Course Name: Newtonian Mechanics With Examples
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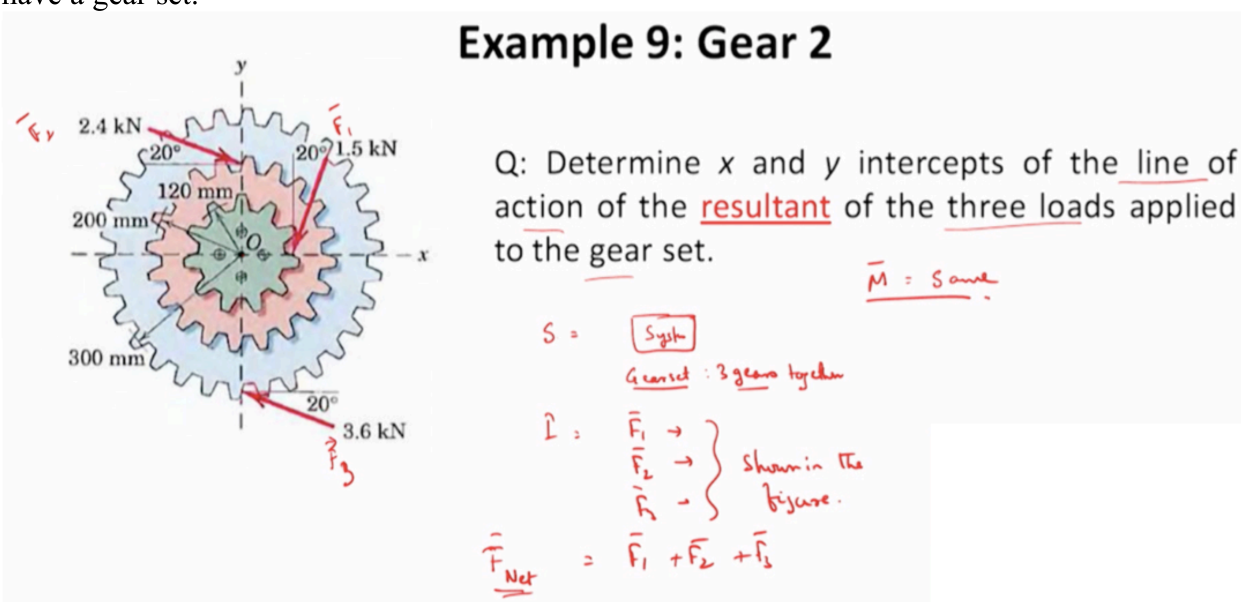
Week 03
Lecture - 10

