Course Name: Newtonian Mechanics With Examples
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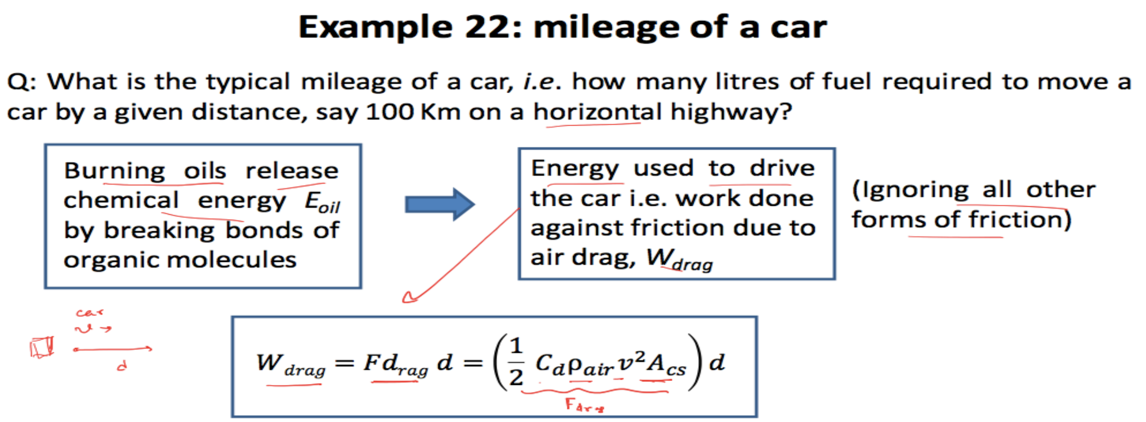
Week 06
Lecture - 27

Welcome to the sixth week of this course on Newtonian mechanics with examples. This week, we are going to discuss a topic that really goes to the very heart of physics. This week, we are going to discuss about the conservation laws. So, here is the plan for the week. We are going to first discuss the conservation of energy. So, we are going to discuss the work-energy theorem and then define the conservative forces.

Some of you may or may not be familiar with it from high school. And then we will talk about the law of conservation of energy. After that, we will discuss the momentum balance or the law of conservation of momentum. Then the plan is to sort of consider a few examples, which are motivated by real life, where we are going to use everything that we have discussed so far.



Example 22: mileage of a car

Q: What is the typical mileage of a car, *i.e.* how many litres of fuel required to move a car by a given distance, say 100 Km on a horizontal highway?

Burning oils release chemical energy E_{oil} by breaking bonds of organic molecules

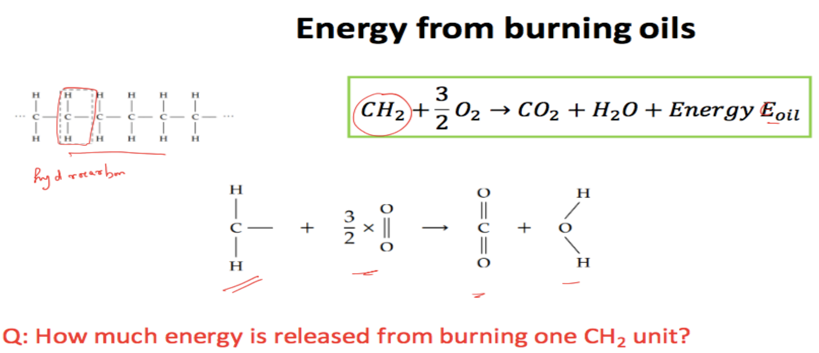
Energy used to drive the car i.e. work done against friction due to air drag, W_{drag}

(Ignoring all other forms of friction)

$$W_{drag} = Fd_{rag}\, d = \left(\frac{1}{2}\underbrace{C_d \rho_{air} v^2 A_{cs}}_{F_{drag}}\right) d$$

So, we are going to use Newton's laws of motion as well as conservation of energy and conservation of momentum. So, let us start with an example. So here is a question. So, let us, we are going to ask that a car, mileage of a car, What is the typical mileage of a car? That is, how many liters of fuel and oil required to move a car by a given distance? Say 100 kilometers on a horizontal highway. So, I put this word horizontal so that there is no consideration of gravity.

