Newtonian Mechanics With Examples

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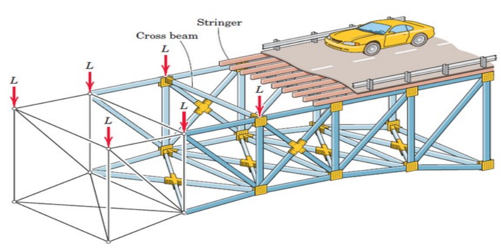
Week -03

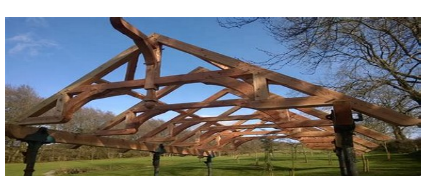
Lecture -14

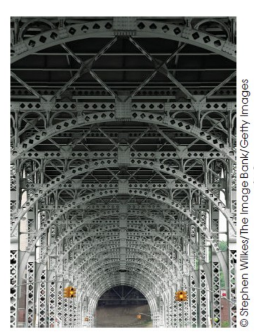
In this course, Newtonian Mechanics with Examples, let us continue our study of statistics, that is, the mechanical equilibrium with different examples. In the last few lectures, we reviewed the tension force. We sort of told about the different properties of the tension force, when it is in form when it is along the line of tangent, and so on. And then, we considered an example of interesting and very common example from everyday life, which is about the ropes, hanging ropes or, hanging cables, or suspension cables. Today, the plan is to take another very interesting and very common example, which has lots of practical engineering applications as well. So today, we are talking about the truss or the framework. So here is the plan for how to go about it.

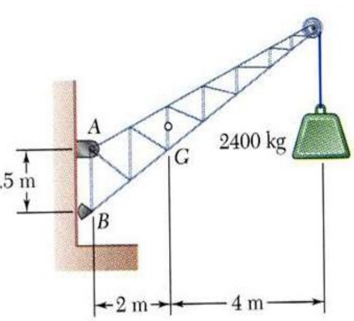
So first, we will take a few examples, and then we define the question that we are going to analyze. Then, we will use our systematic method of analysis and take different worked-out examples to sort of tell you how to solve this kind of problem. Then, we will consider a very important question, which is that under what conditions we can determine this tension forces uniquely? And we will see that this problem is really interesting because this is usually when we talk about truss, the things that come to our mind are from mechanical engineering problems or civil engineering problems.

But we shall take a very different situation and example, and we will show that this kind of analysis is very generally applicable to other kinds of problems, which you may not usually think of from your engineering curriculum. And I particularly like this problem because it sort of gives you a very interesting way to see the relation between basic physics and basic physical principles and the engineering point of view or the application point of view. So, let us start. So here I show you four different pictures.

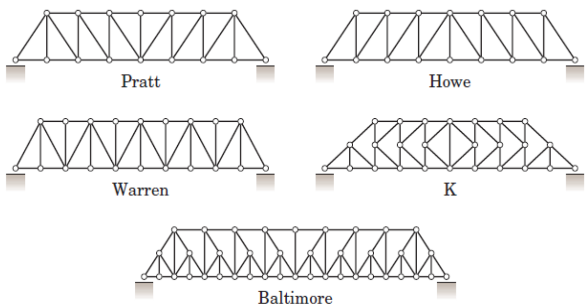




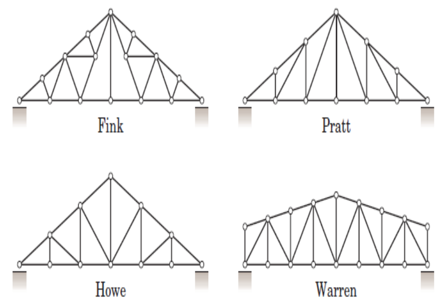




So, one picture is a part of a bridge. The another couple of pictures, this one and this one, are about roofs and this is a picture of a crane which is supporting a load. Now, what is common about these pictures? So what is common about these pictures and in all these cases there are some sort of an external load, such as this car or whatever tiles you are going to put on this roof or this roof and this roof here, this load here. And to support this load, the engineers have designed a framework of beams joined together. So, such as what is shown in this picture here and this the wooden beams that are connected. So these are the, and this crane is made up of these connected beams.



