Newtonian Mechanics With Examples

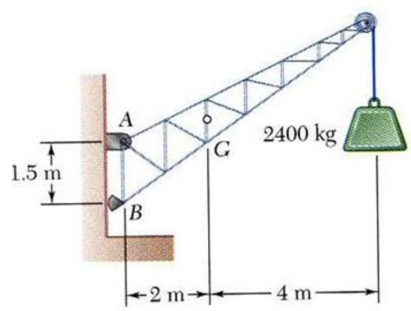
Shiladitya Sengupta
Department of Physics
IIT Roorkee

Week -04

Lecture -18

Okay students, so let us get started with today's lecture. Today we shall consider one more example of the application of this free-body diagram method and applying force and torque balance to solve statics problems. So this is an example that actually you will, this quite familiar to us like you see this in various construction sites.

A fixed crane has a mass of $1000\ kg$ and is used to lift a $2400\ kg$ crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G. Determine the components of the reactions at A and B. (Take $g = 9.81\ m/s^2$)



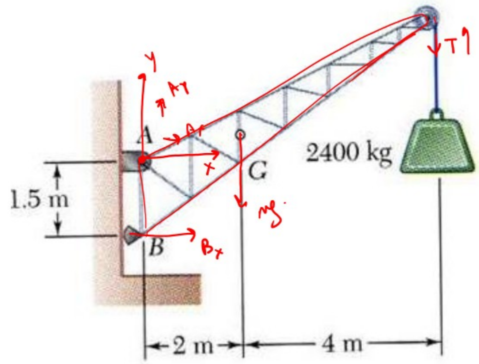
So what we have here is a crane, something that you often see around us, and this crane is fixed at one place, and this is fixed in this picture. It is simply modeled by this support A and B. Now, support A is a type of pin joint support, so it is fixed in the wall and support B is a rocker, which means it can sort of move up and in the vertical direction.

So the whole thing is in the motionless condition, static condition. For our simplicity, so the centre of gravity of the crane is our centre of mass or the centre of gravity of the crane is given. So this point, G mentions the center of gravity through which the weight of the entire crane acts. So that sort of the line of action of the weight of interaction between the crane and the earth passes through this point G. Now what is the question? The question is to determine the components of the reactions at A, and so it is quiet; since this is very common, hopefully, we will learn something about real life by analyzing this problem.

So what I am going to do is that I am sort of going to set you up with writing, I am going to identify, and analyze the problem using our systematic method and then write down the equations and then I will leave it as an exercise for you to sort of solve the equation and calculate the numbers. So, let us get started. So, what we have here is that, so what we are going to take as my system. So I am going to consider this crane as my system. Then what are the surrounding?

So the surroundings, what are the stuff that are relevant parts of the surrounding that is interacting with the crane? So, it is the second question that you should ask yourself.

So the first one is the earth. So, the earth is exerting a force on the crane, which is its weight mg, and this force is known; it is downward. What else? If you take this crane as the surrounding, as the system, so there is a contact interaction by at support A, contact force at support A. Now let us going to define our coordinate system as follows. Let us take, I am going to take point A; it will be clear why it is so, as my origin of the coordinate system and this direction, I am going to define as x and this direction I am going to define as y.



The vertical direction is y, and the horizontal direction is x. So, even though the crane is actually a three-dimensional object, from the figure, it is clear that there is nothing happening in the z-direction, so it is effectively a two-dimensional problem. Now this contact force as A because of the type of joint, it can support, so the joint is fixed, so, in general, we do not know what is the direction of the contact force, so, it has two unknown components, A_x and A_y. Now the other external force is a contact force at support B. Now this is a rocker-type joint, so it means it will, it cannot support a vertical force.

It will start to move if you apply a vertical force, so that means the only force that is possible is B_x. Now what else? Is there any other force on the crane? Yes, so on the other side of the crane, there is this rope, and this rope is exerting a force downward, and that is the tension in the rope. So, the crane is also exerting the rope upwards by Newton's third law with a force which is on the rope, the force is up by the crane is upwards, on the crane by the rope is downwards. So, that is tension force by the rope and let us call that T. So we have how many unknowns? We have 4 unknowns.

Then in order to solve for all of these forces, we need uniquely, if you want to calculate them uniquely then we need four equations. So, next, what are the physics principles? What are these four equations? So, the first equation is the force balance condition in the x-direction, that sum over all the forces in the x-direction must be 0. Now the only force you see from this picture, so this is the force on our system. So, there are only two horizontal forces on the crane, one is at the support A, A_x and at the support B, B_x. So, that is our first equation.