That will keep the problem analysis simple. So, how are we going to understand or even estimate this question? So, of course, see the challenges or the trouble is that most of you, including me, We do not know much about, we are not experts on the car or how a car moves on a road, What is the mechanism that goes on inside the car? But this is a quantity that is very useful and very important. If we are using a car or thinking of buying a car, So then, so I am going to show you that we can use even without knowing lot of details, We can get some estimate which are reasonably good. Now, to do any analysis, We need some basic models about what is going on in this situation. So, here is our simple description of the process.



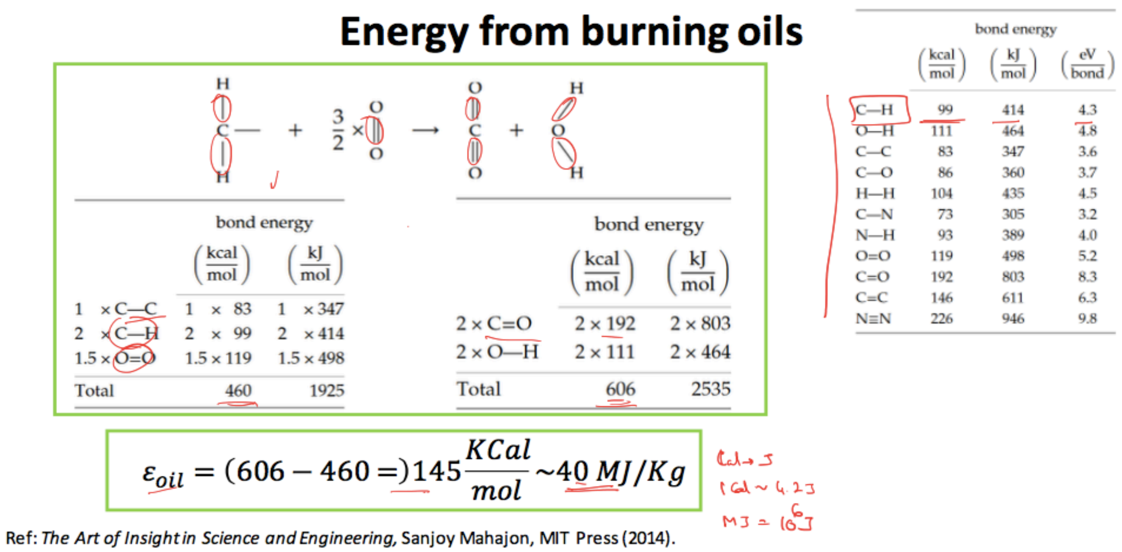
Energy from burning oils

$$CH_2 + \frac{3}{2}O_2 \rightarrow CO_2 + H_2O + Energy\ E_{oil}$$

hydrocarbon

Q: How much energy is released from burning one CH_2 unit?

So, the process means, I mean that a car starts from point A on a highway and It moves 100 kilometer and stops. So, what happens? So, we are going to assume that. So inside the car, so there are some oil, the fuel which can be petrol or diesel. So, this is some oil, and burning this oil releases some chemical energy. So, what is happening? So, this oil consists of some organic molecules because of the chemical reaction that is going on.



Energy from burning oils

	bond energy (kcal/mol)	bond energy (kJ/mol)
1 × C—C	1 × 83	1 × 347
2 × C—H	2 × 99	2 × 414
1.5 × O=O	1.5 × 119	1.5 × 498
Total	460	1925

	bond energy (kcal/mol)	bond energy (kJ/mol)
2 × C=O	2 × 192	2 × 803
2 × O—H	2 × 111	2 × 464
Total	606	2535

	bond energy (kcal/mol)	bond energy (kJ/mol)	bond energy (eV/bond)
C—H	99	414	4.3
O—H	111	464	4.8
C—C	83	347	3.6
C—O	86	360	3.7
H—H	104	435	4.5
C—N	73	305	3.2
N—H	93	389	4.0
O=O	119	498	5.2
C=O	192	803	8.3
C=C	146	611	6.3
N≡N	226	946	9.8

$$\varepsilon_{oil} = (606 - 460 =)145\frac{KCal}{mol} \sim 40\ MJ/Kg$$

Cal → J

1 Cal ~ 4.2 J

$MJ = 10^6 J$

Ref: *The Art of Insight in Science and Engineering*, Sanjoy Mahajon, MIT Press (2014).