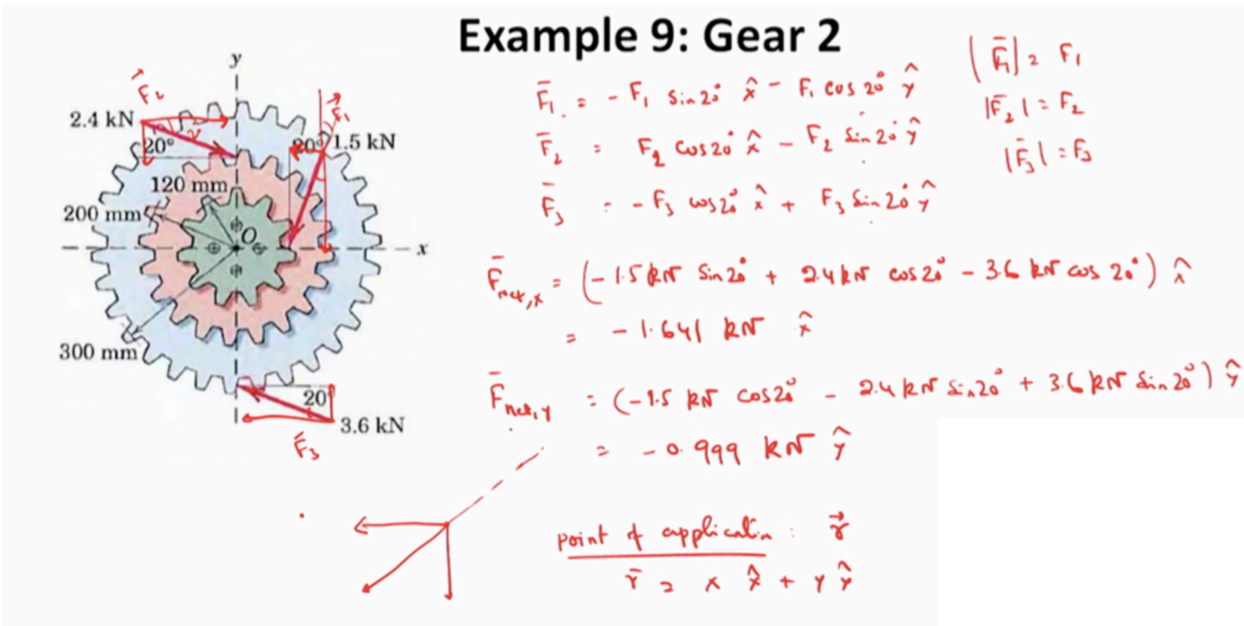
So, this is the third week of the course on Newtonian mechanics with examples and We were discussing the condition of mechanical equilibrium, or statics. So, let us get started with some examples. So, in the previous lecture, we calculated the moment due to on a gear and We calculated using several different methods to highlight different aspects of the concept of the moment. So today we are going to take a few examples. So, the first example is again a gear, but this is a slightly more complicated situation than the previous example. So, instead of one gear, we now have a gear set.



So, it is a system of three gears, and all of them have the same center. So, they are a concentric system of concentric three gears. So, the problem says that there are three forces acting on this gear set, as shown by these arrows. So, one force each on a different wheel.

The question asked was: determine the x and y intercepts of the line of action of the resultant of the three loads applied to the gear set. So, let us analyze what is required slowly. So, the first thing is that, as we say, we need to define our system. So, in this case, our system, so this is our box, which is our system, and this system is the gear set, which is the three gears together. So, we are going to choose these as our system.

Because, obviously then all the forces will be external forces. Note that these gears are stacked on top of each other. So, their rotations are independent of each other. So, the interactions are the three forces. Let us call the inner gear this force as F1, the second gear as F2, and this third gear as F3.



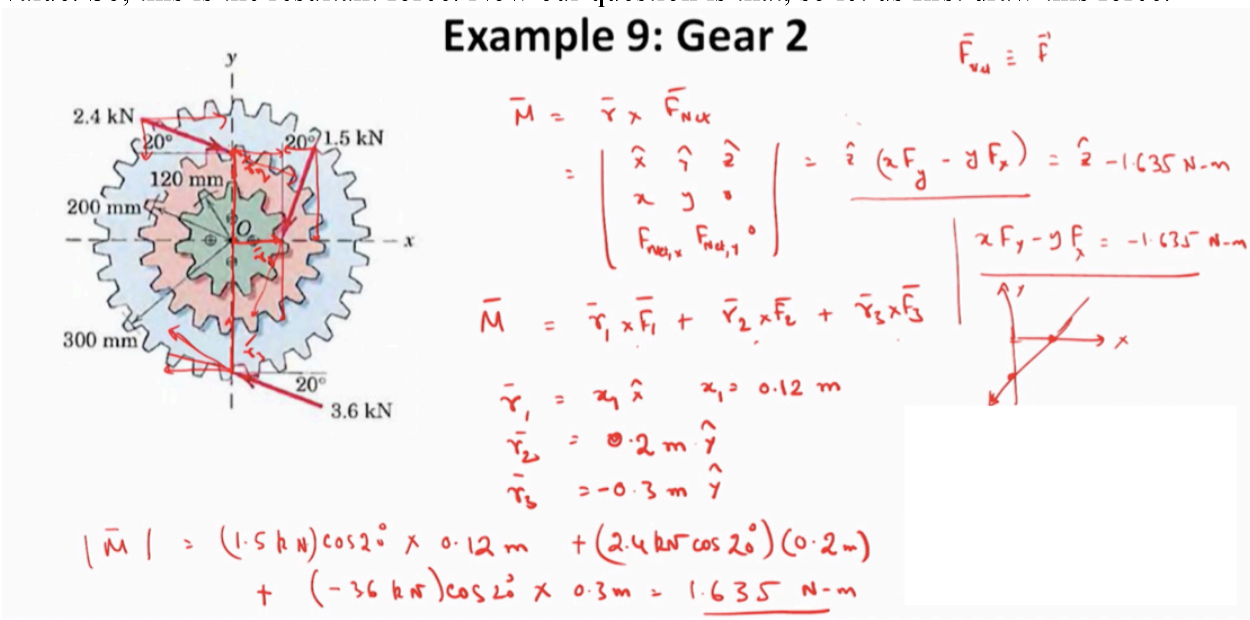
So, the interactions are F1, F2, and F3 shown in the figure. So let us understand the problem. What do we need to calculate? We need to calculate the resultant of these three forces. So, what is the total force acting on our system? So, the total force acting on our system, Let us call that Fnet, and this is simply F1 plus F2 plus F3, so this is fine. But this is not what the problem is; the problem is finding the x and y in the line of action of this force, Fnet.

