Course Name: Newtonian Mechanics With Examples
Prof. Shiladitya Sengupta
Department of Physics
Indian Institute of Technology Roorkee
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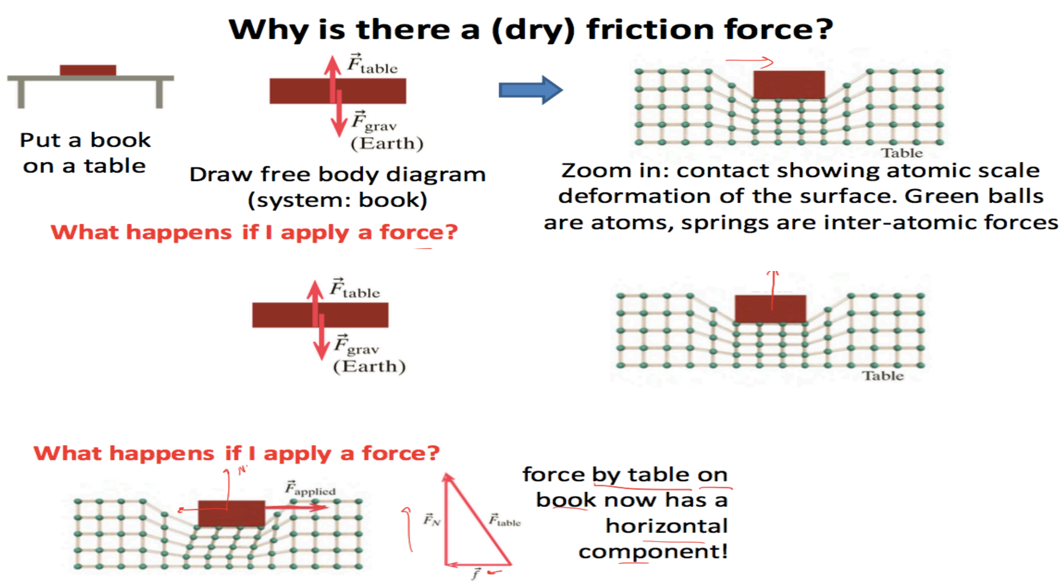
Lecture - 23

Let us continue our discussion of Newtonian mechanics with examples. In the last couple of weeks, the examples that we considered were mostly in situations, Where the objects were motionless in static condition. From today onwards, we are going to relax that condition and we are going to consider examples. Where the system under study can be motionless as well as moving. So we are going to consider the more broader situations covering Newtonian mechanics. Today's topic is going to be friction.

We are going to critically review the basic concepts of friction. Now, why is friction important? It is important because, remember from our earlier in the context of Newton's law, We said that it is very hard to appreciate Newton's first law of motion because It is very hard to imagine or realize in everyday life a situation which is completely force free and two forces that are almost always present in any real-life situation: One of them is gravity, the interaction with the earth, and the other force is friction. So, our plan is the following: So first, we are going to look at friction in two different situations. In the first situation, we will consider the friction between two solid interfaces, which we are going to call dry friction.



And then, our plan is to critically review your so-called Coulomb's law of friction that I am sure you must have already studied in your high school, But we will see that there are some of the things that are usually not highlighted well enough. We are going to cover those aspects of the laws of friction force. After that, we are going to look a little bit at the friction between a solid and a fluid, which is called a drag. Both of these situations are very common in various daily life scenarios as well as in various engineering applications. So you should have a good conceptual understanding about the nature of these forces.

First, let us ask this question, why is there a frictional force? And in this case, I am going to consider only situations where two solid surfaces are in contact with each other. So let us take this simple example: I am putting a book on a table. So let us say imagine as before that this is my table and this is a book. So I put a book on the table, and let us say this book is at rest. Now, what are the forces on the book? So we are going to assume, consider the book as our system.

