Course Name: Newtonian Mechanics With Examples
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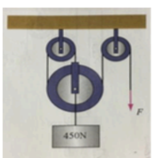
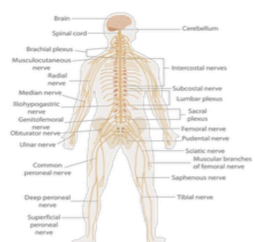
Week 03
Lecture - 11

In the last couple of lectures, we started talking about the static problems, that is, the mechanical equilibrium situations. So, in the last couple of lectures, we discussed the conditions of mechanical equilibrium, like force balance and torque balance. So, today we will discuss our first real examples of a situation, where we are going to apply those principles to learn something about real life. So, the example that I have in mind for you for to discuss today is a very simple system, which is a rope or a cable. So, when we talk about rope or cable, so in your high school, perhaps you are familiar with an example of this sort, where there are pulleys which are connected by ropes, and You are sort of interested in pulling the weight up or down using those ropes. However, if you look around yourself, ropes are very common, not only in the pulley but in various other situations.







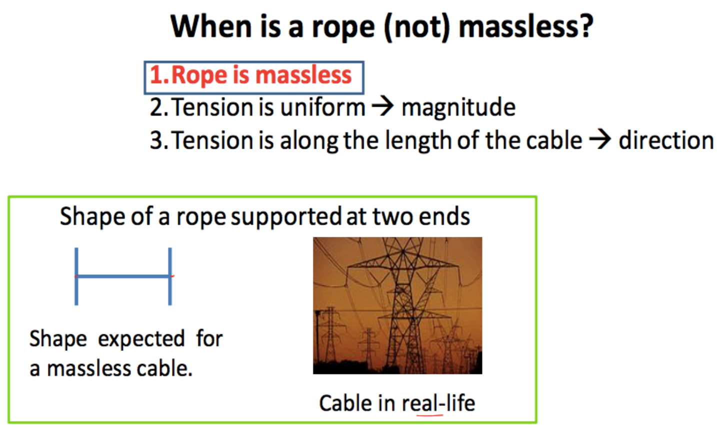



For example, you will find them in some gym machines, or they are used for binding something, like electrical cables, etcetera. And their applications are found for a wide variety of situations, starting from, For example, man-made structures, such as this famous Laxmanjula Bridge in Haridwar, and even inside our body. So, for example, if we look at our muscles and nerve cells, So, you can think of them mechanically as nothing but rope, and even inside the cells, There are DNAs that also look like rope. In fact, when we talk about the rope, we talk about the tension force, and in fact, the plain, the everyday language outside physics.

We also, you know, use this word tension, like say situation is under tension, etcetera. So there, we are actually referring to our body, like the muscles become stiff when there is an emergency situation, and So the muscles are under tension, and that is the origin of this plain English word tension in common situations. So I am going to call this rope an engineering structure, it is a very simple engineering structure. Which are used in various kinds of device and machines, such as bridges and suspensions. So here is the plan that I want to follow.

So first, I will talk about the tension force, and We will review the nature of the tension force critically. Then I will sort of take an example and define the problem, and then we will apply our systematic approach of how to solve or attack any mechanics problem and to work out some examples. So the first thing is that let us start by critically, The word critical is crucial. examine three common assumptions that we normally make in high school physics courses when you, which involves solving problems that involve ropes, for example, this pulley problem. So, the first thing you say is that the rope is massless, and the second thing you say is that this tension force is uniform everywhere along the rope or cable.

So, this is something we are talking about: the magnitude of the tension force. The third thing is that the direction of the tension is along the length of the cable or the rope. So let us review these assumptions one by one. So the first one is that the rope is massless. So now I want you to think about real-life situations.



Do you ever see a rope which is massless? So if the rope is perfectly massless, and then Suppose I put this rope along the support; then what we expect is that, so this rope basically means that, so we have this one vertical force, which is the weight of the rope, So, consider a small part of the rope and then let us say it has a length which is dl and Let us say this W naught is the weight per unit length. Then the weight of the full rope is W naught times dl; this is the weight, and this weight is acting vertically downwards. Now, if there are any other forces in the vertical direction, which could be some external load, For example, imagine a situation where you have a rope and you are hanging clothes from the rope. Then those clothes are exerting some force, but in a vertical direction. So now compare these two forces: the external vertical force and the weight of the rope, and when I say compare, there are two ways to compare: either take difference or take ratio; in this case, take ratio.