So, the point is that if we replace these three forces by a single force on our system, then Where are we going to apply this force? This is the question, and how do you determine? Because it is basically arbitrary. So, we can determine by saying that, So we are going to replace this force and we have to demand this condition that The resultant force must generate the same amount of moment as the one generated by these individual forces. So, the total moment must be the same. Then the state of motion will be same if we replace these individual forces by a resultant force. So, this is the question, and the aim is the same.

So, let us proceed further. So, first, we need to calculate the total force. Now, this is the total force, and this part is easy. So, let us write down the individual forces first. So, F1, So, this coordinate system in this case is provided for our convenience.

So, all the forces have only the x component and y component, there is no z component. So, we said that this is our F1. So, it has two components; this is our F2, It has two components, and This is our F3, which has three components. So, the F1 from this diagram, it is clear that this is 20 degree, this is 20 degree. So, F1 is minus F1 sin 20 degree x hat plus F1 cos minus cos 20 degree y hat.

F2, so it has a positive x component, so F2, so this angle is 20 degrees. So, F2 cos 20 degree x hat and minus F2 sin 20 degree y hat. So, F1 is the magnitude of F1 is, so these are the meaning of the symbols and F3 is minus F3, so this is again 20 degree cos 20 degree x hat plus F3 sin 20 degree y hat. So, the resultant is just a vector addition, so this is, so the x component, let us call that x, so that will be given by minus 1.5 kilo Newton times sin 20 degree plus 2.4 kilo Newton times cos 20 degree and minus 3.6 kilo Newton times cos 20 degree. Now, so this k, k, kilo Newton, k n represents 1000 Newton. So, now if you plug in the values, you will get final answer is 641 kilo Newton. Now if we similarly calculate the y component, so we add this plus this plus this, you get minus 1.5 kilo Newton times cos 20 degree, minus 2.4 kilo Newton times sin 20 degree and plus 3.6 kilo Newton times sin 20 degree, So if you plug in the values, you will get this value. So, this is the resultant force. Now our question is that, so let us first draw this force.



So, both of them have negative components. So that means this is the x component and this is the y component. That means the force must be in this direction. Now, does it make sense? So, you can see from this picture that the x, the vertical direction, the y component of the largest magnitude of y component of the force coming from F1 and from the x component, the horizontal components are F2 and F3. So, you should make sure that if you draw it to scale.

Then you should get that the x component and y component of the resultant force are as shown here. Now the question is, so this is my pivot point. Now what is it? Where should I apply the force to generate the same amount of torque or total moment? So, suppose our point of application of the force is this point. So let us say the point of application is some point. The position vector is r, and this r has an x component and a y component.