Then in the surrounding, there is earth and the table and there are forces. The gravitational forces by on the book by the earth which is this F-graph which is acting downwards through the center of mass of the book and the force F-table, which is upwards through the book by the table on the book. Now this is what we have done before. We have drawn the free body diagram of this problem and applied Newton's third law, second law, etc. to analyze this situation.

And we know that the book at rest means these two forces cancel each other. These are two distinct forces, but they happen to be equal and opposite because of the book. We know that because the book is at rest. Today, let us try to look at the nature of the contact force by the table on the book. So imagine if you could zoom in to the contact between, so if this is your book and These are the contacts between them, so let us say these are the contacts between the book and the table.

Now if you could zoom in or look at only at the contact areas, let us say under a microscope or something, Then perhaps you would get a picture like the following: So what it is showing in this picture is that the green balls are the idealized version of the atoms. So this table is a solid object and to a very idealized picture, We can imagine this table as made up of atoms, the components of the table, whatever atoms like green balls and These are placed in regular positions and in periodic positions, that makes the table. Let us say this is a metallic table or a wrought iron table and There are forces between these atoms, which are represented by these little springs. So the atoms are connected by springs to represent that they can exert forces on each other and On top of that, if two atoms comes close to each other, there is an equilibrium distance. A preferred separation between two atoms when represented by the fact that the spring is unstretched.

Then, if the two atoms come closer to that separation, they will try to repel each other. If they go away from each other, then they will come back to this equilibrium, in the unstretched position of the spring. So this sort of qualitatively represents and describes the force between the two atoms in a real solid material. But note that this is a very idealized, idealistic version. This spring does not represent the actual force, it is just a crude way to imagine the atomic scale.

Now let us ask this question: what happens if I apply a force, a force on the sideways direction? Well, here is a schematic picture of that. So if I put a force like shown by this red arrow on the sideways position on the book, then there will be further distortion on these atomic arrangements of the table. And it is easy to see that because of these new distortions and new kinds of deformations, Now, the force by the table on the book will have a horizontal component because It will also try to remove the distortion or deformation in the sideways direction. And so the contact force now has two components. So to contrast, in the previous case when the book is simply at rest without any external force, The contact force has only a normal component, which is normal to the surface of the table.

Now in this case, because the nature of deformation is now different, it has a horizontal component as well as a normal component. So this horizontal component, so the net resultant contact force is no longer perpendicular, Let us call that contact force Fn or N and the horizontal force. And as you can expect, the horizontal force will be a direction to remove this deformation and hence it should be in the direction opposite to the applied force, So it would be on the left if the applied force is towards the right. And this horizontal force is called friction, as simple as that. So a contact force in general between two solid surfaces in contact, the contact force has two components.

One is perpendicular to the surface of contact, and the second is horizontal to the surface of contact. And by convention, the normal force is called the normal component and the horizontal component is called friction. So this contact force in general has a component normal to the surface and a component parallel or tangential to the surface. And this, so this is what we know from experience: that this tangential component of this contact force is present. If the two surfaces have a relative velocity if they are trying to move past each other, which will cause deformation in the contact area or a tendency for relative velocity.

