Course Name: Newtonian Mechanics With Examples
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Lecture - 16

In the last class, we discussed some examples of frameworks as an example of systems under mechanical equilibrium. In that case, the emphasis was on the examples that we learned, and the emphasis was on calculating the unknown forces at the support. The contact forces at the support or the tension forces along the beam or element of a truss, Today, we are going to look at a slightly different question. So, we are going to ask, given a system of framework or a truss and there are lot of unknown forces, which are the forces that are acting on the joint, a contact, a support or the forces, the tensions, and the internal forces that are acting on the beam. So, these are the unknown forces. Only thing we know is the external load acting on the framework and maybe the weight of the load itself.

So, we ask now: what is the condition under which we can uniquely determine all these unknown contact forces or tension forces? So, what is the general principle? So, the general mathematical principle is very simple. So, if you have this number and a bunch of unknown forces, then you need to supply some equations. If you have enough number of equations, then you can solve this equation and find the unknown forces. So, that is the principle.

So, what are the sources of these equations? So, suppose this truss or framework is made of N joints. In that case, for each of the joints, this is what we learned. In the previous lecture, working through the examples, that in order to compute the unknown forces, you have to go node by node or joint by joint and if you take your joint as your system and everything else is the surrounding, Then you can write down two equations, which are equations of force balance. So, I am talking, so what I have in mind here is a 2D truss. So, this is a condition in two dimensions where all the forces are acting on the same plane.

So, you can write down two equations of force balance per joint. So, let us say if you take this as your x axis and this as your y axis, So, if you choose this Cartesian coordinate system as your coordinate system, Then you have a force balance equation for all the forces in the x direction. So, the sum over all force in the x direction, x component of all forces is 0 and similarly, the sum over x and y components of all the forces is 0. So, these are the two equations that you can write down for one node or one joint. Now, if there are N joints, then you have a total of 2N number of equations.

Now these equations, now if you consider the full truss structure as a whole, like then Right now, you are looking at only the external forces, for the whole structure Again, you can write down two equations of force balance as well as choose a pivot point and then You can write down an equation. One equation for torque balance Why one? Because remember the definition of torque, so if all the forces are acting in the same plane, The torque is going to be a cross-product of the location of the line of action of the force with respect to the pivot point and the force itself So, the torque is the moment; the torque, or the moment of the force is going to be perpendicular in the direction normal to the plane. So you have only one component of a moment, it is a simple situation. Now, why did we write down the equation of force balance for a node? In that case, you can simply think of this way: let us say that we take the node itself as the pivot point and Then we are looking at all the forces that are passing through that node. So, as we have learned before, the torque due to each of those forces should be 0 about that node itself.

So, the point is that for the entire structure, you have two equations of force balance and one equation of torque balance, so it is all three equations. Now, these three equations, because you can sort of write down the 2N equations for each node, then these three equations are sort of not independent. If you sort of combine all the total 2N equations for each of the node, then you can show that you will. It already includes these equations of force balance. So this is what we have seen in the example that we worked out last time.

