Course Name: Newtonian Mechanics With Examples
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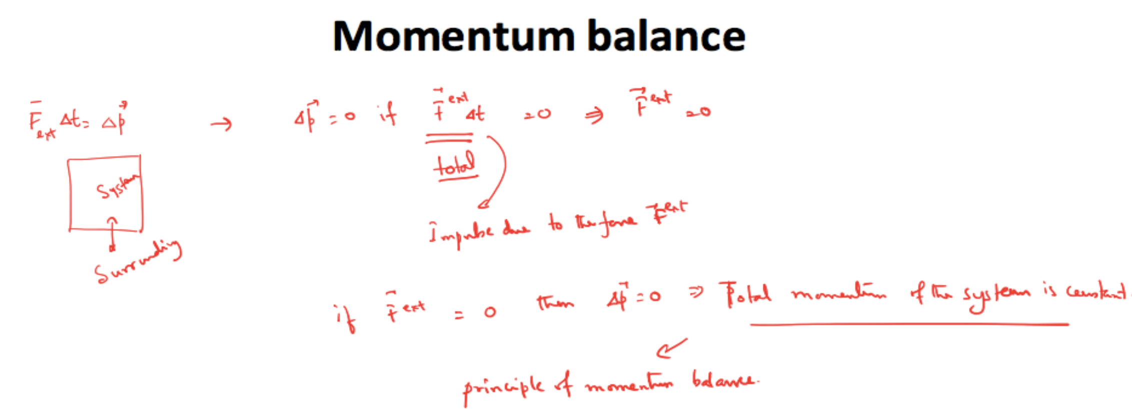
Lecture - 29

Let us continue our discussion of conservation laws at applying the law of conservation laws at applying conservation laws and Newton's laws of motion to solve various problems. So in the last couple of lectures, the focus was mainly on the conservation of energy. In today's lecture, we are going to talk about momentum balance, that is conservation of momentum. So this is what we have remembered in the week where we talked about Newton's laws of motion. The review of Newton's laws of motion. There, we derived and wrote down Newton's law of motion in three levels of increasing generality, and in that context, the most general way of writing Newton's laws of motion is the following: So suppose that we have our system, so recall that first we talk about when we analyze the system mechanics problem, the first step is to identify or define a system.

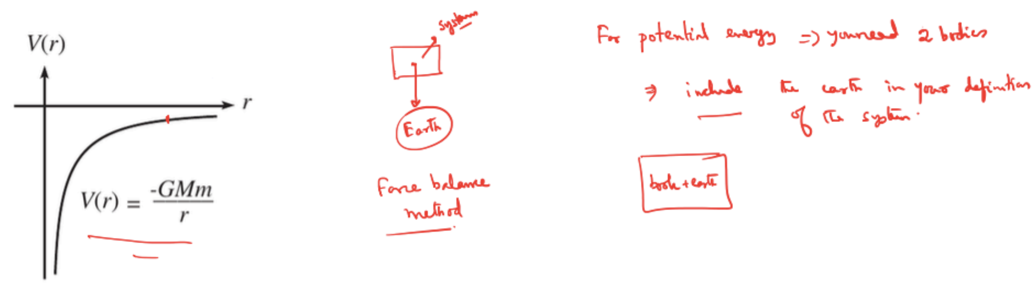


So this is our system and then the, so this is our system and the rest and This is the surroundings, and the system is interacting with our surrounding, so this is our basic box model picture of any mechanics problem. Now the interaction, because of the interaction the, if there are some external force acting on the system, let us say there is some external force F. Which is acting on the system. For a time interval delta t, the net effect of that external force is to change the momentum of the system by an amount delta p.

So this is the most general version of Newton's law. So here, F times delta t represents impulse, so this kind of force can be continuous force. It can be constant force; it can also be something like very instantaneous large impulse force, so all of these things are covered within this simple expression. So then there is an immediate corollary: when the momentum of the system is not changing, so delta p is 0. If the total external force, remember that this is the total force.

So if there is more than one force acting on your system, then you have to take the total. Find out the total force, or rather, the total impulse on the system. So this quantity, this F times delta t is called the impulse due to the force F external. So if there are multiple force, then calculate the total impulse and so if delta p is 0. That means the total internal force is 0, or in the reverse way.