So then, what is the moment of this total force acting at r? So that we can calculate from this definition. So, we take the cross product. Let me write it here, and you can verify There will be only the z component, x component, and y component, both will be 0. So, the z component would be x times Fy minus y times Fx. So, I have simplified that Fnet is F.

So, I have removed or omitted this index net to deduce writing. Now, how much torque is generated by these individual forces? Well, you can apply. that will be given by r1 cross F1 plus r2 cross F2 plus r3 cross F3. So, this is r1. this is on the second wheel, so this is r2 and This is the point of application of the third force; this is r3.

So, we get that r1 is x1 x hat. r2 is, so x1 is equal to the radius of the first inner wheel, which is given by 0.120 millimeters, which is 0.12 meter. r2 is acting on at this point, which is on the y axis.

So this is given by the radius of the second wheel. So, the radius of the second wheel is 200 millimeters, and r3 is the radius, which is acting again on the y axis, So, this is the radius of the third wheel, the largest wheel, and this is in the negative direction. So we can see that if we apply the r cross F definition on individual force. We can see that all of them produce the moment or torque in the same direction. So, we can, in this situation, since the points of application of F1, F2, and F3 are only on the x or y axes.

So our calculation is simple. So, our moment comes from each individual direction. We can simply calculate the magnitude, so we can say that this is M1. The first term is nothing but 1.5 kilo Newton times, so as we can see, we discussed it in the previous examples. For the F1 only the y component is going to contribute.

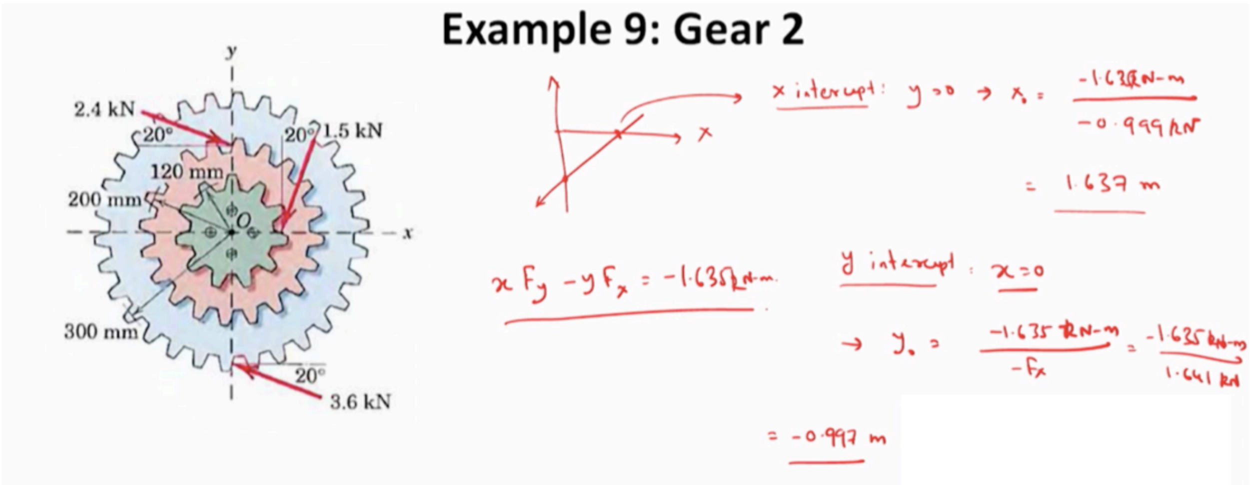
So we have written the y component here, F1 cos 20 degree times this distance. Sorry, so we have to put the force here, so only the y component is going to contribute times 0.12 meter. So, let us say that the second term will be r2. So, in this case, only the x component because the line of action if we sort of move the r to here, and F2 here.

We can see that the line of action of the y component is passing through the pivot point O. So, only the x component will contribute, and the x component was F2 cos 20 degree. So, this is 2.4 kilo Newton cos 20 degree times 0.2 meter, and similarly for the force F3.