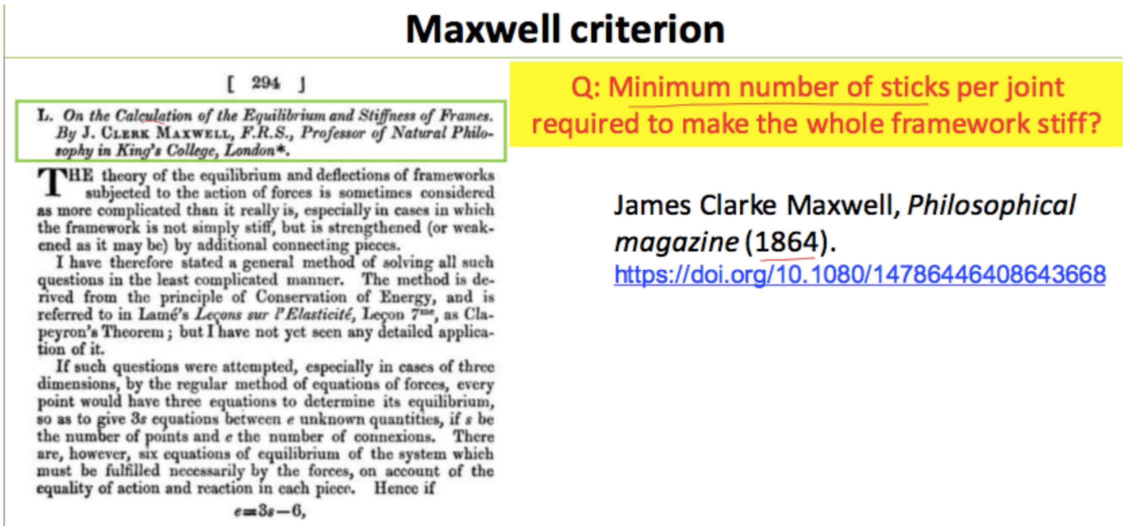
So, the number of independent equations is actually not 2N, but 2N minus 3. Now if there are how many unknowns, so if there are M beams or truss elements, then and we assume with all the terms and conditions that we discussed in the last few lectures that we are going to assume that only internal force is acting along the length of the beam, then the only thing that is, so the direction is known. If I know the truss structure, then the direction of the force, the tension force, is known. Only thing that is unknown is the magnitude. So, there are M unknown magnitudes of internal tension forces.

Here we are assuming that the beams are massless, so we can ignore all other elastic forces. So then we can have M unknowns and 2N minus 3 equations. So then we have three possibilities. The first situation is M, the number of unknowns is less than the number of equations. In that case, it physically means that there are less beams or truss elements than necessary.

So this is a case where the structure is unstable and may collapse under an external load. If the M number of unknown forces is equal to number of equations, then we can solve, So these are algebraic equations, so we can solve those equations and calculate all the unknown forces, unknown magnitude or tension forces uniquely In that case, we say that the structure is rigid and statically determinate because We can compute, we can determine all the unknown forces. The third situation is where the number of unknowns is greater than the number of force balance and torque balance equations. How is that possible? It basically means that there are more beams than necessary. So the minimum number of beams as is necessary to make the structure rigid or rigid means In that sense, it can support the external load.

But then you can always add more beam elements given this minimum structure and Then you arrive at the situation where M, the number of unknowns, is bigger than the number of equations. Which means that some of those beams are redundant. That means that if you remove some of the beams, the structure will not collapse. In this case, all internal forces cannot be uniquely determined because you have more number of unknowns. So this situation is called statically indeterminate.

Now here, I want to sort of mention nice piece of history and this is something that I want to say it because it illustrates the relationship between the engineering point of view and the physics point of view. So this framework, as you know, is designed with suspense, and bridges, roofs, etc., are very old. So it is a very old engineering problem. And engineers like civil engineers and mechanical engineers, who are, have the task of building those structures.