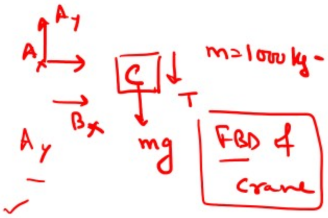
Crane = system.

Surrounding: earth mg ↓

contact force at support A → A_x, A_y

contact force at support B → B_x ✓

Tension force by the rope → T.



on System:

$$\sum F_x = 0 \quad : \quad A_x + B_x = 0 \quad (1)$$

$$\sum F_y = 0 \quad : \quad A_y - T - mg = 0 \quad (2)$$

Similarly, we can have the total force in the vertical direction must also be 0. So, this principle, these equations are coming from the condition of mechanical equilibrium. Now if we do that, then what are the forces on in the vertical direction? So, we have the A_y, so on the crane. So, this is coming from the support A, by the support A on the crane. No vertical force at support B, then we have the tension force T. So, I am going to assume the tension force is downwards and, of course, the force by the earth on the crane, which is its weight mg equal to 0. So, this is equation 2.

Now we also need that the crane, the whole system is not rotating, which means the torque, total torque on the system must be 0. Now, when you calculate torque, again, we need to specify the pivot point, about which point we are going to calculate torque. And in this case, I am going to take point A as our reference point or fixed point, about which we are going to calculate the forces of torque due to all the forces. Why? Because since the line of action of the contact forces A_x and A_y are passing through point A. Note that I am sort of defining these signs in this way, but they can be negative. I am defining the positive sign of A_y upwards, which means if A_y turns out to be negative, which means the force is downwards. And similarly for A_x and B_x. So, these two forces are immediately eliminated; they don't contribute to torque. So, that we will simplify our algebra.

Now there are other forces, and line of action of B_x does not pass through A, so it will contribute to the torque. The line of action of *mg* does not pass through A, it will contribute to the torque and line of action of tension T does not pass through A, so it will also contribute to torque. So, in effect, it is a 2D problem, so the torque is very simple. It is just a magnitude of the force times the perpendicular distance of the line of action from the pivot point A. So, from the given figure, it means, so this distance is so B_y and I am going to apply the right-hand thumb rule.

So, if the torque is coming out of the screen, I am going to take it as positive and if it is going inside the screen, I am going to take it as negative. The torque by *Mg* is in the opposite direction and this distance is 2 meters, and torque by the tension T is also in the, going inside the screen. So, that is our third equation.

$$\sum M_A = 0 : \quad B_y \cdot 1.5\,m - mg\, 2m - T \cdot 6m = 0 \quad (3)$$

So, we have three equations and four unknowns, cannot solve it. We need one more equation. So, where can I get that equation? So, in this equation, we have one more clue in the problem which I have not used yet is that the weight of the block that is at rest is 2.4 newton or 2400 kg. So, this must be important to determine the amount of tension in the rope.

So, let us take, analyze the equilibrium of the block. Now I am going to take the system as the block or in the problem, this is called the crate. So, the free-body diagram of this is very simple. So, let me draw the free-body diagram. So, this is our crane, and it has a force A_x acting in this direction, force A_y acting in this direction, then you have a force B_y, so I am assuming it is acting in this direction. It could be negative, and then you have its weight *mg* acting downwards and a force T acting in the downward direction. So, this is the free-body diagram of the crane. I am going to take the free-body diagram of the block. So, if I take this block or the crate, it has one force mg, which is in the downward direction and another force T, now the force on the block by the rope must be in the upward direction to cancel the force on the block by the earth. So, then T minus *mg*, if I do a force balance in the vertical direction, we get T is equal to $m'g$, so m' is 2400 kg and m is 1000 kg. So, we have another condition that -

System: Block (crate)

$$T - m'g = 0 \quad (4)$$

So, now we have four equations and we have four unknowns. So, I now leave it to you at this point, and you can solve these equations and calculate the unknowns, all the unknowns A_x, B_x, A_y, and T. So, let me summarize what we have learned so far about our first tool to solve mechanics problems. So, the first thing we should do is to choose our system.

So, we were so far discussing the frameworks or truss. So, in this case, the system will be one joint if we are interested to calculate the contact forces at that joint or the framework that is the truss, a whole truss or the part of it if we want to identify what are the external forces acting on the whole framework or the truss, the external load. So, we have taken three kinds of load. A point load, such as a car sitting at a fixed point on the bridge, distributed load and, in this case again, something like the weight of the crane and the weight of the block that it is supporting. Then, the next part is to list out once you know your system, then you automatically do whatever is not part of your system belongs to the surroundings.