If we move it along the line of action, then the y component of this force F3 is passing to the pivot point. It will not contribute to the moment. Only the x component will contribute and that x component was minus F3 cos 20 degree. So, minus 3.6 kilo Newton cos 20 degree times 0.3 meter and this will turn out to be one. Note that I am adding this magnitude, so this is possible because the direction of each of these torques is the same. They are all directed along the z axis; hence, we can simply add the magnitude to calculate the total torque. So, the vector addition in this particular case becomes just a simple scalar addition. So then we have this condition that we have calculated the torque.

So this is the torque due to the resultant force and this is the total torque by the individual forces. So they must be the same, so if we equate them, then we get the condition that x times Fy minus y times Fx, So now if we put the, we can see that the from the thumb rule, The direction is going into the page, so I added a minus sign to denote that. So we demand that these two torques be the same, so that is how we determine the line of action. So then we get, so note that in this case we cannot determine the exact point of implication. There is no meaning to an exact point of application; we can only determine the equation of a line.

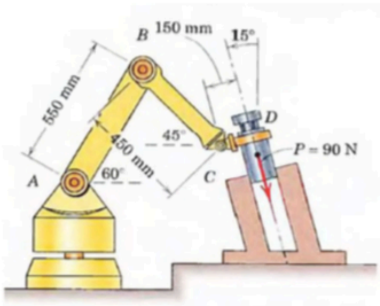
So, denote that this is an equation of a straight line. So, along this line of action, you can apply force at any point and it will give the same amount of torque. So, you cannot determine precisely the point of application. All you can determine is the line along which you should apply the resultant force to generate the same amount of torque. Now the question asks for the x and y intercept, so what do I mean by that? It means that suppose this is my x axis and this is y axis, I draw again and so this force is directed along, We determine the direction of the force, so this must be the line of action in some sense of the force.



So, then, we need to know and determine the position of this point. Where it intersects the x axis and the position of this point where it intersects the y axis. So this is the direction of the force, so this point, we can demand that at this point, This is the x intercept, so the condition of x intersecting x axis is y coordinate must be 0. So, let me write down the equation again: x times Fy minus y times Fx is equal to minus 1. 637 meter. So, note that I am deliberately writing the units so that we do not make any mistakes with the units. Similarly, the y intercept is given by x equal to 0, and that is this point. So let us call that y naught, and this will be given by the condition in this equation you put x equal to 0. Then this is given by -1.635 kilo Newton meter divided by minus of Fx, so minus of Fx is 1.635 Newton meter. So, if I put y equal to 0, then the value of x, let us call it x naught, is given by -1.635 Newton meter divided by the Fy, so the Fy was computed here. So this is our Fy, which was minus 0.999 kilo Newton, so minus 0.999 kilo Newton, so and here, I am sorry must be a kilo Newton meter, So, the value of this turns out to be 1.641 kilo Newton. So, which is equal to 0 points, almost like 1 meter, so note that we get the correct sign because If we expect that the force is along this direction, then we expect the x intercept to be positive and The y intercept is negative and this is precisely what we get as the after the calculation. So this is another important point: whenever you calculate something, the first thing you check is the answer. So you have to ask yourself: does it make sense, Does the answer make sense? So let us take another example. So this is what I am going to set you up with as a take-home problem. Challenge as an exercise for you, so let me explain the problem, and I will give you some hints about how to compute it and how to solve it.

So given a diagram, do not worry too much about the diagram at the moment. So it says that there is this yellow color that is a robot, and this robot is used to make a hole. So, it is used to put this cylinder, this blue cylinder, inside this circular channel, this circular hole and It exert certain force, which is fixed force of 90 Newtons, and the problem says that determine the moment of this force P, as shown here. So this is the point of application of the force and this is the line of action. This is the magnitude and this is the direction about points A, B and C, so different points are given, so you choose three different pivot points and calculate the moment.