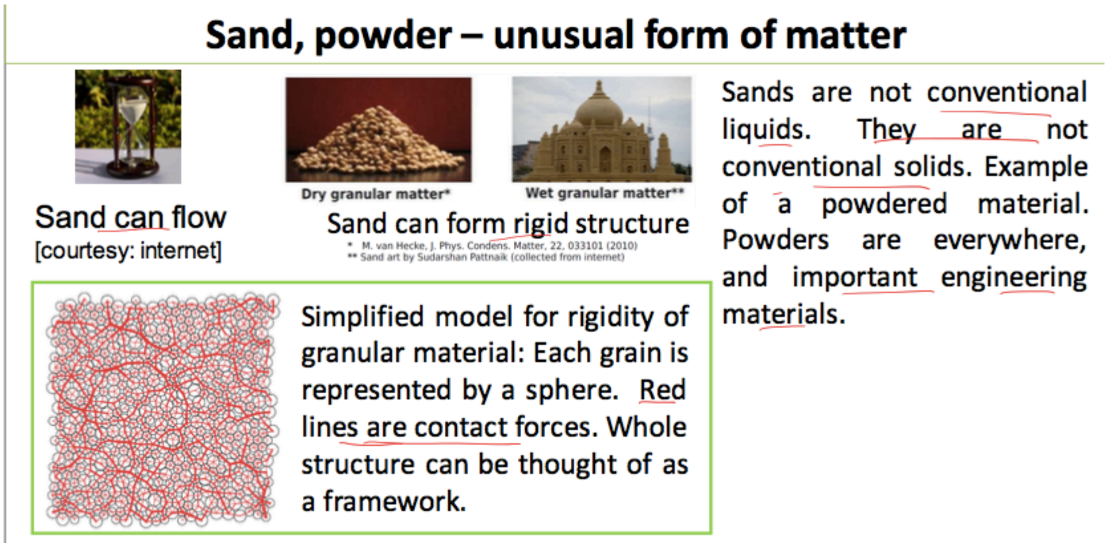


So they are focused, or they are interested in knowing what forces are acting on each of the supports. how much force or support it can withstand, and so on. Now there is a Maxwell, James Clark Maxwell. He was a famous physicist. So those of you know that he is the same Maxwell who studied Maxwell's laws of electricity and magnetism and those of you who are taking thermodynamics and statistical mechanics course, They will be studying Maxwell's relationships.

So it is the same Maxwell. So he considered this problem and he wrote a famous paper which is very old, 18, published in 1864 in Philosophical Magazine I have provided the link so that if you are interested, you can go and check out that paper; it is available online. So he considered this same problem in the calculation of the equilibrium and stiffness of frames. So the frame is the same as the task, truss, but his focus was on a different question. So he posed an interesting question.

So he was not interested in calculating each individual's unknown contact force. But he was interested in some sort of device, a general principle by which you can address this question. So his first question is: what is the minimum number of sticks required per joint to make the whole framework stiff? So this is a kind of typical question from a physics point of view. So physicists always try to work out and try to find out a general principle which can apply to all situations, Whereas engineering's point of view is to take a specific example and find out the specific design criteria, or for a particular design, the specifics, like solving the unknowns. And he went on, and then he sort of gave a general method for how to solve this problem.

If you look at it from a general physical point of view, which is applicable to all sorts of designs with any framework, Now I mention this thing because, in modern times, there is this particular approach, so let me mention this. So this particular criteria that we found discussed is nowadays called Maxwell's criteria. So now I mention one more example from note, not now not from civil or mechanical engineering, but perhaps relevant for chemical engineering and definitely physics. So this Maxwell's criteria and Maxwell's way of approaching this problem nowadays finds a very surprising application in a particular field, which is called powders. So if you want to study structures made of powders or grains like sands, So, sands and powders are very common materials; they are everywhere around us, but they are somewhat unusual forms of matter.



Why? Because, as you know from experience, sand can flow, for example, in a sand clock flows like a liquid. However, if you put lot of sands in a high density packing then they can also form rigid structure such as a heap of sand, a heap of rice, or a heap of food grains. How do we know that this forms a rigid structure? So next for example, if you take a bottle of, let us say sugar and you try and If it is well packed, then if you try to insert a spoon inside it will prevent, So, it can support the external force that is exerted by you on the this heap of sugar. So in that sense they are rigid and this is also a picture of sand art, which is definitely a rigid structure. Now these kinds of materials, so sand is an example of a powdered material.