Now, if you can neglect it, this is the ratio. Now if this ratio is very small compared to 1, So, if they are equal, then the ratio is 1. If the weight is bigger than this vertical forces, then this ratio is bigger than 1 and If the weight is smaller than the vertical forces, then the weight is less than 1. So if the weight is less than, if the ratio is less than 1, so that you can ignore the weight of the cable in comparison with other external vertical loads, then that is the meaning of the word the rope is massless. Easily, you can understand that you have to determine, in a given situation, whether this is justified.

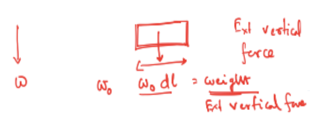
Our next statement that we often find in, let us say, simple textbook problems is a tension is uniform. That is, the magnitude of the tension is the same everywhere along the rope. So this is what we normally expect from the force balance on any small part of the rope. However, there are situations where, if the external force is present, you can control those external forces and you can make the tension non-uniform along the rope, and this is even for a massless rope. So this is a very common application of that fact.

So let us say this is what we when we tie a rope around, So, let us say this is my rope, and this is some pole, and I am tying this rope around this pole. So this is a common strategy because, if I want to keep something tied or bound, then this is a common strategy because what happens is that so in this case there is a friction force between the surface of this pole, Imagine this is a pole and this is the rope; between the surface of the rope and the surface of the pole, there is friction force. Now, if you take the rope as your system, then this is an external force acting on the rope and we shall analyze this problem in detail when we discuss friction and we will see that in this case, the tension is actually non-uniform along the rope and that is why we can bind the the rope along the pole, and if we need to exert a small force at one end and let us say this is something bound to the rope and it can support a large force at the other end. So here is what I am trying to sort of analyze more carefully: So, this is a small piece of rope, so this shaded area is a small Now let us say this is my system, so everything else is surrounding. So the neighboring part of the rope is part of the surrounding area, so this is surrounding area.

When is a rope (not) massless?

1. **Rope is massless**
2. Tension is uniform → magnitude
3. Tension is along the length of the cable → direction

In real-life often you deal with massive cable.
Compare *i.e.* *take ratio*, of the weight of the cable with other external *vertical* load.
Then you decide if you can ignore the weight of the cable.



And it exerts tension on the shaded area; similarly, this part, which is also surrounding, also exerts tension. Now, if we call this tension, I have shown the same picture here. So, if I call this the tension on one side T1 and the tension on the other side T2, Then suppose these are only two forces; there are no other forces present. Then our Newton's law says that the acceleration of the shaded part or system, the total force on it is T1 minus T2 because they are in opposite directions, and this must be equal to m times A. Now if we assume that the rope is much less, so m is 0, then we find that the tension is the same.

Now in the example that I given here, so in this case in addition to this T1 and there are another force present, which is coming from interaction between the pole surface and the rope; Hence, this equation is incomplete. In that case, we will discuss this example later. But only if these two, only the tension, are the only force, then if the rope is much less, then it follows from Newton's law that the tension must be uniform everywhere. So, this is about the force condition. Now we come to our third assumption: what about the direction of this tension? So to understand the direction of the tension, we need to look at the torque balance condition.

So suppose I mean we draw the same force here, Suppose, let us say, that even if we take a mass-less rope in the sense that we discussed before, Now suppose this rope has a component in the vertical direction. So let us say this is a component in the vertical direction and this is the component in the horizontal direction. Suppose this is the case. Now let us say that I take my pivot point at one end-any point on one end of this rope, So this is my shaded region; this is a part of a rope; this is my system, which is a part of a rope. Now if, as in the previous case, we assume that the tension is same or uniform.