Example 10: Robotic arm



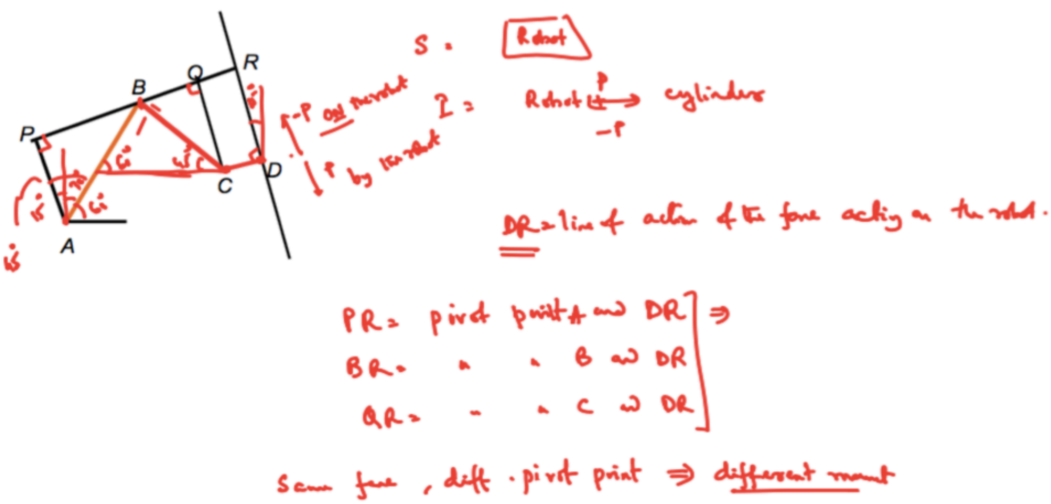
Q: While inserting a cylindrical part into a circular hole, a robot exert a force ***P=90 N*** on the part. Determine the moment about points A,B, and C of the force which the part exerts on the robot.

So the first thing I will show you is that you can simplify this problem and so this diagram, So I have sort of shown you the same diagram in a schematic way, so this is the pivot point A. This is the pivot point B; this is the pivot point C. So this is our simplified version of this robot and then this is point D. Which is the part of the cylinder.

It is going inside the hole. So first, our system is this robot; let us choose our system to be robot, then It is interacting with this cylindrical part, so this robot is exerting an interaction. So, with the surrounding environment, the only interaction you need to consider is the interaction between this entire thing, the robot, and this cylinder, which is pushing. it down with a force P and this, so that means by Newton's third law there is a counterpart. Which is trying to push it up with a force equal to P with an opposite magnitude. So the robot is giving a force P, and the cylinder is giving a force minus P.

that is all you need to know so that the force acting on the robot is P in this direction and This is the force acting by the robot, which is in the downward direction. So given this, now we need to calculate the problem asked to calculate the moments. So, then all you need to see is that the robot has a complicated shape. So, this DR represents the line of action of the force acting on the robot.

Example 10: Robotic arm



So that is DR. So DR is the line of action of the force acting on the robot. So, now we need to calculate the distance of this line of action from the pivot points. So as you can see the distance of the line to the pivot point, pivot point A and the distance from, so B,R in the picture is the distance between the pivot point B and the line of action of DR and QR is the distance from the pivot point C to the line of action DR and As you can see, these distances are different, so for the same force, the moment s are different. So then the only thing that is required is to use the information provided here, So this angle is 60 degree, and this angle is 45 degree, so if you draw a line here, this angle must be 60 degree, and This AP is perpendicular, so this is a 90-degree angle, and this is 90-degree. These are all 90 degrees, then if you draw a line here and a line here.

So this figure says that this angle is 15 degree, so you can see that if you take this line and So from the geometry, this must also be 15 degree, and this is 90 degree, So, this is 60 degree, so this must be 30 degree, so this angle must be 45 degree. So now you know the problem is simply to calculate these distances and multiply the value of the force to calculate the moment. So, the message from this problem is that the same force at a different pivot point gives you different moment, So, this is the purpose of this problem. So I will give you a take-home exercise to work out the distances and calculate the moment.