So we are just going to assume that some bond-breaking happens inside those molecules that releases energy and this energy is used to drive the car. Now let us understand that why do we need energy at all to drive a car? This is because there is friction. Now, there are many sources of friction when you are driving a car. The principle source of friction is the drag due to the air. So, remember in the last week.

We discussed friction, and there we took drag as an example. There are also other sources of friction, such as the friction between the tire and the road or the friction between with different moving parts inside the car. So, to keep the analysis simple, We are going to assume that we are going to ignore all other forms of friction except drag. So, drag is usually the most significant contributor to friction. So, most of the energy is required to do work against this drag force.

So, now if you remember that, so if the force due to drag. Suppose this is F drag and the car moves a distance d on the highway, then The work done against this force and the magnitude of the work done are, By definition, of work is the F drag times distance by which the car moves, so F drag times d. Now that we derived, remember that in the last week we actually derived an expression for this drag force and we found by dimensional analysis that this force must be proportional to the density, proportional to the square of the velocity, so v is the velocity of the car, Speed of the car, and A is the cross section. So, if the car is moving this way, So let us say this box is the car, then the cross section that is facing the wind, that cross section, let us call that Acs; cs stands for the cross section. And the coefficient of proportionality was denoted as the drag coefficient.

So, We are going to write this, instead of C d, we are going to write this half C d by convention. So, the expression for the drag force is given by this particular combination of the density of air, velocity of the car and the cross section of the car times d. So, this gives you how much energy should be used, so this comes to this part: This term that is easy to estimate. Now, we need to know that how much energy is released from burning the oil. Now, for that, we need to know little bit of chemistry.

So, this is the kind of typical molecules, so these are called hydrocarbon molecules. So this is made of long chains of carbon and hydrogen. So, the oil molecules, So, whatever components make up the petrol or diesel, they all look like this, and there could be lot of different chemistry, chemical details, which are not relevant for our analysis. Now, a typical chemical reaction that is going on inside the car is represented by a simple way. So, let us take this particular unit, one unit of what is called CH2 unit, Which is repeating in this molecule and this combines with oxygen and forms carbon dioxide and water vapour and release energy, which is schematically shown here.

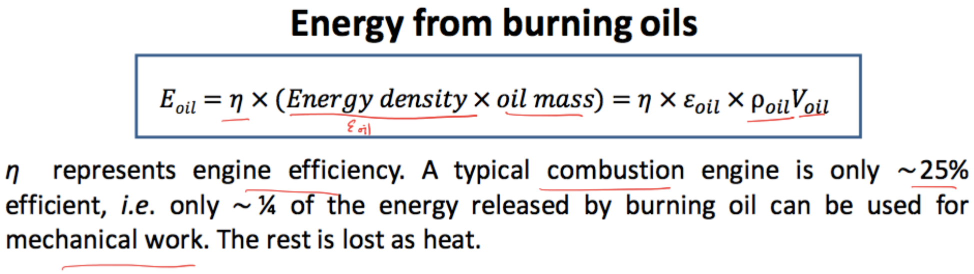
So, this is the CH2 unit which gets repeated in the molecule. One CH2 unit and then it combines with oxygen and this factor of 3 by 2 comes from the balancing the equation on both side and it produces carbon dioxide and water and this releases energy. This energy, let us call that epsilon oil. So, the question is: how much energy is released from burning just one such CH2 unit? So, this can be estimated if we know the bond energies involved in the different components involved in this reaction. So, here is a table which sort of puts some values which chemists have determined.

So, note that we have to remember that these values are kind of not unique. It depends on a particular reaction; for example, this carbon-hydrogen bond energy, This does not depend; this depends on which part, not the molecule and which condition the reaction is going on, etc. So, these are some representative numbers. Now, using this number, you can sort of calculate the bond energies that are present. So, let us say on the left-hand side of this reaction for the reagents, how much is the bond energy of different part of this left-hand side and then if you add all of these bond energies, you get some particular value and note one important point I want to highlight here is that the same energy are written or displayed in different units.