Commonly Used Bridge Trusses

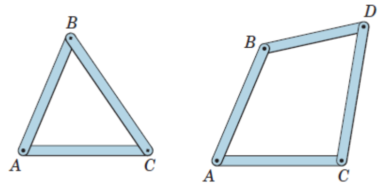


Commonly Used Roof Trusses

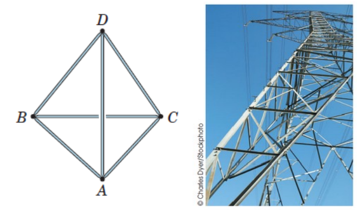
So these are kind of the important engineering structure, and the goal is to keep the whole thing in equilibrium, such that this bridge does not collapse or this roof does not collapse or the crane is able to lift this weight in a safe way. Now so, let us continue more. Now, if you are going to be looking at this problem from a mechanical or civil engineering point of view, maybe you are going to be interested in designing this kind of framework. Now, I should clarify here that in this course, our goal is to study this problem from the physics point of view. So our goal is not to teach you how to design this different kind of frameworks but to sort of help you see how you are going to apply basic physical principles, the simple conditions of force balance and torque balance to sort of that will enable you to design arbitrary types of structure.

So, let us define more precisely what truss is. Truss is a framework. Framework means like these beams, which are connected. So, it is composed of members joined at their ends to form a rigid structure. So the word rigid is important, and this is called a truss. And examples are like, as you shown before, the bridges and roof supports etc.

Now, if I say in a plain English then you can simply from physics point of view, you can simply think of the truss as some sticks, such as match sticks, joined at the ends. Like if you take some match sticks and join them by glue at the ends and then, you can build this kind of frames and that is our toy example of a truss. Now note that there are two types of frameworks possible. One in which these sticks joined and these frames are forming a planar structure, which means all the sticks or beams they are in the same plane.



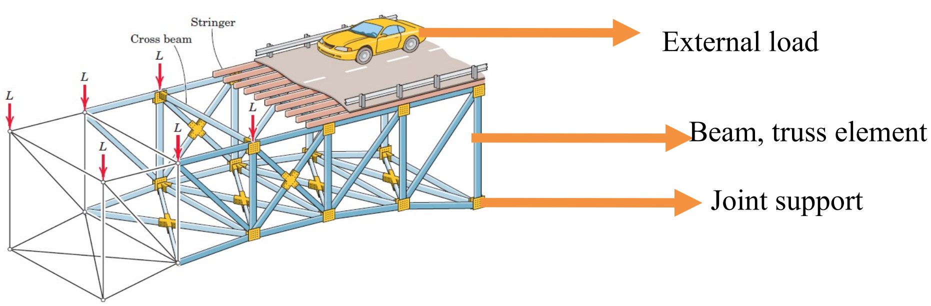
Plane or 2D truss



Space or 3D truss

So, this is a plane or two-dimensional truss. The other possibility of course, is that, so let us say, for example, using four sticks in plane, you can make a like quadrilateral, but in three dimension, you can make a pyramid. So this is an example so: 1, 2, 3, 4, 5, 6, there are six sticks here. So, this is an example of a three-dimensional framework or, which is also called a space truss. Now, for simplicity, we are going to take examples with two dimensional truss, it is easier to draw and analyze and, again but the method of solution will remain more or less same if you go to three dimensions.

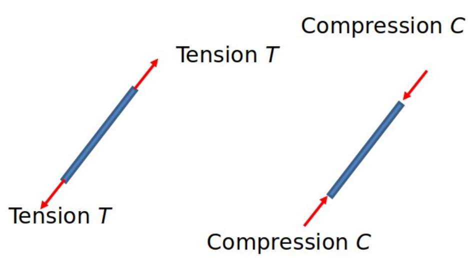
It is just the calculation becomes slightly more complicated. So let us now define the question, so what we are interested in is that in a given, we have considered this situation where there is some external load such as this car on the bridge, and there is a structure, such as this framework of beams which makes this bridge. So which is supporting this load, and this important point of the structure is that these are not a continuous structure; it is made up of sticks and the sticks are joined connected at some joints, so there are some supports from the joint. So these are the three pieces. Now we want to know that, so we want to analyze how this external load can be supported by this framework so such as the questions we can be interested in is that so to support this load there will be some internal force or, which we call the tension force which will appear in the beam.