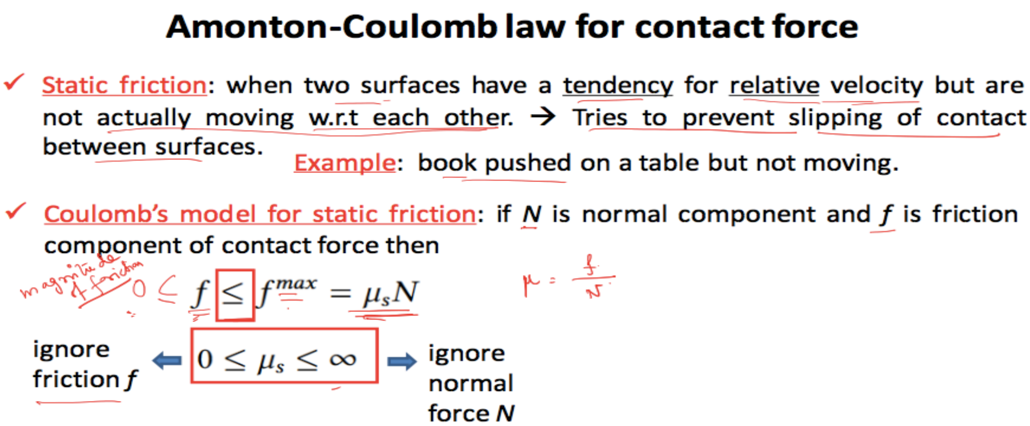
So it may happen that this is crucial because it may happen that if I apply this force, Some horizontal force on this book, but if my force is small enough, the book will not move, but it will have a tendency to move. So this tendency is crucial. And so if there is a tendency for relative velocity or actual relative velocity, then This horizontal component is the friction. Now note that in this picture, this is a very, very idealized version of the picture and the picture is not complete in the sense that, even in nowadays, There are a lot of questions whose answers are not known about the nature of the frictional force at the atomic scale resolution and people are still doing research on this area. So now some definitions, static friction.

So when two surfaces have a tendency but not the actual relative velocity, For relative velocity, and the word is crucial that the velocity must be measured with respect to each other. So it may happen that your two, the table and the book, are moving with each other. Let us say they are in a moving train, so they are both moving in the same with same horizontal velocity, If you are observing them standing on the road, but there is no relative velocity is 0. If the relative velocity is 0, then there will be no frictional force. This is crucial to remember.

So there is a relative velocity, but are not actually moving with respect to each other. So with respect to each other is the crucial point. So then they are still static, motionless with respect to each other. So in this case, the effect of this is still a horizontal force which is trying to prevent the contact between surfaces from slipping. So think of these forces as something that is trying, So they are trying to slip past each other or slide past each other, and this force is trying to prevent that from happening.

So as I showed a little earlier that this is an example Whether you are pushing on a book or on a table, the book is not moving. So it has a tendency to move, but this horizontal force is resisting the motion. Now we come to the very important fact: how to describe and how much is the friction? So here what you already know got introduced in your high school course is something called

Coulombs law of friction Or more precisely, the Amonton Coulomb law for a contact force of friction. So what is this law? So I am going to highlight two important aspects of this law. So, as you know, it says that if N is the normal component and F is the frictional component, then this frictional force has, what is the magnitude of this frictional force? So this is the first part: the magnitude of frictional force is not unique; It can have anything from 0, the magnitude, I am talking about the magnitude.



It can be anything from 0 up to a maximum and this maximum value of frictional force is given by the mu times N. Where mu represents the coefficient of static friction. So mu is a constant. So, this mu can be any positive number. So there are two important points that are normally missed, as far as I noticed from the students understanding.

That, you have to remember that the frictional force does not have a unique value. It varies from a range. So it starts from 0 and goes up to a maximum value, and this mu times N represents the maximum value of the friction. But there are situations, very common situation, we will consider this when we take some example, Where the frictional force can actually be less than mu N. This is the first important point: this is an inequality; it is not equality.

The second point is that if I ask students what the value of mu is, I always get an answer that mu varies from 0 to 1. So, what is mu? Mu represents the ratio of the frictional component, the horizontal component and the normal component of the force. Now what? So this, now what is it? What does mu equal to 0 signifies? It basically means that we can ignore the frictional force compared to the normal force. So it is like the surface between the two objects is frictionless or very slippery. If you are walking on ice, ice is very slippery, so the friction is very, almost 0.

So this is, if you sort of think of in an ideal place for an extremely slippery ice surface, So that represents mu equal to 0, for example. Now if these two forces are equal in magnitude, then mu becomes 1. Now, can mu be greater than 1? The answer is yes. There is absolutely no restriction, either from our experience or from experiments, that The frictional component cannot be greater than 1. In fact, in the limiting case, it can be infinity, meaning it can be as large as possible.