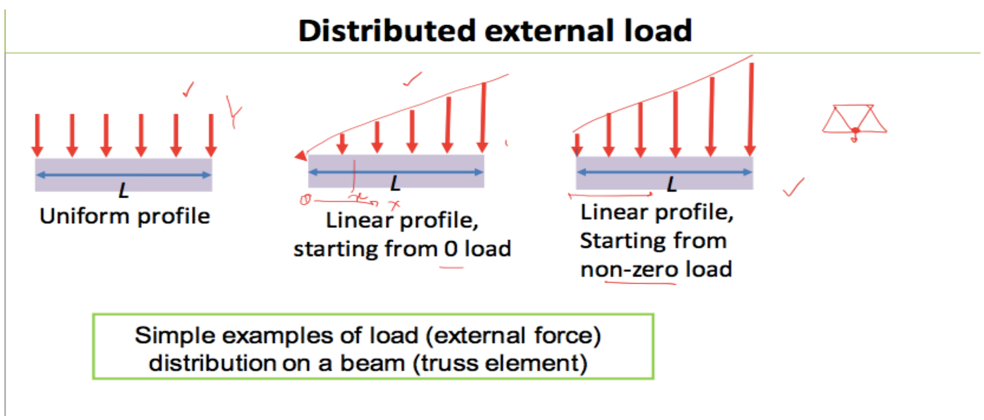
So powdered materials are very important, for example, because a lot of chemicals are produced in powder form such as cements or even if you take a like the capsule, the medicine that you take as a capsule that the medicine is put in the capsule in powdered form. So it is an important question in those kind of situations from practical point of view is that under what conditions will a powder behave like a solid or a rigid body? and under what conditions it will behave as a liquid and that is it can flow. Now I must mention one more thing: sands or powders are not conventional liquids like water because their flow properties are very different from those of typical liquids like water. They are also not conventional solids in the sense that they remain solid bodies. Only when you can sort of compress them enough so that they achieve a high enough density and You have to help them in a rigid form by external force.

If you remove those external forces, then they may fall apart and no longer behave as solids. So this is different from the usual solids that you see around you. But they are very important engineering materials. So, perhaps from chemical engineering, and because of these interesting properties, they are also very interesting for physicists to study. And in this case, the method that we learned about the truss or framework has an analogy.

So what I show here is a simplified model of a granular material. So think of each grain like sand; the different grains have different shapes. So here each grain is represented by the same shape; they are all spheres, or, let us say, if you think today, they are all disks. And the red lines are so that when the two disks are in contact, they form a contact force. So these red lines represent the contact force, and the thicker the line, the bigger the magnitude of this force.

And the direction of the force is assumed to be along the line joining the centers of two disks. Now, if you look at this picture of the forces so you can see that they actually look like a truss or a framework. And in fact, you can use the same sort of analysis to sort of get to use this force balance conditions, etc. to analyze the condition of rigidity of the properties of this kind of granular structure from a truss point of view. So this is something I wanted to mention because, usually, when you think about truss or framework, you think about something like a crane, a bridge or some other manmade structures but Here, I show that this kind of concept can also be used to study various natural phenomena, such as the properties of granular matter. So now we want to discuss another important point about the truss.

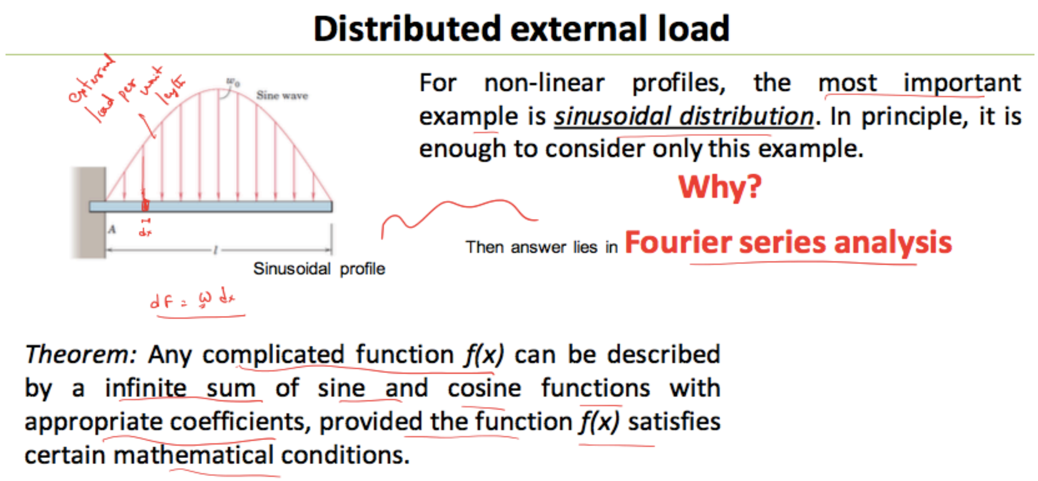
So far, what we assume are the problems that we worked out are these external loads act as a single point, such as for example, So we had this bridge, and there was a car on the bridge and we assume that the car is acting at one of the joints. But in many real-life situations, this is not very realistic. So, perhaps more common will be that these external loads are distributed over a region. So there is a distribution of external loads. For example, here are a few simple examples.