So, how much is the tension force in these beams and how much is the contact force at the joints? So these are the two crucial question whose answer will determine the stability of such structures or rigidity of such structures. Now, of course we need a few assumptions to simplify our problem because, in general, it will be clear in a moment, in general, it is a difficult problem. What makes this problem difficult? So these beams are actually a massive structure like if they are made of steels or irons, they are massive heavy structures, but we are going to assume that these beams are massless. Now we have reviewed this assumption in previous, one of the previous lectures, so it basically means that the external loads are quite high so that the weight of the individual beams can be neglected compared to the load they support.

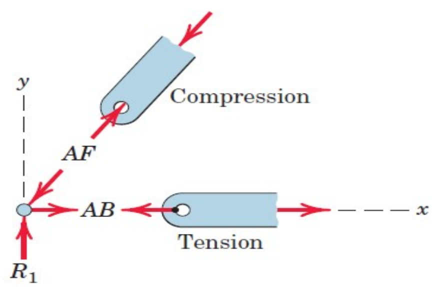
So this is a very simple assumption because we saw that it sort of immediately it follows that the tension force along the beam is uniform throughout, so we do not have to consider the variation of the tension force along a beam. So, in that case, you can sort of think of these beams as kind of a transmitter of force between two joints. The other assumption required to just to very important assumption is that we are going to assume that their lengths are also fixed. So, the distance between two joints is always the same no matter what the external load is. So there is no extension, no contraction, no any kind of deformation or crack or fracture or some sort of a torsion or bending or twist, swing, swing in air, etc.

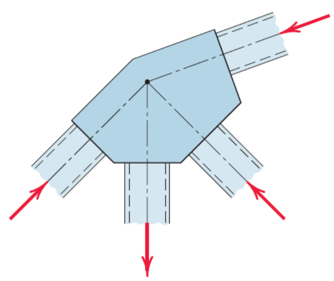
So, all these kinds of motions are possible for real such structures and they are important like if you really want to build such a structure you want to consider all these important effects, but for our analysis, we are going to ignore those effects to make our life simple. So this is actually a very idealistic situation, but this will sort of tell you that our focus is on the like learning free body diagram and applying the basic conditions of force and torque balance, so we are going to ignore all these things. So those of you who are kind of will be actually thinking of doing design of bridges, etc. They will consider all these things in your respective engineering discipline in later years. So if you ignore all those effects, then the only internal force that the beam can support is the tension which is a force that is acting along the length of the beam in the direction it is tangent, and we are going to assume that there is no bending of such beams, these beams are basically a straight line, so there is no curvature of the beams unlike the previous problem that we discussed of suspended cable or hanging rope where the rope was perfectly flexible so that it could, but in this case we are going to assume that these beams are perfectly rigid, inflexible.



So the tension force is always along the direction of the beam, and because we ignore all those other kinds of internal forces, different kinds of deformation of the beams, and the beam is massless, so it immediately follows that tension forces uniform throughout the beam. Now, in this case, our problem is what is called a simple stress so that each beam element, or each element beam, you can sort of think of it as so this is connected at two ends by some support, and the only interaction is happening sort of as if happening only at the support and so this beam is interacting, so if you consider this one beam as your system then the surrounding is and the support so this is if I consider this as my system and then the support is in the surrounding then the beam is interacting with the support at the end. So for the, if the whole beam is in equilibrium is in static condition motionless it is not moving, then it immediately follows that the forces by the surrounding, that is, by the support end of the joint of the beam on the beam, must be equal and opposite so that the net force is 0. Now, there are two possibilities: either these contact forces between the beam and the support is directed towards the support away from the beam in that case it is by convention it is called a tension, or the contact forces, so these are the contact forces, contact interaction between the beam that is our system and support that is the surrounding. The other possibility is that the direction can be in the opposite so that the beam is the support is putting a force,which is into the beam in that case by convention it is called compression.

