Newtonian Mechanics With Examples

Shiladitya Sengupta
Department of Physics
IIT Roorkee
Week -05
Lecture-25

In the last couple of lectures, we were discussing about friction or the Coulomb's law of friction, which describes the friction between two solid surfaces. In this lecture, our topic is drag force, which represents the friction between a solid and a fluid. So, let us start with a story. So, you might have heard about this story that the famous physicist Galileo performed this experiment where he dropped two cannon balls from the leaning tower of Pisa to show one's big one small to show that they both have the same acceleration. It is not clear whether this story actually is true, I mean this experiment ever happened. Most probably no, but let us say if we take a cannon ball and a feather from experience, we definitely know that it does not happen.

That is the feather and the cannon ball both drop from the same height at the same time with zero initial velocity let us say, the cannon ball will drop further and the feather will fall later. So, why does it happen? The point is that we know that the acceleration due to gravity for both of them are same, g, small g. So they should be falling together. In fact, this will be true if we can perform this experiment of dropping a feather and the cannon ball together in back wall.

So, here is a link I provide to a video (https://www.youtube.com/watch?v=E43-CfukEgs) that actually very interesting and fascinating video that actually does this experiment in a vacuum chamber and dramatically show that indeed they fall together. So, I invite you to take a look at this video. But in normal situations in real life, we live inside a fluid that is we are surrounded by air and this air produces some friction. So, we are always feeling this frictional force due to air and this frictional force is more on the feather because of its higher surface area compared to a cannon ball which is more compact. Hence, the net acceleration due to the combined force of the drag and the weight of acceleration, I mean the force due to earth is more for the cannon ball versus the feather.

But if you can take the air out, then you will see a very different thing which is, so, take a look at this video. So, the question is then that is this drag force due to air significant? When can we expect a drag to be significant? This is a very important question if you want to build correct physical intuition and want to solve mechanics problem correctly. The answer actually depends, it is very interesting thing that the answer actually depends on the size of the dropped object. For example, if we fell from a height, often it is fatal to us humans, but from the same height if you drop a, if a cat falls which is a much smaller size, it most of the time survives. So, there is a very another very interesting video (https://www.youtube.com/watch?v=f7KSfjv4Oq0), I put the link here, which asked this somewhat interesting question that what happens if we throw an elephant from a skyscraper? So, this basically performs a thought experiment where if you drop an object of different size starting from the size of an elephant down to a size of an ant from a skyscraper, then whether this is going to survive or not the fall.

The overall point is that as the size of the object gets smaller, its weight Mg gets smaller. So, the drag force becomes more significant compared to the weight. So, that is why whether the drag is significant or not, the answer depends on whether this force is significant or not compared to the weight of the object. For example, a crucial, so let me give you an example of where we expect drag is really important is that if you take a small object such as a raindrop, now you know that raindrop falls from a very high part of the sky, let us say from a cloud which is often more than a kilometer high. Now if you ignore the drag force in this case, then you expect the resultant velocity of a raindrop will be so high that if a raindrop hits us, we will, it is like a bullet hitting us and we will immediately get killed.

But, we know that that does not happen and in fact raindrops fall with a constant speed and that is because the drag force in this case is significant and it cancels the attraction due to earth so that the overall force on the raindrop can be zero. So, then it leads to a question naturally. So raindrop is an example of a projectile motion problem. So in your high school perhaps you have studied projectile motion problem but in a very idealized situation where you have only considered the gravity as the only force determine the motion of the projectile. But, then in real life if you deal with actual projectile, then you have to answer this question, can we ignore the drag force on the projectile motion problem or not? So we are going to address this question in the course of, in the subsequent lectures.

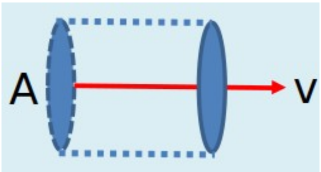
So, before to analyze this problem, what we need is some expression for the drag force. So, then the question is what factors actually determine the drag force? Let us call the drag force on objects is F_d, d denotes the drag. So, this is drag. So, here there is no theoretical way. So, we have to do experiments and build our experience and knowledge.

And from experience we know that it is more difficult to move through water than through air. What does it mean? It means the drag force is higher for higher density of the fluid through which the solid is moving. Second thing, so density is a factor. Second thing that we know that if you are going at a, if you are standing still versus if you are going at a higher speed, for example, in a moving train if you are standing on the, near the, sitting on the, in the window side, air hits us more forcefully when we are going at higher speed. So, this means the drag force is higher for higher velocity of the object moving through.