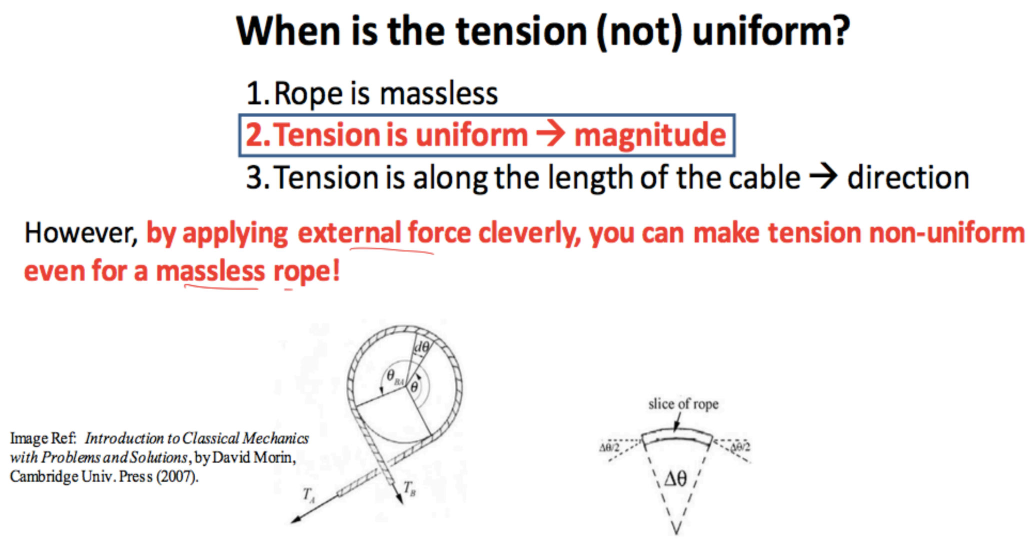
But what if we now look at the torque balance condition? If there is a component of the tension in the vertical direction, then we can see that there could be a torque. Now you may say that if I look at our pivot point here on the left-hand side, then the line of action of this vertical component is passing through the pivot point. So this will not generate any. So now we are calculating the moment about this pivot point and the moment due to this force is 0. So then, the moment is due to the other force, which is the tension, the vertical component of tension.

Let us call it T1v, and on the other end of this slice of rope, the moment due to this force will not be 0. So, that means that if the moment is not 0, then from our previous discussion, we expect that the rope should start to rotate, but this violates our experience. So we are demanding a situation where the rope is motionless. It is static. So that means it follows that this must be 0, and there should not be any vertical component of the tension.

Now you may ask, that okay fine but what about the mass of the rope? This part of the rope has some weight, which we are ignoring. The point is that even if you add a mass of rope here, you can make this slice, So the weight of this rope will be the mass, which is dm, which is the mass per unit length times dx. So, this is mass per unit length, which is also called the linear mass density. This is the horizontal length of the rope, and then the weight is dm times the acceleration of gravity. Then you see that I can make this length smaller and smaller.

So then this mass can be made smaller and smaller. Whereas the tension does not change because it is a uniform or big force compared to the weight. So, the torque produced by the weight is small and the torque produced by this vertical force component of tension will be big. So, they can never balance each other. So the only way that the rope can be static is that we should assume that there is no vertical component.

Now, if there is some external vertical force, then the rope can bend. So this is what we also observe: if we hang a rope between two supports in general, it bends. So, in this case, actually, what happens is that it bends till the point when, So in that case, the tension will not be uniform because the direction of the tension is along the local tangent. Now, if we take a rope which is hanging between two ends, then the tangent direction is continuously changing. So let us see if we take a rope like this, then the tension force is in this direction, and the tension force is in this direction.



So from this argument, what we conclude is that the tension force, when I say vertical component is 0. I am assuming that the piece of rope is perfectly horizontal. Now if, so the more general way, so if the piece of rope is not horizontal and a little bit slanted, Then the more general way of saying this is that this direction represents direction along the rope and this direction represents perpendicular direction of the rope and if the rope is continuously changing its curvature, then the direction of tension is continuously changing. So the tangent direction is continuously changing, so the tension is always along the local tangent. So this is the local tangent direction.

And in this case this, so if you like, so this direction is continuously changing. But even in this case, there is no force on the direction, which is perpendicular. There is no force in the direction which is perpendicular because that will violate the torque balance condition. Now here is again something that is very crucial but which I have hidden. In real life, in addition to tension, if you try to bend a rope, every material has an elastic property.

So, it will try to resist this deformation. So in addition to tension there will be other elastic forces appearing in a bent rope. Now, for simplicity, I am going to assume in this course that we are going to ignore those elastic forces. For example, those forces will be important if, instead of rope, If you consider, for example, the bending of a rigid, more solid rigid body like a beam. In this case, you cannot ignore those elastic forces.

So I am going to assume that in this current example, Our rope or the cable is perfectly flexible so that if those are under those, if those extra internal forces are present, then we will see that the net force acting between our part of the rope and Its surroundings can support a force in a direction which is perpendicular to the local tangent. So this is the local tangent direction. This is perpendicular to local tangent and if there are elastic forces, then there can appear some forces which are perpendicular to the local tangent. But we are going to assume that there is no elastic force. Which means that the cable is perfectly flexible.

There is no resistance to bending. If I try to bend the rope, it will bend whichever way I want. So with this assumption, Now, we are ready to solve some problems, which we will take up in the next lectures. Thank you.