Course Name: Newtonian Mechanics With Examples

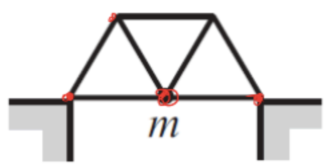
Prof. Shiladitya Sengupta
Department of Physics
Indian Institute of Technology Roorkee

Week 04

Lecture - 15

So, this is the course on Newtonian mechanics with examples. Last week, this is our third week, we started studying the statics problem where the systems are at rest in mechanical equilibrium. So, this week, we shall continue studying those problems with different approaches. So in the last lecture, we started discussing the example of an important practical engineering structure, which is called the truss or a framework. So, just quickly, we described a few examples and defined the problem that we wanted to analyze. So, today we are going to: first, we are going to take an example to go step by step and understand how to analyze this problem.

Example 13: a simple bridge



The bridge in figure is made of three equilateral triangles of beams. Assume that all the seven beams are massless and that the connection between any two of them is a hinge. A car of mass *m* is located at the middle of the bridge, **find the tensions T in the beams**. Assume that the supports provide no horizontal forces on the bridge.

So, this is the problem that just quickly let me recall how we define this problem. So, we will assume that there is some structure like a bridge and the whole structure is at rest. So that we can apply the conditions of force and torque balance, there are two rules that you need to remember: The total force on any single beam is 0, and the total force on any joint, support, or node is 0. So, the strategy that you are going to use is also called the method of joints.

And one more important point: the external loads are given and our goal is to find the internal tension forces in each beam and the forces and contact forces at the joints. So, here is an example of a simple bridge. So, this is a very kind of toy model of a bridge. So, this is a bridge, and this type of bridge is called a cantilever bridge. So, this is different from a suspension bridge, which you can think of as a hanging cable.

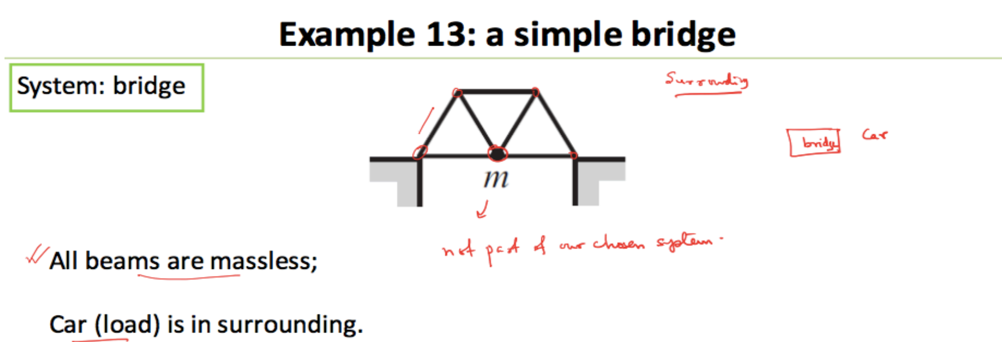
So, here, this bridge is supported on both ends, and it is a very simple structure. So, this is often seen as a railway bridge in our country. So, this is supported by three equilateral triangles, which are made up of 1, 2, 3, 4, 5, 6, and 7 beams. So, assume that all seven beams are massless and that the connection between any two of them is a hinge. So, these are like pin joints.

So, the supports are like pin joints. Now there is an external load, a car, so this black object is a car of mass m, Which is standing still in the middle, precisely in the middle of the bridge. Now, given this situation, find the tension T in the beams. So, this is the question. Now it is also given that we should assume the supports at the end.

These are the supports, the joints that support the bridge on two sides. There are no horizontal forces. So, this is like a roller-type support. So, how to solve this problem? So the first thing that we should note is that we should start by choosing our system. So, in this example, we are going to choose the entire bridge as our system.

Now, if the structure is very lengthy or complicated, then you can also choose a section of your frame wall as your system. Now we note down two important clues given in the problem: that all beams are massless, which means the tension is uniform along the beams.

