the earth and we will draw the free body diagram. So, this is effectively a problem of statics. So, we will do force balance and then we will take a different point of view where we do, we will come at the same result at a slightly different way of looking at it, we will apply the momentum balance. Now one point, crucial point here is that we know the weight of the bird is due to the mass mg, but we do not know how much is the lift force and what is the origin of the lift force. The origin of the lift force is very simple to understand.

It is because the bird is flapping its wing, so it is creating a flow of air downward. So, the bird is transferring momentum to the air. So, and as we discussed before that our principle that change in momentum is equal to the force acting on the system times F. So, if you look at the, if you consider air as the system, then the bird is transferring momentum that means the bird is exerting force on the air. Then by Newton's third law, the air is also exerting a force on the bird and this force is the lift force and this force must act in the direction opposite to the weight of the bird to counter and that is the reason the bird can stand still.

So, this is one way of looking at the problem. Now we need to derive an expression for the how much is this lift force and here I am going to introduce to you a very important quantity. Because the fluid, the air, because of the flapping of wings of the bird, the air is flowing. Now when you have some fluid, something flowing, you have a current and there is a very important concept that is used to measure how much some stuff is flowing. That concept is called flux.

I am sure you have introduced, seen this flux in various other situations. So, what is flux? Simply speaking, imagine that you have a door and this door has some area, area A and then something is flowing through the door. So and this, what is this something? Well for example, if this something is charged, electrical charge, then the flow of charge is called electric current. This something can also be mass. For example, let us say you are watching some, you place some kind of a door in a river and water is flowing through the door.



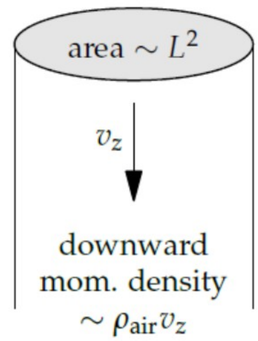


Image ref: *The Art of Insight in Science and Engineering,* by Sanjay Mahajon, MIT Press (2014).

So, it could be mass and then this flux of mass is called normal current or mass current. It could be energy such as heat energy. So, imagine that it is a, this door, the different, two sides of the door has different temperature, then the heat can flow from the hot side to the cold side. So, it could be heat and then this flux of heat is called heat current. So, this flux of water, you can also think of shower.

So, when you switch on the shower, the water is coming through these nozzles and there also these nozzles are like acting as doors through which the mass of water is flowing. So, in general, some stuff, so all these different quantities I am going to sort of call as stuff, something. Then what is the definition? So, we want to quantify the flow. So, what we say is that how much stuff that is flowing per unit area per unit time. So, as you see that more you wait, if you watch the flow for an interval Δt, the longer the time interval of observation, the more amount of stuff will flow through this door.

Similarly, if the door has a higher cross-sectional area, then the more amount of stuff will flow through this door. So, we divide by area, so per unit area per unit time. And here note that, that I have drawn in a way that the flow is perpendicular to the orientation of the door. If there is some component, so let us say if it is moving in some slanted direction, then all you have to do is that you have to take the component of the flow which is perpendicular to the door. The other component does not count because it is not going to flow through the door.

So, it is always the flow perpendicular to the orientation of the door. So, this is flux. Now, another way of writing these flux is that which is clear dimensionally is that the stuff flowing per unit volume times the flow speed. So, I am not going to derive it, it is very easy to derive this expression and show that these two are equivalent. But, I just want to point out that is it clear from dimensional analysis that if I multiply this equation 1, both numerator and denominator by something at length, then length per unit time will be some dimension of speed and then the denominator, there will be area times length which is $length^3$ which are the dimension of volume.

Now, this amount of stuff which is flowing per unit volume is called density. For example, if my stuff is a charge, then there is the charge density times the speed of the charge that constitutes the flux. If it is mass that is flowing, mass of water that is flowing, then this is mass density that is our normal density times the speed of flow of the water. Let us say the water that is coming out of the shower or water that is flowing through here, through a river. If it is energy or heat, then this will be heat density or energy density that is flowing times the speed of flow.

So, this is all we need to know for our analysis. Now, look at this particular problem that of the hovering of the bird. So, then what is this door? What is this door? So, the bird, so this is a schematic diagram. So the bird, I am replacing, representing the bird by a cross-sectional area of area L square, so some length L and then the cross-sectional area is L square. And due to the flapping of wings, there is a flow of air downward.