Let us say this block represents a beam, and then this red arrows represents the profile of the external load. So in this case as you can see that the all arrows are of same height that means The low external load is uniform, and it is distributed over a length of L. In this example, the arrows are not of equal height, which means that the force profile is not uniform but If you sort of join the details, you can see that it is linearly. So it is starting from 0 at one end of the beam Let us say this is at one end of the beam, and it starts at 0, and then it linearly increases. So the load at a distance, let us say, is our origin, and this is our x axis.

So then the load at a distance x is proportional to the distance from the end. And this is a slightly more complicated example. So here again, the load is increasing linearly with the distance from one end. However, in this case, the difference is that in this case, with the sec, then in from the middle example is that In this case, at the end, the load is not 0, it is non-zero. So you can think of this as a combination of the first example and the second example.

Now it is not required that this load profile need not be linear it can it can be non-linear also. Now in the non-linear there could be lot of infinite number of possible load profiles. However, the most important example is a sinusoidal distribution, which means so this is where I am showing the arrows. So, by the way, I must mention this arrows Do not think of this arrows as force per unit area which means if you take a unit area, then the force acting on this The total force acting on this particular area is given by the arrow times the area. That is, if you take, let us say, the length of magnitude dx, then the total force acting on this length dx is this arrow, which is called the load per unit length times the length of the area.

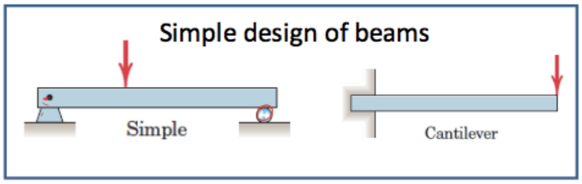


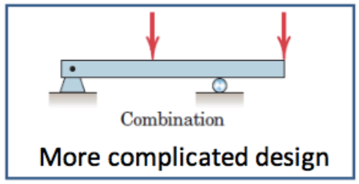
So this is the meaning of this profile. So these arrows represent the external load per unit length. So here, it follows a sine curve. So sine curves and cosine curves are equivalent. So I am assuming a sine curve. So, why is sine the most important example? because of a very powerful and wonderful mathematical theorem called Fourier series analysis.

What is that theorem? So I am going to so you learn this theory about Fourier series in other courses. So I am just going to state that the general principle of that is that any complicated function fx any complicated non-linear function. Something like this can be described by an infinite series or sum of sine and cosine functions. You can always analyze them in terms of components. So each component is a sine profile or a cosine profile with appropriate coefficients provided the function fx satisfies certain mathematical condition.

So if we know the sine analyze, if we understand the sine and cosine profiles, then by combining them We can understand any non-linear profile. One more thing is that when we sort of look at this kind of situations of distributed force like the nature of the support, we found that the support there are different designs of support Similarly, that is also true if you have a beam. So for example this is a beam which is one end is sort of pin joint and the other end is a roller. So this is supported at both ends. This beam is called a cantilever so it is supported at one end and the other part is hanging in the air.

Nature of beam is also important





And then you can get more complicated and this red arrow is again showing the external load. So again, you can have a combination of them and have a more important design. So the question is, are we interested in this kind of situation? So the question that we can analyze the so there could be of course different types of question but what we are interested in is that, given what you want to be interested in calculating, the force on the joint, so there is a practical engineering question. Now, given the external load, this external load is distributed over an area so what we want to do is we want to replace this force distribution by a single equivalent force with the same magnitude. So we want to know the magnitude, direction, and line of action of the equivalent force.

we want to calculate and in order to do that, we want to calculate the moment that this force is generating, and the equivalent means that it will be clear in the moment. So these are the things I will be interested in knowing. So we want to calculate the total torque generated by a distribution of load and it is easier to do that a simple trick is if you can replace this distribution of load by a single equivalent force acting at a single point and how to do that. So this is the question. So this is the question that we are going to analyze in the next lecture. Thank you.