Some subtle points about energy and momentum balance



If the total force is 0, then there is no change in the momentum of our system, which means the total momentum of the system is constant. So this is our principle of momentum balance, or the conservation of momentum. So, I deliberately wrote in the equation in this format because, in the examples that we are going to take, we will see that there are lot of different situations where so the momentum is not simply M b, or the force is not simply mass times acceleration. So it is always important to start with the most general version of Newton's laws of motion to think about the momentum principle. Another point that I want to mention about your choice of system.

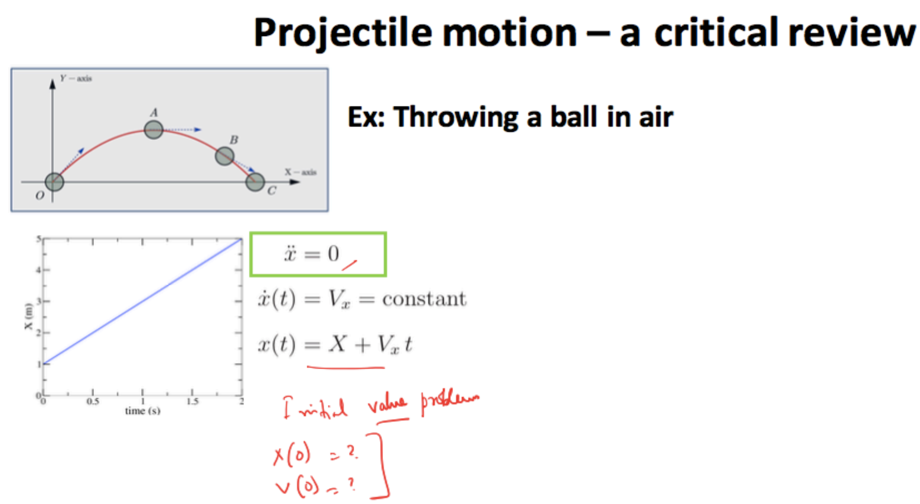
So in your box model, when there is a subtle difference, so this is a general point about energy and momentum balance, so there is certain subtle difference. When we talk about the free body diagram of a system, let us say you have two. So example of two body or particles which are interacting with each other by gravitational interaction. So these two particles can simply be some object like this object and one object. one particle is this object and the other particle could be the earth and they are interacting with certain gravitational interactions, and we have learned before that this is a conservative force.

So you can derive the force of interaction from a potential energy which is shown here where r is the distance between the particles. So, this is our object, so r will represent the distance between the centers of the earth and the centre of mass of this object. Now, normally, when we talk about examples where objects are moving on the surface of earth, then we are basically talking about the, like at one fixed r because we can approximate any distance on the surface of the earth by basically the radius of the earth. So this is a fixed r, and at a fixed r, then the force will appear to be constant. But if we talk about large distances, like interplanetary distance, then the gravitational force, We will see the variation of the force with distance.

So the difference I want to point out is the following: if I take this as an object, and if I want to draw the force-free body diagram, then we usually treat the earth and the interaction between the earth and this object. So this object is our system, and then we can treat the gravitational attraction as an external force acting on this object. However, if we do, this is our force method. However, if we analyse the energy of the system, the gravitational energy, So, for any potential energy, you need two bodies to define the potential energy. So in this case, if you are talking about the potential energy, then you should include the earth in your definition of the system.

So for example, if you usually isolated, if you consider the earth and this object as a part of your system, so your object, let us call it a book, then that forms a closed or isolated system and for which the total energy, the sum over kinetic energy plus potential energy, will remain constant. But here you need to; the point is that you need to include the earth to define this kind of potential energy. So these are certain conceptual points. Now we are ready to apply what we have learnt so far. So we are going to now take some examples, some very important examples and where we are going to apply everything that we have learned so far in the course.