So, identify which part of the surroundings is giving, interacting with the system and identify all the forces. So, this will give you all the external forces in this particular case of a truss or framework problem. And then for the modeling part, if we are dealing with a simple truss, then we can ignore some of the internal forces, such as the elastic forces, the bending forces, shear forces, etc. And we can simplify, and then we specify our simplifying assumptions as two rules, rules 1 and 2. I referred to the earlier lecture to recall you what those rules are.

Then you should draw the free body diagram, which shows the external forces. In this case, one thing that you should keep in mind is that the tricky part, so, according to our rule 1 and 2, the directions are given. So, this will be along the beam or the truss element, but the direction can be tricky. Is it towards the node or the joint or away from the joint? Another thing you should remember is that the nature of the joint determines whether it can support force in all possible directions or if it is a rolling type joint, then if we ignore friction, then it may have a force only in the normal direction or not. So, the direction of the force depends on the nature of the joints.

Then the final step is to write down the force and torque balance equations and solve them to get the unknown forces. So, usually, in the case of a truss problem, we solve this, we go node by node and solve this, solve the unknown forces node by node. So, now we are going to start a new topic. So, this is going to, we are going to add one more method of thinking or method of analysis or a new tool of analysis to attack these statistics problems or the systems, how to analyze systems in mechanical equilibrium. Now why do we need a new tool apart from this force balance and torque balance? Isn't it possible to solve all kinds of mechanics problems simply by starting from the principle of force and torque balance which are derived from Newton's second law of motion? Of course in principle it is possible.

But there are certain situations of practical and engineering importance where those methods can be very tedious. So, there are two issues. For example, if you sort of start doing some truss problem, you will immediately very quickly realize that because there are so many unknown contact forces and tension forces in the beam or for contact forces at the joint, the support that the method of solution is very tedious, it is very very painful, very lengthy because of so many unknown forces. The second thing is that, some of these unknown forces are sometimes not, a lot of these unknown contact forces are not always required, we are not interested in them. In a lot of practical situation, we will take some example, we are not interested in those unknown contact forces.

So, in that case physicists try to think about, ask themselves this question, can we get rid of those unknown contact forces which are like internal forces in the system and deal with directly the external forces which are usually known and given. Then our life would have been very easy. And this question can be, so this is what is the motivation for this principle of virtual work. We shall see that, this is not, it is kind of equivalent to our force balance and torque balance condition, but this is much more easier way in certain situations. So what are those situations where we should apply this principle of virtual work works better than the force balance and torque balance method.

So, here I list the learning objective of this topic, this part of the topic. So, first we shall see, they will, so give you several examples. So, this principle of virtual work is really useful when you are dealing with constraint motion. So, we will first understand what is constraint motion and we will start with several examples from real life and we shall learn about different types of constraints. Now this is not only important, like is kind of very common to our everyday experience, but those who will be interested in some kind of project in robotics or designing robots or mechanical arms or this kind of stuff, there this importance of constraints are kind of the bread and butter, very important to learn about the constraints.

Then once we sort of get to know what we are talking about, then we will discuss two consequences of the fact that the motion can be constrained. First is that we will see that the coordinates are not independent, is the first thing and here, we shall introduce the concept of degrees of freedom and generalized coordinates that will simplify the analysis. Second is that the role of, we shall look at the role of the constraint force and we will see that one trick to get rid of the constraint force is to realize that they do not do any work, and that will lead to the principle of virtual work. So, we shall first we state the principle and then we will consider example workout to explain how to use this principle. So, Newton's law is originally designed to describe point particles, motions of point particles, it is very efficient to do it for point particles, but in real life we do have many situations where you are not dealing with point particles and in this case we are considering, let us start with considering that we are dealing with constraint motion.

Traffic jam (New Delhi – Gurgaon)

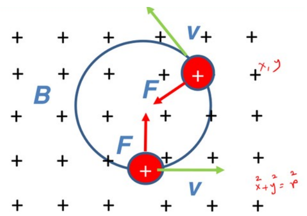
Bus on a road (Munnar, Kerala)

Now, to me at least when I talk about constraint motion, the first thing that comes to my mind is a traffic jam, such as shown in this figure. Now this is an example of, so these cars cannot move because there are other cars surrounded by it. On the right hand side picture, I show another example of a bus which is turning at this turning at a bend. Now in this case what is the constraint? The constraint is the following, it is, we may not think about it always, but it is very obvious that the bus can move only along the road, the surface of the road, which means it is not allowed to jump in the vertical direction. So, in our language we will say that the, in the left hand side example of traffic jam, this is a, the car is, can only move along a lane in one direction, so, this is a motion in 1D, a constraint motion in 1D.