That is, there is a flux of air downward. So, that means in this case, what is flowing is momentum. And what we need to estimate is the momentum flux. And by our definition, this is going to be the momentum density times the flow, the speed of momentum flow, so the flow speed.

Now, momentum is a vector. So, we are talking about, so the relevant momentum is in the downward direction. Why downward direction? Because we are going to see that this is the momentum that is going to balance the weight. So, we are considering the direction in which the gravitational attraction is acting on the bird. Now, what is the momentum? So, momentum as you know is mass times the velocity. So, in this case, assume, consider that the downward velocity that is in the z direction, in the downward vertical direction of the air that is flowing due to the, so then the momentum is the downward momentum.

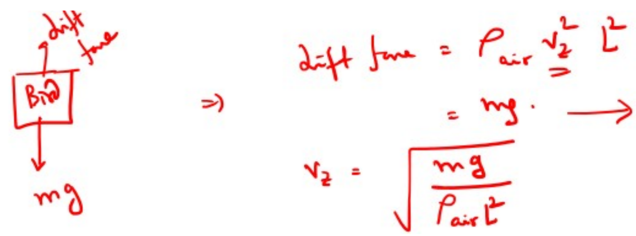
So, let us write downward to sort of emphasize the direction of the momentum will be mv_z and it is a density, so per unit volume. So, this will be mass per unit volume of the air that is flowing. So, that means mass per unit volume is nothing but the density of the air. So, this is the density of the air times v_z represents the downward momentum density. The flow speed of momentum is again v_z. So, that means the flux of momentum is v_z^2. So, this is $\rho_{air} v_z^2$.

So, as I said that this momentum is transferred by the bird to the air, which means the bird is exerting a force on the air. So, how to estimate that force? So, what is the relation between flux of momentum and the force? So, let us look at the dimension analysis again. So, if we look at this momentum principle that the change in momentum is force times the interval over which the force is acting.

So, if we look at the momentum balance principle, then the relation between momentum, So, we find that the dimension of force is momentum/time. Now, if we multiply up and down by area, then what is this quantity? In from our definition, this is nothing but the flux of momentum. So, the flux of momentum times the area through which this momentum flux is passing gives you the force. So, we have already estimated the flux of momentum to be $\rho_{air}\ v_z^2$. That means that this force exerted by the bird on air is given by

Now what is this L? What is the area that the bird is having? Is it the area of the body? No, this is the area over which the bird is generating the air flow and this is what is called the wingspan. So, the bird, if it spreads the wing, then the tip to tip distance of the wing that is called the wingspan. That is the relevant length over which the bird is able to generate the air flow. So, this is the reason that in many planes the wing, so, plane itself is does not have much cross-sectional area, but is some wingspan is usually very large. So, that is the relevant the cross section that the over which the lift force is acting.

So, this L is the wingspan of the bird. So, now you are able to write an expression of the force by the bird on air. So, by Newton's third law, the force by air on bird is also going to be the magnitude of the force, the direction is now in the upward direction. So, then if the bird is standing still, so now we can draw the free body diagram. So, this is our box that represents the bird. So, there are two interaction, one is the weight downwards and the lift force by the air, and then this lift force we just estimated to be $\rho_{air}\ v_z^2 L$ and this must be equal to mg.



Now in this case what is unknown is this problem the L is given, ρ air is given, what is unknown is the mass of the bird is given, what is unknown is the v_z, the flow speed that the bird is able to generate, and the flow speed will be such that they must be equal. So, that gives us an expression for the flow speed due to the flapping of the wing. Now, the question is about the power generated by the bird. So, the power is work done per unit time, which is force times velocity.

$$
\begin{aligned}
P_{ower} &= F \cdot v_z \\
&= mg \cdot v_z \\
&= 10^{-2}\,\text{Kg} \times 10\,\frac{m}{s^2} \times 3\,\frac{m}{s} \\
&= 0.3\,\text{W} = \underline{300\,\text{mW}}
\end{aligned}
$$

So, let us first, and then this F in equilibrium due to this equation we can use either of this expression, so we are going to use the force to be equal to the weight of the bird times the flow velocity. So, let us first calculate the flow velocity.