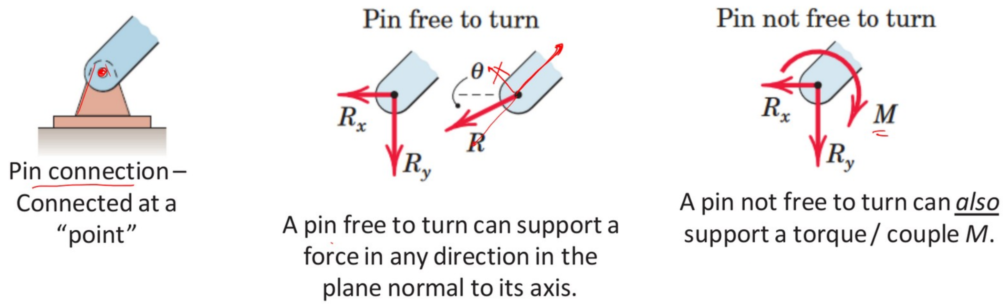
So this is about the direction. There are just two possible directions along a straight line, and for the force balance, these two forces at the end must be equal and opposite. The other thing is that, so just to say it on the from the other point of view, so in returns, because these contact forces are in like real interaction so by Newton's third law if the forces are giving contact supports, are exerting force on the beam in return by Newton's third law beams are also exerting forces on the supports or the joints. So, for example, if I look at this diagram so let us say there are these two beams, which are joined at this point at the support common support like, which is also called a node. So then the AB is the force by the node on this particular beam, and sorry the, this force is the force on the beam by this joint, so by Newton's third law, this is the force which is the reaction force, so this force is by the beam this is a beam and this force is on the beam, this is on the joint, and this is by the joint, so they are action-reaction pair. We are also going to assume a crucial assumption that all the forces, that is if there are more than two or more than two beams, are connected at a common joint, then the line of action of all the forces are passing through the same point.





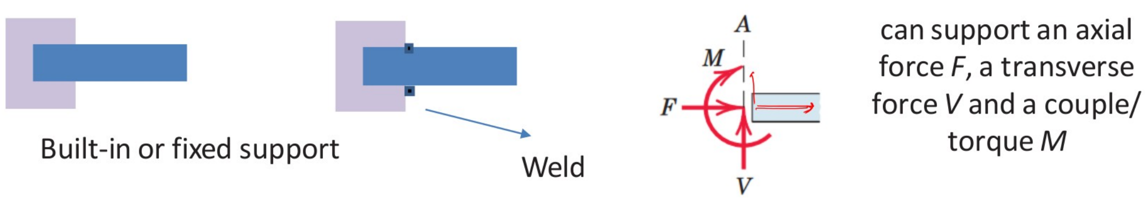
So this is a simplifying assumption. I mean the point is that, in general, this joint covers some particular area but you are going to replace it by a point so we are going to assume that the line of action of all the tension or compression forces are passing concurrent.

Now, the second important point that we need to keep in mind is that the contact forces and the torques between the beam and the support, it depends on the type of joint so there are several different types of joint possible. For example, this type of joint that is shown in this picture is called a pin joint because this is a pivot point it is a common point and this beam is sort of so this red part is the fixed part of the support, and this blue is the beam, and this is connected is kind of hinged at one point as if of the support so this is called a pin joint and again there can be two kinds of possibilities. So for example, this joint can be kind of welded together in a way so that this beam cannot rotate. So, let us consider the case where the beam can rotate so it is free to turn.