Then the third factor is that the flatter object like a feather face more drag force, significantly more drag force than a more compact object, round object like a cannonball of the same mass. So, this translates to the fact that drag force, so the more the cross-section of the object higher is the drag force. So, the cross-sectional area of the object. So, cross sectional area means suppose this is some object and this is moving through air in this orientation, then the cross-section, so it may have some surface area, but what matters is the, if you do a cut of this object perpendicular to the direction of velocity, then what is the area of that cut? That is the cross section. For example, in this way, in this case, this is the cut and this is the cross section.

But if the, if we look at this way, then we can see that the cross-section covers the covers the entire area. So, the drag force, if the, if this object moves in this direction, this way, then a drag force will be higher. And finally, we know that the drag force increases with the property called viscosity of the fluid. So, these are the factors that we expect should determine the drag force. Now before we go to some, write down some formula, you, at this point normally what you usually do is to write down a formula and memorize the formula.

Instead of that, let me sort of emphasize on the physical picture of what is going on about the drag force. What happens when a solid object moves through a fluid? So, here is a schematic. Let us say this disk represents the cross-section A of an object which is moving through this blue fluid with a velocity v.

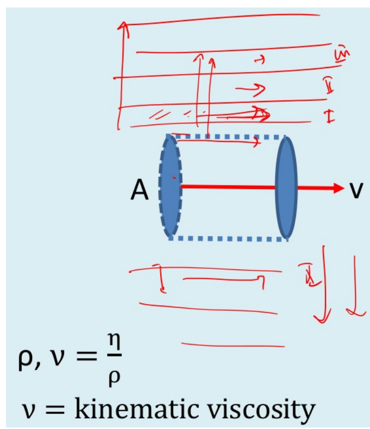


So, let us imagine that initially, the fluid is at rest. So, there are two things that happens. First thing, suppose the fluid is at rest initially, then as this object starts to move, suppose you immerse it at rest and then it starts to move, then it drags the fluid which is in contact with the object with itself, with a velocity v. So, this is the drag. So, it creates the fluid surrounding the object starts to move. Which means, in other words, that at initially the kinetic energy of the fluid was zero at initially and then when the object starts to move, the kinetic energy of the fluid is no longer zero. And where is this kinetic energy is coming from? It is coming from the object.

So, if the object is moving through the fluid, this its, if there are no other forces involved, then its velocity will gradually slow down. It will gradually slow down and it will eventually stop. This is due to the friction, due to the fluid on the object. So, the kinetic energy of the object will convert, will go into the fluid surrounding it. So, it will convert to drag the, create kinetic energy, create motion in the fluid which is in contact with the object.

So, this is the first thing. So, this is the drag. At the same time, so then it transfers. So transfer of KE, kinetic energy from solid to fluid.

This is the first step. But then what happens to that moving fluid? So, if the fluid surrounding it starts to move, then imagine there are different layers in the fluid, then that fluid it starts, will start to drag the layer of fluid in contact with it, with itself. And then there will be a friction between these two fluid layers. And as we know from Newton's law of viscosity that if the fluid is simple fluid or which are called Newtonian fluids, so this, so the, this liquid, this layer, let us say this layer, fluid layer 1, 2, 3, etc. So, this layer 1 will exert a tangential force on layer 2. And similarly layer 2 will exert a tangential force on layer 3, layer 3 and so on so forth.



And this force will sort of, because of this force, this force will be proportional to the, but the velocity of this layer 2 will be little less compared to layer 1. Velocity of layer 3 will be little less compared to layer 2 because of the friction between the fluid layers. So, there will be a velocity gradient in the direction which is perpendicular to the velocity of the object moving. And hence, gradually, so this, all the fluid, remember was originally at rest, but now they are, they will start moving and hence what we will see is that as the object moves from this position to this position, the fluid gets disturbed. So, the fluid in contact with the object gets dragged and then starts to move and then, so, there is a net flow of momentum from, in the perpendicular direction, so that the kinetic energy gained by this fluid in contact with the object is gradually transferred in through the fluid in the direction normal to the velocity of the solid object.