So, this is why I have done it deliberately to highlight the importance of units in any calculation. The second part of the right-hand side, after the reaction, again you have these two bonds and these two bonds and you can compute the bond energy that is for each of these bonds and you get the bond energy of the right-hand side after reagent and you notice that the what is before reaction and after reaction, these values are not same and then the chemistry teaches you that this difference in energy after the reaction and before the reaction is released as the chemical energy from the reaction. So, the energy source that is driving the car and this estimate comes out to be, So the difference is about 140, in particular, using this particular set of numbers, the difference is about 145 kilocalorie per mole, which is approximately, So we are going to convert that into joule. So, we go from calorie to joule; one calorie is approximately 4.2 joule and then, using that, we do the conversion and round it of some round numbers and We get that if we, the energy is about 40 megajoules per kilogram, Which means if we follow just this one sample reaction, then if we burn 1 kg of oil, It is going to release about 40 megajoules of energy.

So, mega joule is 10 to the power of 6 joule. Now, this energy is released, so this is called the energy density. So, this is called the energy density because per unit mass, per unit, per kg. How much energy do you get out of this fuel? Then, if you know the total mass of the oil, which is different for different size of cars, depends on the tank capacity of the car. So then multiplying by the oil mass, you can, you are going to get the full total energy that is released from burning the oil. So, the oil mass we can write thus as the density of the oil times the volume of the oil.

The reason is that when we actually deal with oil, We usually measure in terms of volume. When you go to the fuel station and buy fuel, fill the car, We ask about the volume of the oil, not the mass of the oil. Now here is another important point that we have to factor in. That is that combustion engine. That is the type of car; the engine used in a car is not 100 percent efficient, Which means that not all 100 percent of the energy that is released from this kind of chemical reaction is.



Energy from burning oils

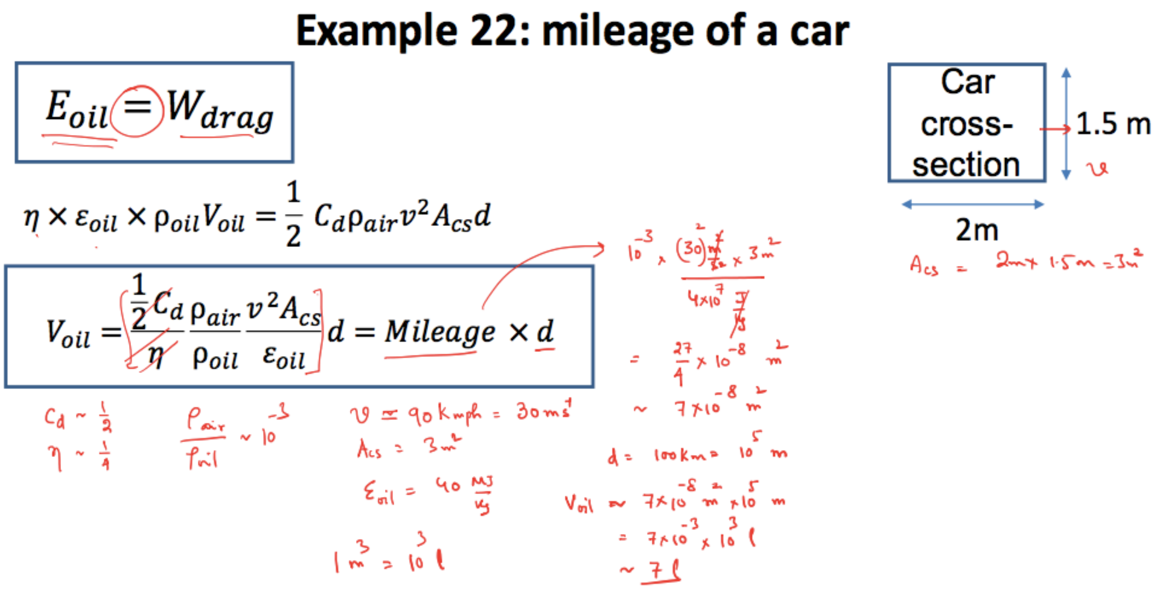
$$\boxed{E_{oil} = \underline{\eta} \times (\underbrace{Energy\ density}_{\varepsilon_{oil}} \times \underline{oil\ mass}) = \eta \times \varepsilon_{oil} \times \underline{\rho_{oil}}\underline{V_{oil}}}$$