So we are going to apply Newton's laws; we are going to apply the momentum balance. We are going to apply the energy balance. So the first example that I wish to discuss is projectile motion. Now my aim here is to take the projectile motion that is typically discussed in the textbook and critically review different aspects of that and compare what the projectile motion is in real life versus what is described in the textbook. So the typical textbook projectile motion is throwing a ball in the air, such as what is shown in this diagram.



So you start from some position, which you call the origin and then you take the horizontal direction as x axis and vertical direction as y axis and this trajectory defines the position and the blue arrow describe the instantaneous velocity of the ball at different points along the trajectory. Because these positions and velocities are vectors, so we can resolve this motion into two independent components, one along the x axis and one along the y axis. And if you look at the x component, imagine that the ball is creating some shadow on the y axis and some shadow on the x axis. So as the ball is moving, the shadows on the x and y axis are also moving. Now in typically you analyze this problem with in the presence of gravity.

So gravity is acting in the y direction. So that means that in the x direction, there is no force. Hence, if we apply Newton's law, we get that the acceleration along x direction is 0 and as a consequence the shadow on the x axis of the ball will move with a constant velocity. And hence, if you plot the x coordinate of the ball as a function of time, you will get a straight line. Now here is one thing that is important to remind you again: Newton's law is an initial value problem, which means if we want to measure the actual position and velocity at any instant, For the ball, we have to mention the initial position and initial velocity of the ball.

Without that, we cannot determine the subsequent position and velocity. So this is about the horizontal motion. And in the vertical direction, what we have is that gravity is acting downward, and as we said, this is a constant force. So you get a constant acceleration downward, and if you solve it, you find that the y coordinate, the shadow of the ball along y axis as a function of time follows a parabola, as shown in this figure. So then you can eliminate time from, so we have x as a function of time and y as a function of time, and then you can eliminate time from both of these expressions, and finally, you can get the trajectory y versus x.

So this is called the space-evolution trajectory of the particle. So this is called a time evolution trajectory, where you show the position as a function of time and this is the space trajectory, the actual path that the ball is taking. As you know, this path is also a parabola. So I have not shown the origin, so you can sort of extend the parabola. So this is your famous parabolic path.

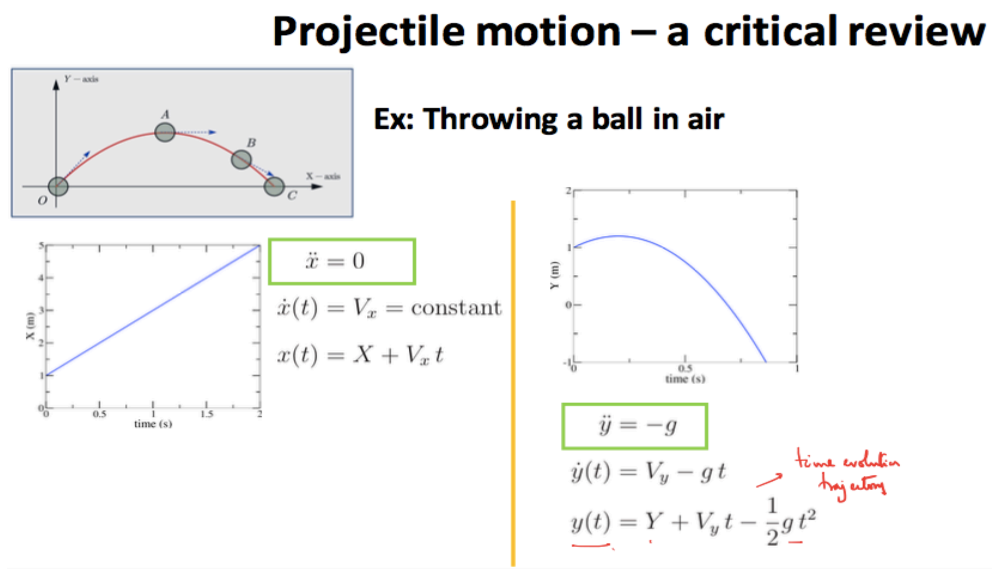
Okay. So let us now, so this is what I am sure that you have already learned in textbook. So let us now critically review this projectile motion. So what happens in real life? The first thing is that there are certain, so this example is kind of idealistic. Which means that there are certain assumptions made in analyzing this problem, and what are these assumptions? So we are going to list them one by one. So the first assumption is that the acceleration due to gravity G is constant.