And this is of course true in both direction. So, it happens in both directions. So, the second part is a loss mechanism by which due to viscosity of fluid, by which this KE of fluid is dissipated in the fluid, which means it gradually gets lost in the fluid in the perpendicular direction. So, these are the two process, physical processes that are, that happens when a solid object is moving through a fluid and both of these processes is, may be important. So now to derive the expression, we will take a simple route that is mathematically simple and we will sort of eliminate all the main details.

So, we are going to use a something called the concept of dimensional analysis and making a, we are going to make a dimensionless quantity to sort of get at the expression for drag force. So, what I have seen so far that the force, the drag force is proportional to ρ and density of the fluid, proportional to the area of cross section of the object and is proportional to velocity, but is it proportional to velocity linearly or square or cube or what? That we do not know. So, let us say that it is proportional to some power of velocity. Then we can combine them to say that the drag force is proportional to this three quantities in this particular combination. And we demand, take the ratio and we say that this ratio is dimensionless, which is a property of liquid, it varies from liquid to liquid.

$$F_d \propto \rho, \quad \propto A, \quad \propto v^{\alpha} \qquad F_d \propto \rho A v^{\alpha}$$

$$\frac{F_d}{\rho A v^{\alpha}} = \text{dimensionless} = C_d$$

And it is basically, yeah, depends on the nature of the object and the liquid, but it does not depend on all these six factors which are already taken care of. This is called the drag coefficient or C_d. Now from this, by now you can do dimensional analysis to determine what is the power of v. So, if this is dimensionless, that means this, the product ρ times A times v to the alpha must have the same dimension as force. So, what is the dimension of force? So, force has dimension, it is mass times acceleration.

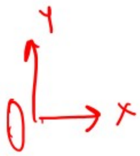
$$[F_d] = MLT^{-2} \qquad [\rho] = \frac{M}{L^3} \qquad A = [L^2] \qquad \frac{M}{L^3} \cdot L^2 \cdot \frac{L^{\alpha}}{T^{\alpha}}$$

Now if we compare the dimension of F_d and the dimension of this product, we immediately see that alpha is 2. So, this is a crucial point that the drag force is

$$F_d \propto \rho A v^2 \Rightarrow \text{dimensional analysis} \quad [v^{\alpha}] = \frac{L^{\alpha}}{T^{\alpha}} \Rightarrow \alpha = 2$$

So, this is the first part which sort of transfer of kinetic energy from the solid object to the liquid in contact with the solid.

But you see that we are missing one factor which is viscosity. So, we need to know now the second aspect of the physical process which is the loss or loss of momentum from the surrounding the liquid that is in contact with the fluid to the other regions of the liquid further away from the solid object. Now here this part as we know that this force that the shear force which is the internal friction, so remember that we have this different imaginary layers of liquid, so which exerts force, tangential force in the direction in which the object is moving, but the different directions has a different velocity, different layers have different velocity. Now how much is the shear force? So in the simple case of a Newtonian liquid, this is given by the Newton's law of viscosity which says that this force is proportional to the area of the tangential area times the gradient of the velocity.



$$F_{shear} \propto A_t \frac{dv_x}{dy}$$

$$= \underbrace{(\nu\rho)}_{\eta} A_t \frac{dv_x}{dy}$$

$$\eta = \nu\rho$$

So, the object is moving in x direction, so then this velocity is as an x component and but the gradient is in the perpendicular direction, hence you changing with the derivative is with respect to y. Now this constant of proportionality, I am going to write it instead of the customary way of writing η, I am going to use the symbol ν which represents the kinematic viscosity. It is a material parameter representing the mechanism of energy loss due to the flow of momentum in the direction perpendicular to the motion of the object.

Now again let us say that what is the dimension of ν? Now the dimension of ν can be found from looking at this expression, so the dimension of the shear force is

$$[\nu] \qquad [F_{shear}] = MLT^{-2}$$

$$\left[\nu\rho A_t \frac{dv_x}{dy}\right] = [\nu]\cdot\frac{M}{L^3} L^2 \frac{L/T}{L} = MLT^{-2}$$

$$[\nu]\cdot\frac{1}{LT} = \frac{L}{T^2} \Rightarrow [\nu] = \frac{L^2}{T} = L\cdot\frac{L}{T}$$

$$\frac{vD}{\nu} = Re.$$

So then the question is that how now we want to make a combination of a dimensionless quantity that represents the effect of this viscous mechanism, viscous loss, the viscosity. So, we can see from here that if I make a combination something like some velocity times some length, let us say which represents the size of the object and traditional and divided by nu, so this is a quantity you can verify that it must be dimensionless and this quantity is called the Reynolds number.