η represents engine efficiency. A typical combustion engine is only ~25% efficient, *i.e.* only ~¼ of the energy released by burning oil can be used for mechanical work. The rest is lost as heat.

can be used to do mechanical work, which means to do the, to drive the car. So, a lot of the energy, in fact most of the energy gets lost as heat or sound, and so on, due to the friction. So, a typical combustion engine is only 25 percent efficient. Which means we can use, the car can use only 25 percent of the energy from the oil to drive the car, to move the car. So, hence, I add the factor eta, which represents the engine efficiency.

That is the correction. Now we arrive at our key principle of physics. The key principle of physics is that this process of transfer, So the burning of oil generates energy that drives the car. So, the amount of energy generated by the oil, corrected by efficiency must be equal to the work done to drive the car against the drag force. So, the energy of oil is equal to the work done against the drag force. If we equate these two, so we have estimated the energy released is eta times, the energy density times the mass of the oil, and if we equate it to the work done, we get an expression for the volume of the oil consumed.

Now let us put some numbers. So, this expression I have arranged it in terms of, as a kind of a combined grouped T similar terms together. So, first there are these constants, One is the drag coefficient, and the second is the engine efficiency. Now for a typical car, I am going to assume that the Cd is about half, and the drag coefficient under the condition we are discussing is about half. The engine efficiency we have already assumed to be 25 percent or 1 by 4. So, using this number, very luckily, these factors of the unknown coefficients get cancelled.



Example 22: mileage of a car

$$\boxed{\underline{E_{oil}} = \underline{W_{drag}}}$$

$$\eta \times \varepsilon_{oil} \times \rho_{oil} V_{oil} = \frac{1}{2} C_d \rho_{air} v^2 A_{cs} d$$

$$\boxed{V_{oil} = \left[\frac{\cancel{\frac{1}{2}} C_d}{\cancel{\eta}} \frac{\rho_{air}}{\rho_{oil}} \frac{v^2 A_{cs}}{\varepsilon_{oil}}\right] d = \underline{Mileage} \times \underline{d}}$$

$C_d \sim \frac{1}{2}$

$\eta \sim \frac{1}{4}$

$\frac{\rho_{air}}{\rho_{oil}} \sim 10^{-3}$

$v \simeq 90\, kmph = 30\, ms^{-1}$

$A_{cs} = 3 m^2$

$\varepsilon_{oil} = 40 \frac{MJ}{kg}$

$1 m^3 = 10^3 l$

$10^{-3} \times \frac{(30)^2 \frac{m^2}{s^2} \times 3 m^2}{4\times 10^7 \frac{J}{kg}}$

$= \frac{27}{4} \times 10^{-8}\, m^2$

$\sim 7 \times 10^{-8}\, m^2$

$d = 100 km = 10^5 m$

$V_{oil} \sim 7\times 10^{-8} m^2 \times 10^5 m$

$= 7\times 10^{-3} \times 10^3 l$

$\sim \underline{7 l}$

$A_{cs} = 2m \times 1.5m = 3m^2$

Now, how much is the density ratio? So, then we have this expression: the ratio of density of the air. Which is outside, and oil, which is inside the car. Now how much is the density of the oil? We may not know the numbers, but we can sort of make a guess that the density of oil will be very similar to the density of water. And the density of water is approximately thousand times more dense than air. So, you are going to put this rho air by rho oil as something like 1 by 1000.

And then this speed of the car, so you are going to assume that the car is moving on a highway. So, let us assume that the speed of the car is about 90 kilometer per hour, Which is about 30 meter per second. That will make our calculation easy. Now what about the cross section? So, a typical car, so here is a picture of a very schematic car. So for us, the car is nothing but a box which is moving in this direction and some velocity is speed v.