So, this is about 0.3 watt or 300 milli watt. So, is it a big number? So, here it looks like 300 milli watt is not a big number. However, we must also realize that the bird has, is very small object, it is only 10 gram its mass. So, we should look at the power per mass, per unit mass. So, if we compare power per unit mass, then from this expression what we get is

$$
\frac{P}{m} = g \cdot v_z = 10\,\frac{m}{s^2} \times 3\,\frac{m}{s} = 30\,\frac{\text{W}}{\text{Kg}}.
$$

Now is this a big number? Now to make sense of this number, one way to make sense of this number is to compare with what is the human, how much power human can generate. Now here is an very simple exercise that I give you as a take home experiment. So, you can easily estimate how much power you can generate by doing the following exercise. Go to a building which is at least 3 storey, 4 storey high, and then find out the stairs, and then run, take a stopwatch and run from the ground floor to the top floor and measure how much time you took.

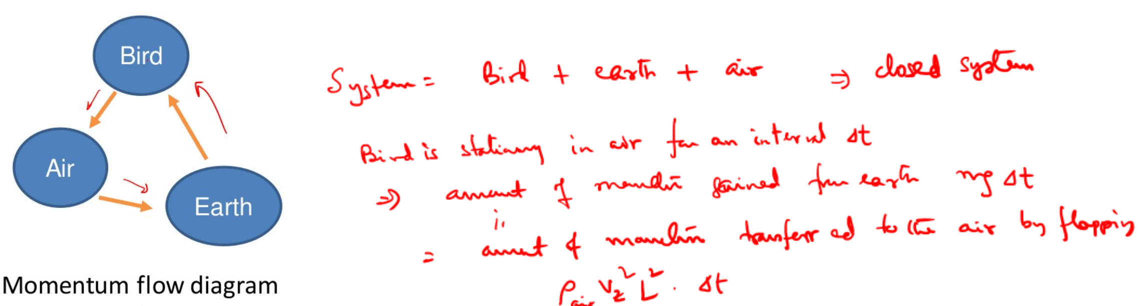
So, the net displacement in the vertical direction, so you have done work against the gravity, so, you can calculate by how much height you have covered, gained by going from ground floor to top floor. So, times your weight will give you the work done by you against the gravity, and divided by the time interval that you have followed will give you the power output of your body. So, mgh divided by the time interval will be your power output. So, then if you sort of divide it by your mass, that will give you the power output of your body per unit kg. And if you compare that, you will find that this number is actually very significant.

So, now we are going to look at this problem in a slightly different way. So, so far we have used the free body diagram technique to analyze the problem where we took the bird as a system. There is another way of looking at this problem is that instead of bird as the system, take the (bird+earth+air) as our system. So, this is a closed system or isolated system. So, there is no external force acting, if this is our choice of the system.

So, then the total momentum is 0. But now you can look at the, there are momentum transform from one part of the component of this system to another part. How? Because the earth is giving force, exerting force on the bird, So, earth is transferring momentum to the bird. Now, so if the bird does not do anything, because of this force, that earth is giving to the bird, it will fall down, it will simply fall. So, the bird has to transfer this momentum somehow, so that it does not fall. So you can think of the bird is flapping the wing to transfer momentum from itself to the surrounding air.

So, bird is flapping wing, transfer momentum from itself to the surrounding air and it is gaining moment, if it gaining momentum due to the gravitational attraction from the earth. Now, if it is able to transfer the amount of momentum that is gaining from earth, full amount to the air, then it can maintain the height and it can over steal at one place in the air. So, this is a slightly unusual way of looking at the problem. But this is a very powerful approach which has, you can use in many, many difficult situations. So, you track the, consider an isolated system and track the momentum flow.