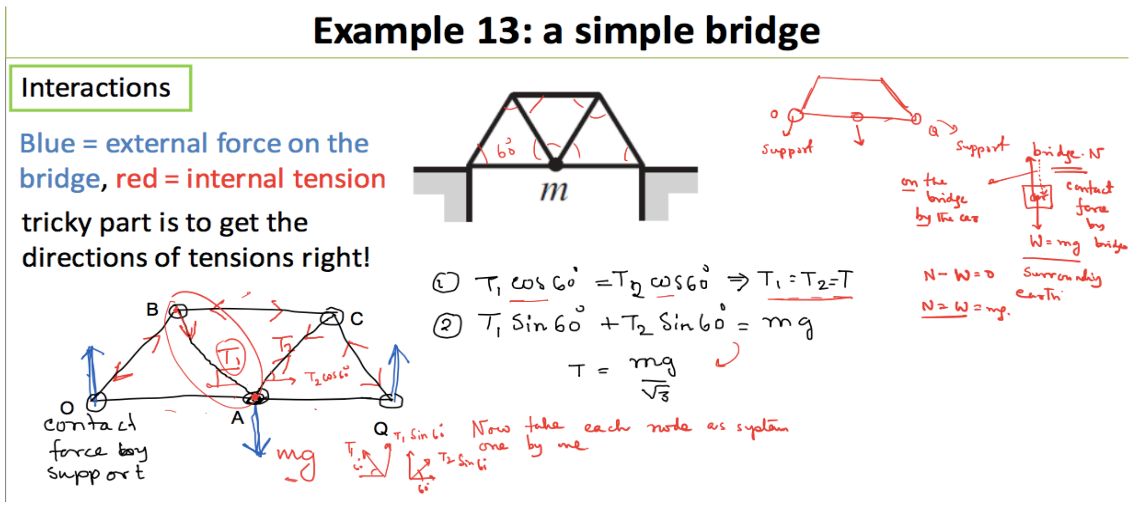
So all the tension forces are uniform along the beams. Now there is this external force. What are the external forces? Now, once we determine the system, we automatically include everything that is outside our system is our surroundings.



So this load is not part of the system. So this car, which is of mass m is not part of our chosen system. So when you analyze the problem, you should make this kind of mental map to sort of highlight this point. Now, once we define the system, we need to list out all the interactions. What are the forces interacting in the system? Now here, let us say if I take the entire bridge as the system, then the forces are actually there are some forces, as we show in this picture.

So there are force on this joint; this is a support. So, it exerts some force on this bridge, and similarly, on the other side, let us call that Q. So in this case also, it exerts some force on the bridge. On top of that, the car is also exerting a force downward on the bridge, which is a contact force. Now that this force is also unknown, we can determine the contact force, and you can replace it with the weight of the car.

Why? Because suppose this is my car, and then let us draw the free body diagram of the car. Then on this car there are two forces, so one force is its weight, which is mg. So then this is an interaction with the surroundings of the car. In the surrounding of the car, not the bridge, but the car, there are two important objects: One is the earth, which is pulling it and exerting a gravitational force, mg and the other is this. So, the car is also sort of interacting with the bridge, and let us say this force is N.



So then, this force is a contact force by bridge; now I can draw it more clearly. So, this force is a contact force on the car and I also draw another force, So this force is Newton's third law pair of this contact interaction between the car and the bridge. So this is the force on the bridge by the car. So, this is the force that is required for our analysis. Now if the car is at rest, then we know that the total force on the car is 0.

So, that means N minus W must be 0, which means N equal to W, so this is the magnitude of the contact force and Hence, we know the magnitude of the force on the bridge by the car which is also W that is mg and Hence, the direction must be downward, so now the direction is downward. This is important because, in this problem of truss, the directions can be tricky. This is important. So you must pay close attention, and that is why in order to reduce, To avoid confusion, you must have a very clear idea about how to define your system. Then the directions will be very clear.

So if I take the whole bridge as a whole as my system, then these are the external forces. In the given information, I have drawn this diagram here to denote these external forces in blue. Now that this is given, the problem asks to calculate the force, internal forces, and tension forces in the beam. In this case, the strategy is, so now you sort of take each node as your system and each node as your system. If you take each node as your system and then we need to draw the free body diagram of that particular node and apply the conditions of force and torque balance.

Then we can write down some equations for the unknown tension forces. Once we find all the equations, we can solve them to calculate the unknown forces, so this is the general strategy. So now, which node should you start with? You should start with a node on which the external force is acting; that is, always simplify. That is an important trick to remember. So, here I am starting on this particular node.

Now, in this frame of stress problem, it is tricky to get the direction of the internal force tensions, right? Which means at a given node whether the force and along the beam are along, like away from the node or towards the node. So, whether this is in our language as defined earlier, whether this is a tension force or a compression force, Now in this particular case, let us say we take A, the node A on which the car is standing. Let us take that as our system, Then there are three forces acting at this node: One force is evidently the force on the bridge supported by the car. We have just found that this is equal to mg, and it is directed downward. The second force is the tension along the beams, so there are two beams, which are sorry, there are 1, 2, 3, and 4 beams.

