Course Name: Newtonian Mechanics With Examples

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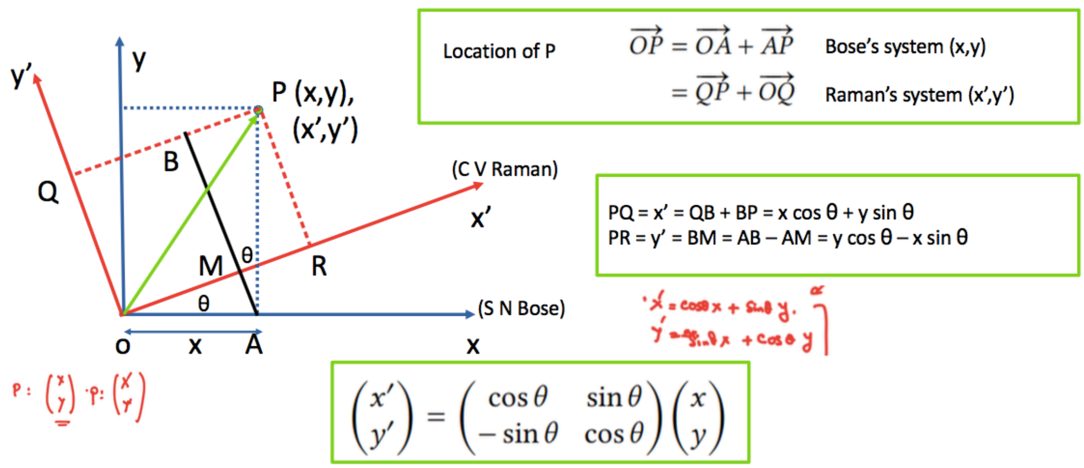
Week 01
Lecture - 02

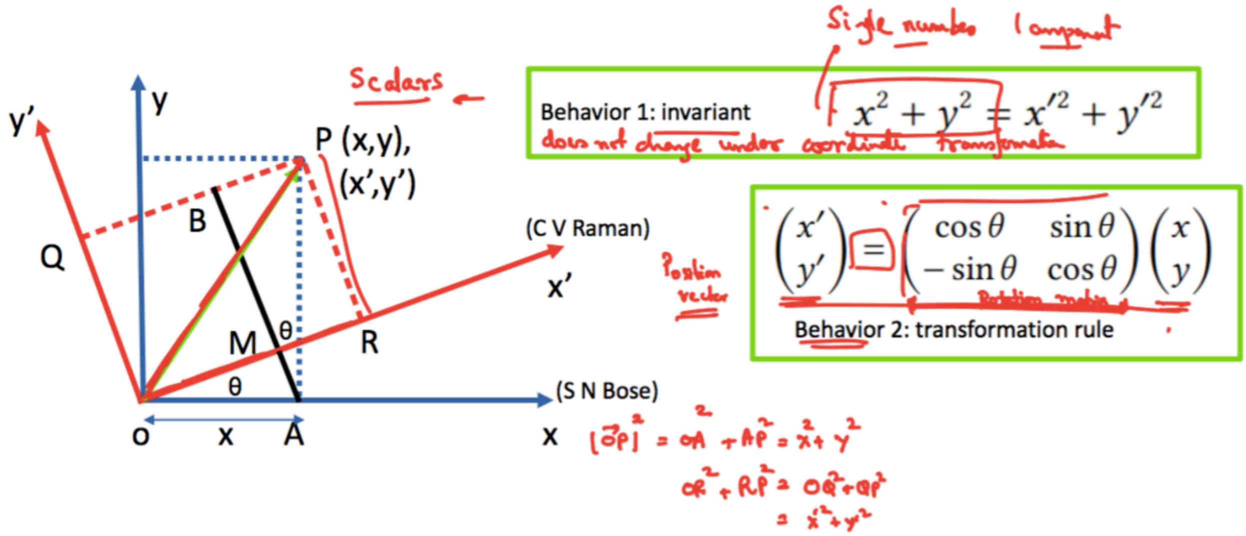
So, let us continue with our review of scalars, vectors, and tensors. In physics, the most general way to think about scalars and vectors is the principle that the laws of physics should not depend on the choice of the coordinate systems. How a quantity behaves under coordinate transformation determines whether it is a scalar or a vector or some more complicated object called tensor of higher order. Now, what do I mean by all these things? So, let us illustrate these concepts through an example. Suppose there are two friends, SN Bose and C. V. Raman and they are trying to locate a point P could be the location of a house. So they are standing at point O, and they want to know the location of point P from O. In this example, we are assuming that they are on a plane. So, there is no Z axis.