On the right hand side picture, the bus is allowed to move along the road. So, at the turning it can, if it is going very fast then it can skid sideways. So in, so this is why this is a 2D motion. So, this is a constraint motion in 2D. Now in real life, so this may not be always discussed in a typical course of physics and engineering.

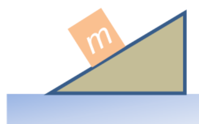
But in near future, but I just want to mention it for the sake of completeness that some of this constraint motion in real life, there exist in real life examples where the constraint motion are, can be difficult to describe by equations, by describe mathematically. So, here is an example, so there is a lioness who wants to chase her food, so, she is looking at the food, but her movement is restricted to protect her cub. So, she cannot go far away from the cub. Then the cub might be in danger. So, this is a situation, so her motion is constraint due to the fact, due to this scenario, but this is a constraint which is maybe not obvious how to describe mathematically.

Now, we took some example from real life, but these are also familiar to you. For example, you often in textbooks, you often have some scenarios, situations where a particle is moving in a circle. Some particular curve, such as in this picture where there is a charged particle which is, and there is a entering of say this region where there is a perpendicular magnetic field. So, in this case, from your magnetic electricity mechanism course, you know that the Lorentz force is such that the force is always perpendicular to the direction of velocity and hence the particle will start to move in a circle which is known as the cyclotron motion. Then, this is a constraint that the coordinates of the particle, so, let us say this x and y are not independent.

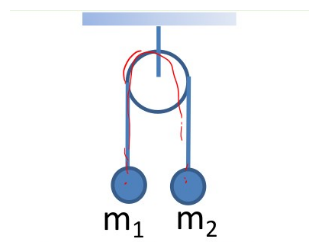


Charged particle in a perpendicular magnetic field moves in a circle: Cyclotron motion

$m v^2 / r = e v B$



Block on an inclined plane: block can not jump up from the plane.



Atwood machine: rope length is fixed

You have, they will satisfy this equation that $x^2+y^2=r^2$ (r is the radius of the circle and this radius of the circle is determined by the amount of force the magnetic field exerts on the circle), on the particle which is given here, the product of the charge times velocity and the magnetic field. Another example is a block on an inclined plane, a very famous problem. We are going to analyse it soon, perhaps in the next week. So, in this case, the block cannot jump up from the plane, so, it is constrained to move along the inclined plane, so it could be a one-dimensional or two-dimensional motion, but not three-dimensional. So, this is the constraint which is not usually stated but and very obvious, but this is there.

The third example is the pulley problem, another famous problem you must have encountered in your high school physics course. So this is formally called an Atwood machine, and here the rope length, the rope length, rope that goes from here to here, this length of the rope is considered to be fixed. As you know, this puts a constraint on the motion. So, the motion of mass m_1 and m_2 are not independent, for example, if the mass m_1 starts to move downwards, the mass m_2 must immediately move upward to keep the length of the rope constant. For your various disciplines of engineering and physics courses in the undergraduate level, may be something that is more interesting will be some sort of a robotic mechanism.

So, here I give you a couple of examples. On the left-hand side, what I show you is a scissor mechanism. So, this is a very common mechanism to sort of move an object. So, now it's kind of, it's more familiar perhaps, it's a collapsible gate, and this is a table. So, this is a kind of X-shaped beam or rod, and there is a hinge as you can see in this moving image. So, this hinge, there you can see, it's clear that these rods are not allowed to move freely because they are connected at the hinge.

A scissor mechanism to move objects

An excavator machine is made up many pieces joined together. The movement at the joints are constrained.

The other example is this kind of excavator machine which is made up of various pieces and all these pieces are again the hinges, and because of the hinges, they are not free to move. There are certain constraints. A very common example of a hinge is our arm. So, our arm is allowed to move only up to a horizontal level, but it cannot move, allowed to move in a downward direction.

It can move only upward direction. So, this angle between this part of the arm and this part of the arm can be a maximum of 180 degrees but not more. Now that we have taken some examples of constraint motion, in the next lecture, we shall sort of first study how to classify, or what are the different types of constraints that are possible. Thank you, see you in the next class.