So, let us say we are going to assume its width is about 1.1 and half meter, and the length is about 2 meter. So, then the cross section is going to be about 3 square meter. And the energy density we have just estimated is about 40 mega joule per kg. So, if you plug in this value, we get, so, by the way this whatever combination in bracket, We can term it as something called mileage, because that is going to give us the amount of oil volume of the liter, amount of liter of oil consumed per unit distance.

So that if I multiply by the distance moved, then we get the total volume that is consumed. Volume of oil that is consumed. So, we get it is about, so this is 10 to the power minus 3 times 30 meters per second. So, v square times 3 square meter divided by 40 mega joule which is 4 into 10 to the power 7 joule per kg. So, you can verify this joule per kg is same as the meter square per second square.

So, this is going to give you about, This is a factor of mileage. So, this is going to give you about 27 by 4 into, so This is 10 to the power minus 8, and so your mileage turns out to be a dimension of area. So, this is approximately 7 into 10 to the power, and now so the car has given, It is given that the car has moved for a distance of 100 kilometer. So, this is 100 kilometer Which is about 10 to the power of 5 meter. So, then the v oil is approximately 7 into 10 to the power minus 8 meter square into 10 to the power 5 meter.

So, meter cube is, so 1 cubic meter is 1000 liter. So, 7 into 10 to the power minus 3 into 1000 liter which is approximately 7 liter. So, our estimate is that it will take about 7 liter of fuel to drive a car for about 100 meter, 100 kilometer. So, which is more or less like a roughly around the same number that typically we expect for the mileage of a big four wheeler of this much dimension. So, the point here is that this example illustrates an important point.

The important point is the following: the sort of, sometimes, conservation laws can be very powerful tool of analysis of practical situations. Why? So, this is, the reason is the following. So, any physical process, so we can, so this is a point, Our point of view is that any physical process you are, we are going to think of as a kind of a time evolution of our system and think of it as kind of a steps, series of transformations from one time to an instant to another time. So, think of it as looking at the process of the car moving as kind of a movie and then each frame gives you the state of the car at a certain time instant and then you are going from one frame to the next frame; that is the process. Now if we apply Newton's law of motion directly, first of all, We need to know a lot of details about all the forces that are involved in this process.

Lot of forces. Lot of things that, we do not know. More importantly, we must go step by step. So, in Newton's law, If we start with some t equal to 0 and we want to know the state of the system after sometime t. We have to go for one step, next step to next step and so on. There is no shortcut and We need to know lot of details in order to go from frame 1 to frame 2 to frame 3 to frame 4 and so on.

The conservation laws, such as what the energy conservation that We are discussing now gives you a shortcut. So, we are sort of; the point is that we are looking at a process, Something is changing. The conservation laws points out there are some quantities which does not change, and some physical quantities which does not change. Now if we focus on something that does not change, then we can connect any frame to any frame by an equation. And most and the power of this approach is that we can skip lot of details about the system.

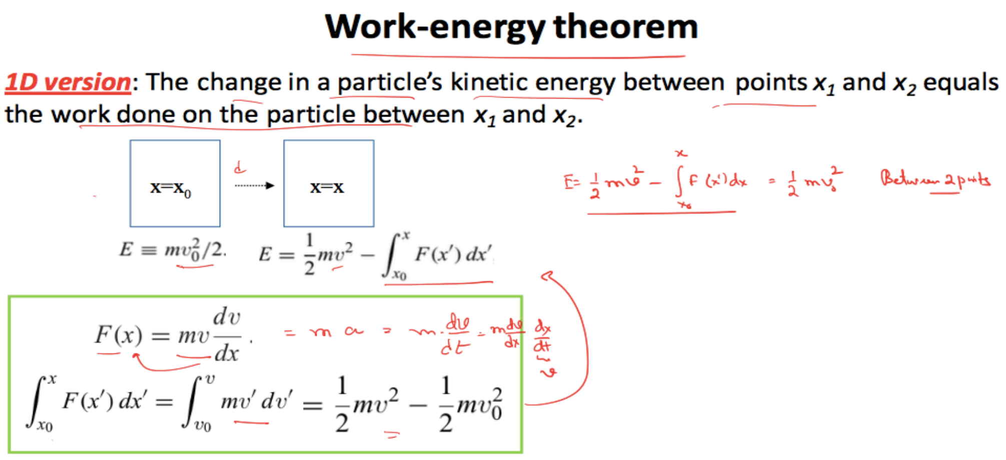
So, for example, we do not know exactly how this chemical reaction is going on in this, during the car is moving. We only need to know that there is some physical process by which this energy gets transferred. And we need to know that this energy is not getting destroyed. It is just transferred from the chemical energy stored in the oil to the work done against a drag force. And this equality, this symbol of an equation, signifies that something is not changing; the energy is not changing.