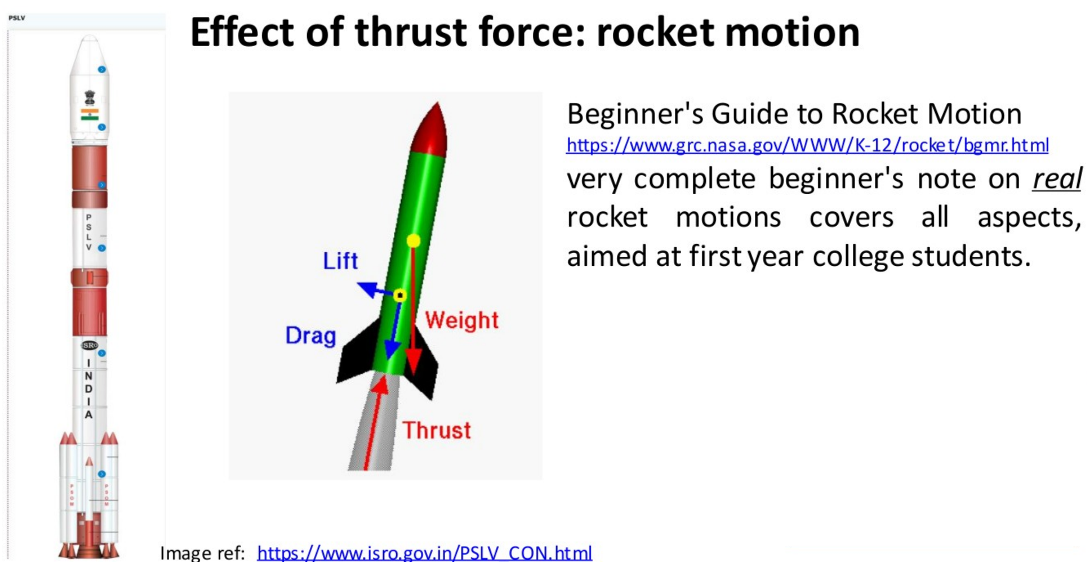
So again, at the steady state, so if the bird is stationary in air, that is the hovering, this means the amount of momentum it gains, gained from earth, so this is the amount of momentum that is gained in the downward direction, which is mgΔt in an interval for an interval Δt, must be equal to the amount of momentum transferred to the air by flapping, which we have just calculated to be times, this is the flow and times Δt will give you the total amount shifted. So, if you equate them, you get back the same equation. Now, what happened to the momentum that is gained by the air? So, if we ignore all the frictions, so we can simply assume that air transfers this momentum, because air is in contact with the earth, transfers this momentum back to the earth. So in our, these three component system, there is a flow of momentum that goes from one component to the another.



Momentum flow diagram including air

And this is an example of this momentum flow diagram. Again, the point is that in this way, we may not, you can ignore lot of details about our specific details about the system and still can gain lot of important insight and analyze problems. So, this is the power and beauty of applying the conservation laws, the conservation of momentum, conservation of energy, etc. Now our next example, we are going to look at the third force that is important for projectile motions, that is the thrust force. And we are going to look at the effect of thrust force on the rocket motion. Now again, I am going to emphasize on real rockets as opposed to textbook rockets.

Now here is an example of an actual rocket. So, this is called PSLV, which is the standard, the most used rocket by ISRO, India's space research organization to launch satellites in the space. As you can see that it has lot of complicated design and in fact, these modern rockets which are used to launch satellites have different stages. And if you go to ISRO's webpage, the link is provided below. You can sort of get to know more details about the design of the rocket. But the main important point to remember here is that the actual satellite is only at the head of the rocket.



It occupies very small portion of the rocket. Most of the rocket is actually fuel, is a tank for the fuel. Now what do we mean by a real rocket? So, here is a schematic diagram of a real rocket. As you can see, there are several forces and when the rocket is an example where all of these forces are important, so it has a weight, so the gravitational attraction is definitely important. But in addition, when it is, so the rocket which is taking, I mean, so all the rockets that we launched so far lifts up from the earth's surface. So, it is actually and it moves at a very, consider high speed through the air.

So, this drag force is definitely important. The lift force also acts on the rocket and of course the thrust is important to lift the rocket and also in outside, in deep space where there is no atmosphere, etc. That is the only mechanism to accelerate the rocket. So, there are several. In addition, I must mention that look at these extra wing tails. So, because of these, so far, we have discussed about the momentum conservation, energy conservation, but the rocket can also spin.

It can also spin through the air. So, the angular momentum of the rocket is also very important. And in fact, if the, in a real rocket, if the, if you want to aim for a certain location where you want the rocket to go, so then you have, must consider the spin of the rocket. And the spin of the rocket is because of the torque by the surrounding air on the rocket and this is because of this design of the rocket. So, if you want to know more about real rockets, I have provided a link to a very good place to start. So, this is a web page from NASA's website and it has a very complete beginner's note on real rocket motions which covers all aspects of actual rockets and the most beautiful part is that is aimed for first year college students.

So, it is not very complicated. So, we are going to take examples of the thrust force in the context of rocket motion in the next lecture. Till then, see you. Thank you.