There are in the XY plane for simplicity. So how do they describe the location of point P from O? What can they do? Let us see, The first one is SN Bose (one of them), he will consider this point O where he is standing, as the origin, and he will draw a coordinate system, let's say the blue coordinate system. So this is the X axis of the blue coordinate system, this is the Y axis of the blue coordinate system, and then he can go from O to A along the X axis, and then from A to P which is parallel to the Y axis, then he will reach point P. So, in his coordinate system, he will analyze by writing down that the location of point P from point O can be described by a vector so by an arrow. So the length of this arrow represents the distance from point O to point P and the direction of this arrow represents the direction in which the point P is located from point O.

So he will write it as an equation, this arrow, this vector, which is called the location or position vector of point P from O as OP and this can be written as a vector addition of two pieces, one along the X axis and the other along the Y axis. This is along the X axis, this is along the Y axis. Then the length of this vector OA is X and the length of this vector AP is Y. So he will say that the point P, the location is given by these coordinates and these coordinates are X and Y, so two numbers. So the X component and Y component that represents the location of point P.

Now his friend C. V. Raman, he can do the same thing, but suppose that he chooses a different orientation of the coordinate systems. So he chooses this is the direction of X axis in his choice, which is denoted as the X prime to make it different from X, the blue one, and we denote this as the red coordinate system. And similarly, this is the Y axis of Raman's selected coordinate system, the red coordinate system.



In the red coordinate system, he will go along this X axis along OR to a point R and then he will go from R to P in a direction which is parallel to the Y axis of the red coordinate system and he will reach the same point P. So again, he can denote the position vector of the point P from O. That is the sum (vector) of two pieces, so OR and then RP. So you can also think of this way, so you can first go along the Y axis, so along OQ and then in a path which is parallel to the X axis of the red coordinate system, which is QP. So this is the equation written here, so this is QP plus OQ.

So from the figure you can see both path you can reach the same point P. So in Raman system, this length either this or this which are equal is the x component of the coordinate of the location of point P and this represents the y component. So in Raman system, the coordinates of the same point P are different. So in the Bose system, the coordinates are x, y, and in the Raman system, or the red system, the coordinates are x prime, y prime. Now let us see what is the relation between the coordinates in the blue system and the coordinates in the red system.

$$\begin{pmatrix} A'_x \\ A'_y \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{R(\theta)} \begin{pmatrix} A_x \\ A_y \end{pmatrix}$$

Red coordinate system — Blue coordinate system

From this figure, suppose I draw a perpendicular from A to QP. So it intersects QP at B. Now you can see that suppose I want to write down how much is PQ, So PQ which is by definition the x prime, is given by the piece QB and the piece BP. So this is QB + BP. Now how much is QB? So this QBMO is a rectangle.

QB is the same as OM. If you consider this triangle OMA, in this triangle, OA is the x, and the angle OMA is 90 degrees. so OM is x cos theta. Hence QB is x cos theta. Now, how much is BP? Consider this triangle ABP.

Now you can easily show by applying geometry that the angle between the x axis in the red frame and the x axis in the blue frame for Bose's choice of coordinate system is theta. So angle AOM is theta. So if angle AOM is theta, then angle BAP is also theta. This I leave as an exercise. This is easy to show.

Now if angle BAP is theta, we know that AP represents the y coordinate in the blue system. So BP is y sin theta. So this is BP. So we get an equation that x prime is cos theta times x plus sin theta times y. Now how much is y prime? So y prime is this distance.

So y prime is the same as OQ, and in this rectangle (ORPQ), y prime is the same as PR. PR is the same as BM because it is a rectangle. BM is AB minus AM. Now how much is AB? If I again consider this triangle ABP, we know this AP is y and this angle is theta, then AB must be y cos theta. And if I consider the triangle OAM in which OA is x and this angle is theta, so AM must be x sin theta.

So I can write y prime as minus sin theta x plus cos theta y. So I am writing the same equation here just in a slightly different way because if you use a matrix notation, so some of you may be familiar with the matrices. So I can immediately write this in a very beautiful way in using matrix notation. So I will write, denote the point P, the coordinates of point P using a matrix with a column vector with two components. The top component is x, bottom component is y.

The same point P can be written as another column vector, x prime and y prime. So then this geometry shows that there is a relation between the components or coordinates in the red frame, and with the coordinates in the blue frame, And so I can write it as a product of two matrices. So this represents the coordinates of the point P, the location vector of point P in the blue frame and this represents the coordinates in the red frame. And this is a product of this matrix which represents all these theta dependent part. So I can separate the theta dependent part and the coordinate part.

Now let us try to understand what did we do here. We know physically that the positions of points P and O are fixed. We consider the same point O and both S N Bose and C B Ramon, they started from the same point O and reached at the same point P. That means that if we want to describe the location of any point on this plane using these coordinates and if we demand that we can choose any direction as our x axis, then if we take two such set of coordinate system, the coordinates must have some relation from the system, let us say the blue system to red system. In other words, suppose I know the coordinates of the point P in the blue system and I want to know that what will be the coordinate of the same point P from the same origin O, if we choose a different set of coordinate system, that is the red coordinate system, then this is the formula or relation that gives me the new coordinate system.