So, what do I mean by that? It is very easy to see. So you already, I mean you know, so think of it as two surfaces which are glued to each other. So there is gum. We apply some gum to glue one surface to another, like you put some gum and close and envelope. So this gum in that condition, because of the presence of this adhesive, like a shallow tape or something, the frictional force is extremely high, and the horizontal component is extremely high.

So that represents a scenario in which mu can be very, very high. So remember that there is no upper limit on mu and it will lead to some surprises. Which we shall show you in the next lecture, even in a very familiar example. So mu can be, there is no upper limit of mu has to be 1, it can be greater than 1. Now, I want to highlight one thing that this friction force is then in principle unknown.

The reason you always take F equal to mu n is then you have some expression for F, the friction. But in reality, friction is a self-adjustable force, which means its magnitude is not always known from the situation. This is a reality. You cannot ignore the facts. This is also consistent with our experience as we shall show in an example.

Amonton-Coulomb law for contact force

- ✓ Sliding or kinetic friction: when two surfaces are sliding or slipping with respect to each other.

Example: book being dragged on a table.

- ✓ Coulomb's model for kinetic friction:

$$f = \mu_k N \qquad \mu_k \leq \mu_s$$

All complications about surfaces are represented by a *single* number μ → no separate dependence on contact area or relative velocity (kinetic).

Now what about the direction? The direction of mu is given by the fact that it opposes the relative motion between surfaces. This comes from our physical picture: this friction, the horizontal force, Is that aspect of the contact force that is trying to remove this deformation. Hence, the direction of the friction is given by the simple rule that it opposes the relative motion or In this case, the tendency of relative motion between surfaces and that sets the direction of F and Usually, the direction of F is clear from the situation of the problem that you are trying to solve. Now the third important point about this Coulomb's law of friction is that it is amazingly simple law. The ratio says that this ratio of the frictional force and the normal force, Which is this coefficient mu s, it does not depend on the area of the contact.

So this is a very, very strange and surprising aspect of this law that we do not know actually this contact force, in reality, can be enormously complicated and we really do not know those contact forces, but it just says that you do not need to know all the details of atomic-level details of what is going on in the contact; Just measure the normal component and the horizontal component and the ratio is a constant, which means if you increase the normal component, the horizontal component will automatically increase. Although I should mention that this mu s do depend on the nature of the surface, So, the frictional forces between the book and wooden table and between the book and the wrought iron table are going to be different. For completeness, I will also mention the sliding or kinetic friction when two surfaces are actually sliding or slipping with respect to each other. For example, if the book is actually being dragged on a table, then Coulomb's law say that If now the friction is given by again basically the same form that this friction is proportional to the normal component and the coefficient of proportionality is now called the coefficient of kinetic friction. Experimentally, it turns out that this value is either equal to the coefficient of static friction or slightly less than that.

So again, I emphasize that this simple formula, this simple-looking formula, is easy to state, but It is actually very difficult to sort of give a first-principle proof of or derivation of this formula. That is why it is to be the case. So all because this formula says that all complications about surfaces are represented by just a single number mu, nothing else, no contact area, no relative velocity, no dependence on contact area or the relative velocity in the case of kinetic friction. It is an amazingly simple law. So to summarize, I sort of reviewed Coulomb's law of friction and There are three points that I want to highlight.

When I did the critical review,. First is that the frictional force can be anything between 0, minimum value is 0 and maximum value is mu times N and it is a self-adjustable force, its magnitude can vary within this range. The second thing is that the deduction is given by the tendency to oppose the relative motion Or the actual relative motion in the case of kinetic friction and the coefficient of friction mu. It can be any positive number between 0 to infinity. So it does not have to be restricted from up to 1. And the third part is that I want you to appreciate the simplicity of this law of Coulomb friction that absolutely do not know or have any idea about the nature of the contact force, What are the positions of these atoms? But still, we can describe the frictional force without knowing anything about that.

So in the next lecture, we are going to consider examples. Thank you.