It is just sort of transferring from one form to another. And then we do not need to know. We can skip a lot of details and sort and that is why we can use this law to estimate without almost knowing practically nothing about our car. So these kind of quantities that does not change are called, technically called invariants in physics. So, amazing fact that in Newtonian mechanics in 3D, we discover there exists such quantities that do not change even though a lot of things are moving in a complicated way.

And there are seven quantities in three dimensions. So, these quantities are one number, so seven quantities means seven numbers; the one number is the energy, three components of momentum and three components of angular momentum. So, for each of them,. We have a conservation law. So whatever process is going on in the system, if we ignore friction, then the energy is going to be constant. So we are going to look at this statement and make this statement more carefully.

But let us sort of formally say what we have just discussed through this example of estimating the mileage of a car. So we are going to first say it in today's lecture in 1D, and then we will generalize it to higher dimensions. So the formal statement that we have just done to use this analysis is that the Suppose there is a particle, so the change in the particles kinetic energy when it moves between Points x1 and x2 equal the work done on the particle between x1 and x2. So, for example, the particle is like the car, so it was the initial position and This is the final position, and this distance is let us say 100 kilometer. So x naught is your initial position, and x is the final position.

So suppose the car was moving with some speed v, v naught at x naught position, then and at some point, after in the final position, the speed is, let us say, v. Now the kinetic energy at initial position should be half m v naught square, and the kinetic energy at the final position is half m v square. Now the difference is the work done against the force to move the particle. So the kinetic energy changes because there is some force acting on the particle. For example, in this case, the drag force is acting on the car and the work done against those forces is the reason.



Work-energy theorem

1D version: The change in a particle's kinetic energy between points x_1 and x_2 equals the work done on the particle between x_1 and x_2.

$E = \frac{1}{2} m v^2 - \int_{x_0}^{x} F(x')\,dx = \frac{1}{2} m v_0^2$ Between 2 points

$E \equiv m v_0^2/2.$ $\quad E = \frac{1}{2} m v^2 - \int_{x_0}^{x} F(x')\,dx'$

$$F(x) = mv\frac{dv}{dx}. \quad = m a = m.\frac{dv}{dt} = m\frac{dv}{dx}\underbrace{\frac{dx}{dt}}_{v}$$

$$\int_{x_0}^{x} F(x')\,dx' = \int_{v_0}^{v} mv'\,dv' = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

So the change in kinetic energy is transferred to the work done. So this is called the work energy theorem, and this is very easy to prove, just two lines of proof in 1D. So suppose this F, we know that F is m times A, where A is the acceleration and A is by definition the rate of change of the speed or velocity in 1D, they are basically the same and This we can write as m times dv dx times dx dt. Now dx dt is v and hence we can rewrite this equation as mv times dv dx and then we can take the dx and multiply both sides by dx and do the integration from x naught to x and then the right-hand side is just the integration over mv which is the momentum and it turns out to be equal to the change in the kinetic energy. So now, if we rearrange the term, We get this particular equation, which means that between two points, there is some quantity we can calculate as, so between two points, this is the statement, so this quantity remains invariant between two points.

So this is formally called the work energy theorem in 1 D. So in the next lecture, we are going to discuss the conservative forces and the generalization in 3D. So, before that, let me summarize what we learned in this lecture. So we took an example of, practical example of work application of work energy theorem and we used it to estimate the mileage of the car and this examples sort of highlights emphasizes the power and beauty of applying conservation laws to solve mechanics problems that we can skip lot of details, and then we can formally and technically express this concept about the invariance that something is not changing when some physical process is happening as formally as the work energy theorem. We discussed this theorem in 1D and in the next class, we are going to generalize further. Thank you. See you in the next lecture. Thank you.