So, so how do I go from the blue system to the red system? So I rotate the blue system, the x axis of the blue system by an angle theta. So this particular operation, this called rotation by an angle theta, this is an example of a coordinate transformation. And now I summarize what we found so far that if we apply a coordinate transformation which is then the location of the same point in two different coordinate system, the coordinates will be different, but they will be related by a particular rule. And this rule is a combination of this cos theta sin theta. So in 2D, this is an example of this rule and this 2 by 2 matrix which contains the information about how much rotation is required to go from one the blue frame to the red frame, this is called the rotation matrix, that represents this particular coordinate transformation.

so this particular fact that they are related is the meaning of the statement that the laws of physics should not depend on the choice of coordinate system. In this particular case, the position vector of a point P from another point O should not depend on the choice of coordinate system. Now I want to highlight two points. Now if I look at the distance between from point O to point P. This distance is represented by the length of this side OP, and this OP square is equal to the OA square plus the AP square.

So OA square is X square, and AP square is Y square. Now the same distance OP can also be represented by OR square, which is in the red coordinate system, plus RP square. Which is same as OQ square plus QP square, so this is the first kind of behavior. There are some quantities that remain invariant, which means they do not change if you do a coordinate transformation. So they are does not change under coordinate transformation.

There are some quantities, for example the position vector which has two components, so X component and Y component and they do change. So the components in the blue coordinate system and the red coordinate systems are different, but they are related by a simple rule: take the components of the blue coordinate system, Multiply by this rotation matrix, and you will get the component in the red coordinate system. So this is the behavior of the position vector under coordinate transformation. Now I must be careful here, I said here that the laws of physics should not depend on the choice of coordinate system. So, I have not mentioned which laws of physics.

What are the laws of physics that I should follow? and demonstrate that this remains the same under this coordinate system. But instead of that, I am going to take this approach and assume the laws of physics remain under coordinate transformation and then I will use this as a definition for a kind of way to classify these two kinds of behavior. So I will say that there are some quantities which behave as the distance between two points. In the sense that if you do a coordinate transformation, those quantities does not change, They remain invariant and this type of behavior, so those quantities are scalars. For example, if you take an object with some mass and then rotate it, So let us say if I take this rock and then rotate it, so each of this rotations is kind of a transformation or coordinate transformation.

but it does not change the mass of the object, so mass is a scalar. Another class of material shows this kind of behavior, so they have more than one components, So note that this is a single number or one component. So another class of quantity that has more than one component, so they are described by more than one number but these components follow a certain rule. The relation of the components in the blue coordinate system and in the red coordinate system.

They have the exact same relation. Which means that suppose I take a quantity A whose components are Ax prime and Ay prime, so this is a component in the red coordinate system and the same object has two components in the blue coordinate system. Now if I do the same coordinate transformation, which means that I rotate the blue x axis of the blue system by an angle theta, then precisely the same matrix, the same rotation matrix, so I have to apply the same rule that you take these components in the blue coordinate system and multiply by same matrix, So this matrix is exactly the same as what we found when we looked at the coordinate transformation for the position vector. which means that they behave in the same way as the position vector. Since position vectors has a sense of magnitude and direction, so these quantities are called vectors. They also have a sense of magnitude and direction and can be represented by an arrow.

So, that is why we usually say that vectors are quantities which have both magnitude and direction. Now, using this method of classification We can easily scale, so this method is scalable. So when you study for example, engineering or even in science, you always try to find a method or concept which can be easily scalable or generalizable. So this concept is easily generalizable in the sense that now we can define more complicated object quantities with more number of components.

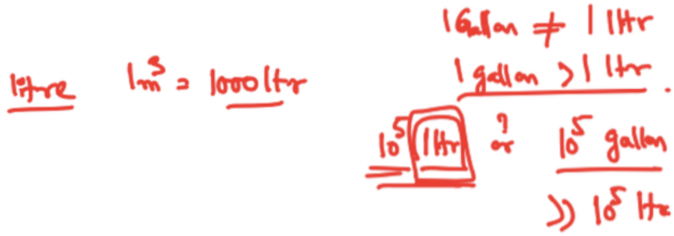
So if I want to write down the components. For example, a quantity like stress or moment of inertia. We need two of these rotation matrix. Then we may need three of them or we may need four of them using a particular certain format. In general, any quantity that we will encounter in this course is classified as a tensor of rank N. where N represents that how many of this coordinate transformation matrix, you will need to write down the relation between the components in the blue coordinate system and in the red coordinate system.