Now, as I mentioned, if you apply Newton's laws of gravitation, then you get the expression for the acceleration due to gravity and where this capital M is the mass of the earth and capital R represent the distance between the center of the earth and your object, and on the earth's surface, R is basically a constant, and G is a universal constant of gravitation. Hence, this acceleration due to gravity is constant. However, when we talk about projectiles, we should not restrict motion only to the surface of the earth. We should also be able to analyze satellite motion, for example. And if we want to include the satellite motion, then this condition is no longer true.

In fact, it is easy to show that if you assume that G is not constant but is given by this more accurate version of the gravitational force, then you can never get a parabolic trajectory. The trajectory that you get are two kinds of trajectories. One, which is a closed trajectory. As with the trajectory of the earth around the sun, this is an ellipse. The other is an open trajectory, which is not a closed circuit.

So these trajectories are either an ellipse or an open trajectory, which is a hyperbola. So this is called a Kepler problem, or Kepler's two body problem, where two bodies, which are interacting with each other via Newton's law of gravitation. Then what is the space trajectory that they are following? So this is a very standard problem in classical mechanics, in a more advanced course on mechanics. And the answer is that either you get a closed loop, which is an ellipse or you get an open loop, open path which is an hyperbola. Many comets follow this kind of trajectory, but you never get a parabola.

So a parabola is really an approximation of an ellipse or a hyperbola. The second assumption, which is also very standard, is that usually we talk about the when we throw the projectile, we stand on horizontal ground. But this need not be true. For example, you can also stand on an inclined plane and throw the projectile. And in this case, all your standard results about like what is the distance of throw, angle of throw such that it will move the longest horizontal distance.



The standard answer is 45 degree, that will no longer be valid if your ground itself is inclined. So these things you can, I mean, you can sort of still include in your analysis. It just makes your analysis a little bit more complicated in terms of the mathematics. However, there is a very crucial point that makes this textbook example completely ideal and not very realistic, which is that we assume that gravity is the only force. Now, this is misleading and not fully correct.

For example, we have already seen that in our real life we are immersed inside a fluid That is the air around us, or it could be water if you are talking about projectile motion under the sea. In this case, the effect of the fluid. You should never ignore it. We have already seen in the context of our discussion of drag force that this effect. So, this friction due to the fluid leads to something called the terminal velocity.

Which is not present in our ideal example of a projectile. So now, when I talk about projectiles, we are no longer only thinking about a ball. But there is also something like a raindrop, which is also an example of a projectile. I also want to mention but this is beyond the scope of this course, that sometimes the forces, the random forces, also play a very important role, and I am talking about the example of a coin toss. A coin toss is also an example of projectile motion, but it is an unusual example in the sense that here we do not know what is going to be the actual trajectory in the sense that We do not know whether we are going to get a head or tail as an outcome of a toss.

So in this case, random forces play an important role. So what are the additional forces that are present in a real-life projectile motion? One force, as I already mentioned, is the drag force. So this is the friction between a solid and a fluid. Another force: there are two important forces that are also present for various kinds of projectile motion. For example, one force we are going to talk about is the lift force.

So this is again an effect of the fluid through which the projectile is moving. So, this is a force which is responsible for, and due to which, If you think about how a bird flies through air or a plane flies through air, their motions and their flies are possible. So here again, I want to emphasize that a projectile is not only about a ball, so the bird flying through the air is also an example of a projectile motion. And if you want to analyze and understand those motions, then you also need to include the lift force. So we are going to take an example of that, perhaps in the next lecture.

Another important force that we are going to talk about is thrust. So this force is essential for rocket motion to be possible. A rocket is a projectile that can move through a deep space where no fluid, such as the atmosphere, is present. In that case, there is no lift or no drag, but still need some force to accelerate the rocket, and the origin of that force is thrust. So there are other, and of course, various other additional forces that might be present.

So we mentioned these forces, which are sort of very important for in real life and We are going to understand their role through examples in the next lecture. Till then, see you. Thank you.