So, here this D represents some length scale, some linear size, linear dimension of the of the object, so let us say the diameter of the object. Now the point is that we have now four quantities and so and from which we have found two dimensionless parameters, one is the drag coefficient which is given by this expression and the other is Reynolds number which is given by the vD/ν. So, the importance of forming dimensionless parameter which is a very standard way of analyzing problem in fluid mechanics is that now we know what are the control factors in the problem and we know that there could be different infinite number of combinations of density, area, velocity of the object, but what matters is this particular combination. So, this dimensionless parameter sort of gives you the real control parameter that determines the mechanical properties of the motion. Now it turns out that these two parameters are not independent of each other, they are not independent, yeah.

So, this we can write express this drag coefficient as a function of the second parameter that is the Reynolds number. Now we do not of course know what is the exact inter function is, but here we sort of get a insight by looking at extreme cases or the limiting cases. So, first one limiting case is when this kinematic viscosity tends to 0, which means the liquid is almost free of friction, so this is a frictionless condition, so there is no, the viscosity of the liquid is very very less, almost you can ignore the viscosity. So, in this case you expect, you can ignore the viscosity, then you expect that this drag force to be independent of viscosity.

$$C_d = \frac{F_d}{\rho A v^2} \qquad Re = \frac{vD}{\nu}$$

$$C_d = f(Re)$$

$$\nu \to 0 \quad (Re \gg 1)$$

$$C_d = \text{const} \Rightarrow F_d \propto \rho A v^2 \quad Re \gg 1$$

So, this is a case, if ν is very less, this is a case where the Reynolds number which is inversely proportional to ν is very large compared to 1.

Whereas let us take the other extreme case where this ν is very high and where ν is very high, that is the Reynolds number is very small compared to 1. Now in this case our experiment shows that we expect that this, this drag force is proportional to v.

$v \to \infty \quad Re \ll 1$

$F_d \propto v \Rightarrow C_d \propto \frac{1}{Re}$

$$\frac{F_d}{\rho A v^2} \propto \frac{\nu}{v D}$$

$$F_d \propto \nu \rho \left(\frac{A}{D}\right) \cdot v$$

$$Re \ll 1 \quad \boxed{F_d \propto \eta R v}$$

So in that case the drag force becomes linearly proportional to the velocity. So this is the correct way to think about the velocity dependence of the drag force, that it is not that the small velocity means that the drag force is proportional to the velocity, which is usually many textbooks describe in that way. Small velocity means drag force is proportional to v and large velocity means drag force is proportional to v^2. But the experiment shows it is not the velocity only control factor, the drag force is determined by all these other factors, the density and the cross-sectional area and velocity and kinetic viscosity. So, the controlling factor is the Reynolds number.

So, when the Reynolds number is small, then the drag force is proportional to the velocity 1, that is the correct statement. And when the Reynolds number is high, then the drag force is proportional to velocity square. And these are the two limiting case that is the simple, simple limiting case. And then if you, so note that I have written a proportionality, if you replace, then you get some constant. And this constant, to calculate this constant, you really need to solve the Newton's law of motion starting from Newton's second law for the fluid and object in detail, which is a mathematically complicated problem and beyond the scope of this course.

→ Stoke's law of viscosity

$$F_d = c\, \eta R v$$

6π, 4π] depends on stick or slip boundary condition

But it turns out this constant is just a number like 6π or 4π and it slightly depends on what is called the boundary condition. But apart from this factor of 6π or 4π, our dimensional analysis can actually capture the correct dependence on velocity and the size of the object. And this by the way is called the Stokes law of viscosity. In the next lecture, now we have some expression to summarize what we have today is that we learned how that what is the, when the drag force is significant and second, what is the form of the drag force, how it depends on velocity of the moving object. And we saw that there are two extreme limiting cases, one which where the Reynolds number is small.

In that case the drag force is given by the Stokes law expression or is proportional to linear lean velocity to velocity and proportional to the viscosity, so it is determined by the viscosity and in the other case where the Reynolds number is high, then it is proportional to v^2. And you have to remember the correct physical picture that there are two mechanisms involved in this case, one is a transfer of energy from solid object to the liquid and then the transfer of energy from the liquid to faraway region of the liquid. So, in the next lecture, we are going to illustrate these ideas with worked-out examples. Thank you.