So if they are invariant, you need zero of them that represent the invariant behavior, and then these are scalars. If you need one of those coordinate transformation matrix, then these quantities are called tensors of rank 1. In this course, for most of the quantities, we will use either scalars or vectors, that is, tensors of rank 0 or 1. Perhaps we will only encounter one quantity: the moment of inertia, a more complicated object that is a second-rank tensor. Which means If I want to write down the components of moment of inertia in the blue frame and in the red frame, then the components in the red frame are related.

So if I want to know the components of moment of inertia in the blue frame. Then, to calculate the components in the red frame, We need two of this coordinate transformation matrix. So this exact rule, and how to write down these components, We will discuss this in due course as we progress in the course. Now I want to mention a couple of comments on notations. So the first thing, so this is based on my experience of teaching first year students the mechanics.

So usually, the students do not pay enough attention. when you write the symbols of the quantities, but this is a mistake. Why? Trust my word that if you can write down a quantity correctly, It indicates that you have understood the concept well. It reflects how your level of understanding. So second thing is this, if you pay attention to the notations, then it helps you to catch mistakes during problem solving when you are doing a calculation.
So here is a sort of very quick and review of about notations. So my first point is that when you write down a scalar, like mass, energy, power, etc., Always remember to write units.

It is very important. So imagine a situation. As I mentioned before, Why? the units are very important, because, as you know, there are different unit systems. To give you a very simple example, we mentioned that volume is a scalar. Now usually in India, we use the matrix system or SI units. So we measure the volume in meter cube, cubic meter, or more commonly in liters.



So one cubic meter is 1000 liter. So let's say you have a vehicle and you are trying to fill the tank with oil. So if you go to the petrol pump, you will ask for the amount of volume of oil in liters. But in other countries, for example in US, they use a different unit system. So they use something called gallon. So if you go to the petrol pump in the US, you will ask to fill your car with a gallon.

And a gallon is not the same as a liter. In fact, it is bigger than in gallon. Now imagine instead of car, you have a airplane which is flying from India to US. And imagine that this is a nonstop flight. And suppose your job is to fill the tank of the airplane with oil. And suppose you know the tank capacity is something like for example, let us say 1 lakh.
And say that you do not remember and the what is the unit? Is it 1 lakh liter or 1 lakh gallon? And so, if you do not pay attention and assume that since you are in India, It must be 1 lakh liter. So you fill the airplane with 1 lakh liter. But suppose the pilot who asked you to fill the tank with oil is from US and he meant 1 lakh gallon, which is actually much greater than 1 lakh liters. Then what will happen is that when the plane starts to fly, it does not have sufficient amount of fuel inside the tank. So it may have a danger of running out of fuel in the media.

And that is all that happened, because if you do not pay attention to the unit and this as you can see that this is a very dangerous situation. So always try to remember write the units. Second is about the notations about vectors. So I would like to summarize the different ways to write vectors. I am sure some of you, or maybe all of you, have come across this in your high school curriculum.

So most of this is for the types of notations that will be used in this particular course. So the most common type of notation is to represent so take the vector as a momentum, which is a vector it has a magnitude as well as a direction. Now you can write this as P represents the momentum with an arrow on the top. So this indicates that this is a quantity that has an magnitude as well as a direction. Now, if you read the textbooks or even sometimes in this course, The vectors are denoted by boldface.
So this is a vector. So this is a boldface. So there is a dark thick color. So this is a vector. Then sometimes these vectors are represented in terms of the components. For example, in three dimensional space the momentum has three components, three independent you need three independent numbers to describe momentum. So these numbers are let us say the x component, y component and z component.
So they are denoted by Px, Py and Pz. Now as we have seen in today's lecture that the third way of writing the representing the vectors are as a column vector. So in this case, so the this is also representing the components of the vector. Now this in modern days, this third way of writing, I mean you can write it in a row vector or column vector. So both will be used in this course. But the point is that in these days some of you or maybe all of you have learned some amount of programming.
And in programming you may have come across the concept of array. So an array is a variable with more than one component. So an array behaves in many situations like a vector. So this way

of representing the vector as a row vector or column vector is kind of very useful when you are writing some programs and doing some calculations in computer. Especially some of you may be interested in learning about data science or data analysis. So there when people represent a very complicated object with many, many different components.

So this is I am talking about an example which goes beyond the usual realm of physics. So usually, in physics the number of components is determined by the dimension of the space, whether in a plane you need two numbers to describe a vector. In this room, you need three independent numbers to describe a vector. But like in the example that we discussed before about the fruit in a basket, you can come across in real-life situation a very complicated object in order to describe that quantity you need many, many different components. When I say many, I mean that it could be hundreds, thousands, millions, or even billions of components.

So those kind of quantities for example, can be useful to describe for example, a medical record of a patient, and so on and so forth. In those cases this notation is array thinking of the vector as an array of numbers or a column of numbers or a row of numbers is very useful. Thank you.