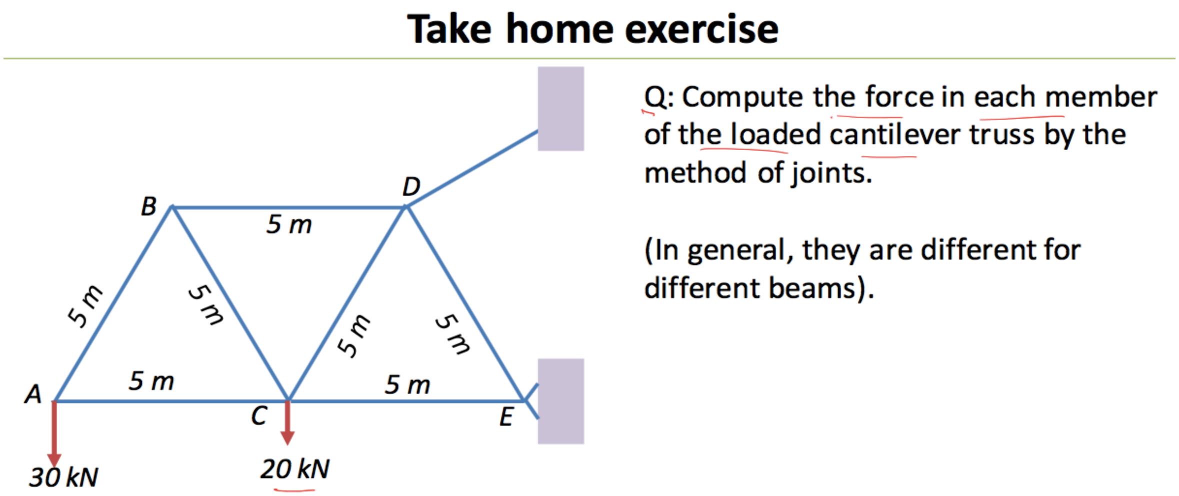
Which are connected at this node A. But in the given problem, it is given that in these two beams there is no horizontal force. Which is given in the problem. So we ignore these two beams, so there are only two beams remaining. Let us consider that the tension forces in the beams are T1 and T2, respectively in beams AB and AC.

Then from this picture, by geometry, so all of these beams are of equal length, So, all these triangles are equilateral triangles, so the angles are all 60 degrees. Then we consider the force balance equation for this node A to balance this force mg, which is downwards. Intuitively, it is clear that the direction of this force is T1 into T2, which must be away from support A. So, the horizontal component of this T2 must be T2 cos 60 degree. Because this is 60 degree and similarly, the horizontal component of the force T1, which is acting in the beam AB should be T1 cos 60 degree.

And since the node is at equilibrium, so if that means if we do a horizontal force balance, So what we find is that these two force must cancel each other, so that is T1 cos 60 degree must be equal to T2 cos 60 degree and It follows that these two tension forces are equal in magnitude, and let us call that T. Now if we do a vertical force balance, it is a force balance in the vertical direction. So then we see that the vertical component of this force is T1. So, this T1 is sin T1 sin 60 degree, and similarly for T2, this is 60 degree, so this is T2 sin 60 degree, and there is a force mg downwards. So, these two vertical components must be balancing this downward force mg by the car on the bridge.

So, this is not the weight of the car; it is the contact force by the car on the bridge. So then we can write down the vertical force balance condition and if we apply that T1 is equal to T2, which is equal to T, then it follows by solving this equation that this unknown tension at each of the tensions T must be equal to mg by root 3. Now I will give you an exercise that you can sort of see and verify. Now take this another node B as your system and then there are three beams which are connected at node B and I have shown; now remember that if we apply our rule 2 that, So if we want to know the force acting on node B, then we need to determine the force direction at node B by the beam and on the beam, and if we apply the force balance condition to the beam AB, the total force by the beam must be 0. So that means if it is experiencing, if the beam is experiencing, if we know the direction of the force T1 at the node A.

Then the direction of the force on node B, which is the other side, The other end of this beam must be opposite, and it must be equal in magnitude as well. So, if we know this force, then we immediately know the magnitude of this force. This force must be mg by root 3 and similarly, you can consider the other nodes and each of the nodes. So, take B, then take C and then write down the force balance conditions. From which you can get what these tension forces are in the other beam elements.