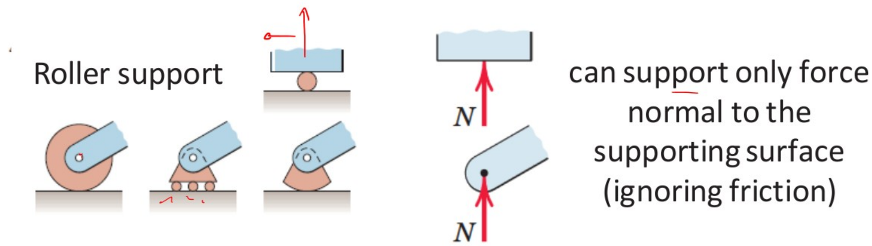


So, in this case, there is the beam you can see that it cannot support any torque because if you apply even a small amount of torque on the beam and then the beam will rotate so this type of joint pin joint will not support any torque but there will be some force and this force in general is can be in any direction so this force is a plane so this support is and the beam together they form a plane let us say the plane of the screen and in general there will be two components and so there will be a force and in general this force is unknown. The other possibility is that the beam is kind of welded in such a way that the beam cannot rotate in that case the difference is that in that case, not only there is a contact force, but also there could be a contact torque which is denoted by this quantity M the moment of the force M.

Now, there could be another type of support is fixed support such as this rod or beam which is inserted in some wall as if it can also be welded, in that case again there could be a, in general, there could be a force which is along the axis and now and there could be a torque so this joint can also withstand a torque applied on it because it is fixed and it can withstand a contact force.



Now, in general, this contact force like in this example, I mean this will be in the plane of this support, but it need not be along the axis it is only when we ignore any internal force which is perpendicular to this axis such as shear force which arises because of the deformation of the beam only then we say we can assume that this force will be perfectly and along the axis. Similarly, in this previous in the pin joint case, in general, the direction of the force can be different from the axis of the beam but if we ignore any internal force which make those assumptions that such that we ignore all other kinds of internal forces like elastic forces etc, then this perpendicular component cannot exist and there is a simple simpler version where we can assume that our force will be along the beam and then there is another when let me mention another type of support which is called the roller support in this particular case so there is a so the the support point the pivot point can is put on a roller and let us and we assume that there is no friction between the roller and the surface on which it is roll it rolls and it can move in the horizontal direction.



So, this kind of support, the point is that there can be only the force in the normal direction because any force in the horizontal direction in a direction in which the support can roll will make the support roll, so it will no longer be in a condition of mechanical equilibrium. So this is a roller support that can support only force normal to the supporting surface, ignoring friction. So these are some of the common types of supports I mentioned because it may be like you will encounter them in real life and various problems. So, given all these assumptions, let us now define the problem and understand the problem that we have the nature of the problem or the type of the problem. So what we have here is that this whole structure is at rest as the first thing.

So you can apply this in the statics problem so it is you can apply the condition of force and torque balance. Now, there are two rules that you saw two assumptions that not assumptions, so this basically it follows from Newton's laws of motion if we apply the condition of force and torque balance is that on any single beam so, no matter how complicated is your given structure or the framework total force on any single beam is 0 that is the first thing and the total force on any joint is also 0. So, for example, if we take a some structure like this, so we have it is made up of two parts; one is called a joint, or which is also we will call the node some point, and then it is connected by beams or some straight line some sticks. So, total at if the whole structure is at mechanical equilibrium. That means that the total force on any of the beams is 0, and as well as the total force on any of the node is also 0.

So these are the two rules that you should keep in mind. Now let us also in this kind of problem you should be very clear about what is given and what there is what is known and what is unknown. Usually, in this problems, the external loads are known. So you are going to design some structure with which can operate within a given load and then the structure responds by generating the tension force along the beam and these internal tensions are kind of so these are essentially the contact forces at the joint at the support and these forces are unknown because the need for the feature of this force so this is a force I emphasize this point again so this is a force contrasted to it something like a weight of the structure. So, if you know the mass of the beam then or the mass of the whole structure, then the weight is always mass times the acceleration due to gravity.

So it is always known, without doing any calculation, that the internal tension forces are unknown, and if we want to calculate those unknown forces, we have to apply these conditions of these force and torque balance and solve those equations to determine those unknown forces. So, we shall continue taking a concrete example and apply this framework in the next lecture. Thank you.