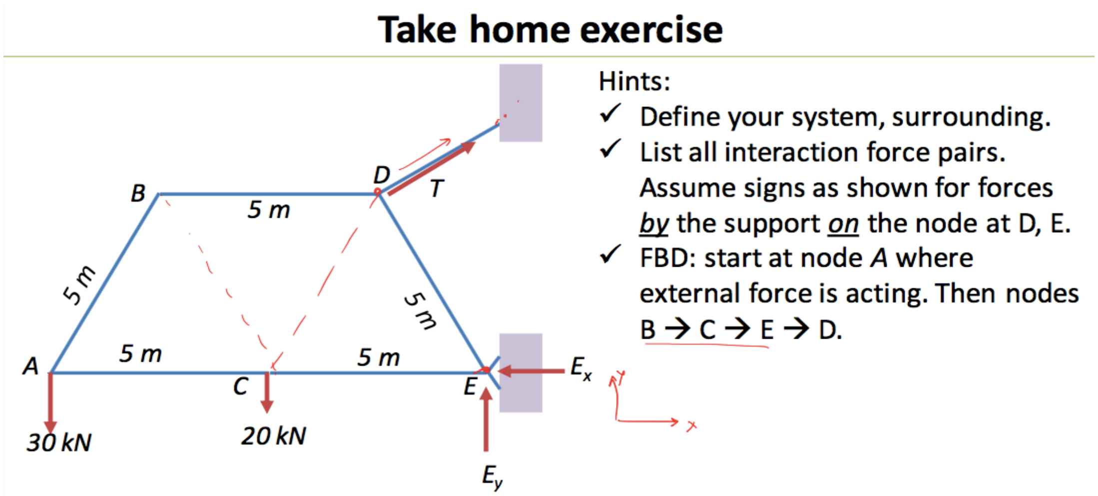


So, I will leave as an exercise another problem, so this is a slightly more longer, Slightly more complicated truss with same number of beams: 1, 2, 3, 4, 5, 6, 7, so same number of beams. But in this case instead of the, so the total external forces are slightly different, So, the supports are now no longer at the two ends and the base, but they are on one side at the two ends of this beam, D and E, so this is the difference. And the other difference is that, like in the previous example, you have an external force. Which is acting on this framework at the node C, which is 20 kilo Newton, But in addition, there is another external force acting at end A, which is 30 kilo Newton. So, note that the red arrows represent the external force both in magnitude and direction.

So, the question is: given this structure and the supports, compute the force in each member of this loaded cantilever truss, So, this is kind of a bridge or a cantilever which is supported. So, cantilever is a structure which is supported at one end and it is the side of hanging; the other part is hanging in the air and by the method of joints. So, one thing you must remember here is that, in general, you should not assume that the tensions in all the beams are the same; you have to assume like we did here. When we started that, the tensions are different and then if the problem showed that, the calculation shows that the tension turns out to be the same, then it is well and good. So, I am not going to solve this problem because this is meant to sort of give you an exercise practice to sort of write down the force and torque balance conditions.

So you do not need any torque balance conditions here in this particular problem at each node by node. So, instead, I am going to give you a few hints on how to approach this problem. So, first, if you take your define your system as before and the surrounding, So, let us say you take the entire structure ABDECA as your system and that will give you an idea of what the external forces are acting on this framework or the bridge. So, there is, of course, the force T, which is because of the support. This particular support which is supporting the structure and then there are other forces.

Due to the other support at the point at the support E at the node E, Now, so I will give you a hint, so assume that it supports the nature of the role. The contacts are the supports, and this supports are such that I mean the support at D and support at E is such that these are the directions of those external forces. So, assume that you can assume that. So at D, there is only one force, which is going along the length of this beam at the joint E, these contact forces two components in general, one which is the x axis. This direction is our x axis, and this direction is our y axis, and then this is a horizontal component and a vertical component.



So now all you need to do is go node by node take each node and then apply the free body diagram at each node and Write down the force balance conditions and make a list of which is unknown. So, the tension forces are unknown and the external forces are given and Then you have an equation, and you solve this equation to determine the unknown tensions. Now, here is a hint that which node you should start with. So again, you should start with the nodes where the external forces are acting. So, let us start at node A and write down the draw, then take node A as your system.

So this is connected by two; let me complete the beam structure. So it is connected by only two nodes two beams. So write down the free force balance equations at node A, then go to node B, then go to node C, and then go to node E instead of D. So if you follow the node, your calculations will be simple. So let us continue discussing some more aspects of this truss structure in the next lecture. Thank you.