Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology, Roorkee
Lecture 09
Nodal Planes, Systems of Thin Lenses

Hello everyone, welcome to my class. Today we will talk about Nodal Planes and System of Thin Lenses. As a revision in last class, we talked about thick and thin lenses, and thereafter we learn what is the unit planes. And while learning unit planes we came to know that unit planes are planes, which gives unit magnification.

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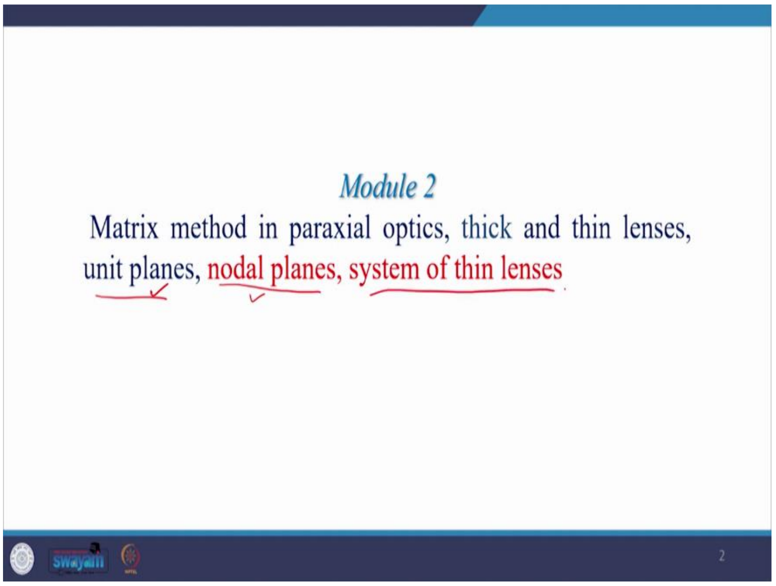

Apart from the unit plane, there are several other planes and one of them is nodal plane, which we will talk about today and thereafter, we will move to system of thin lenses, we will analyze the system of thin lenses, and we will derive an expression for focal length of the combination of the two lenses.

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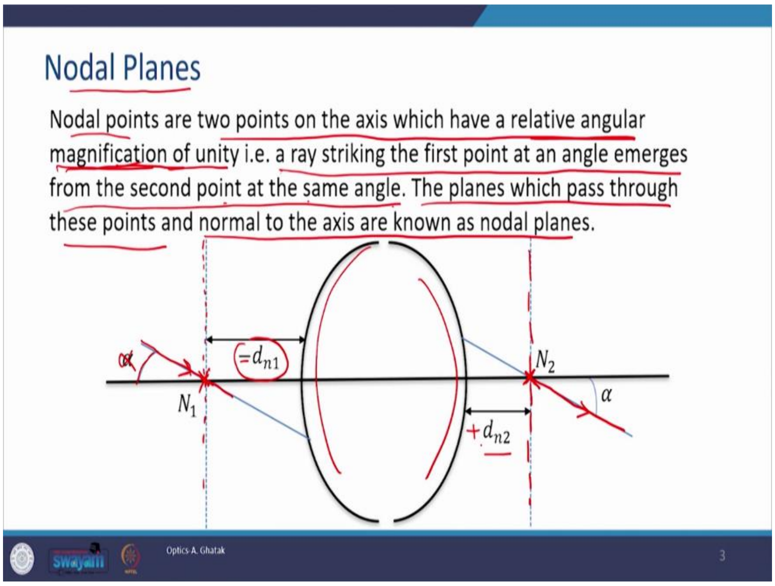

Now, before introducing you nodal planes, let us define what are nodal points, nodal points are two points on the axis which have a relative angular magnification of unity. Now, in case of unit planes, the magnification was unity, but here it is relative angular magnification, here the magnification is angular magnification. And in the case of unit planes, it was linear magnification, the usual magnification. The usual magnification was unit in case of unit plane. So, now in case of nodal plane it is angular magnification, which is unity, what does the angular magnification means?

Angular magnification means if a ray falls at nodal point and this ray makes an angle α (alpha) with respect to the axis of the system, then the ray will emerge from the other nodal point which is on the other side of the optical system. And this emerging ray will also subtend an angle α with respect to the axis, the horizontal axis, this is what it is explained here. A ray striking the first point at an angle emerges from the second point at the same angle, nodal points are two points. And if a ray falls on one of the nodal points, then the emergent ray will emerge from the another nodal point, the second nodal point, but the angles would be the same. This is what it is written here.

A ray striking the first point at an angle emerges from the second point at the same angle. Now, here in the figure, you can see that the N_1 represents the first nodal point and N_2 represents the second nodal point and this is an optical system. Now a ray which falls at first nodal point N_1 at angle α as per the definition of this nodal points, it will emerge from the second nodal pointN_2 at the same angle α, the angle will not change. Now, the planes which pass through these points and normal to the axis are known as nodal planes.

Now, once N_1 and N_2 are defined, we will draw planes which are perpendicular to the axis of the system and these planes are termed as nodal planes. Now, the distance of this nodal plane from the left refracting surface is designated by d_{n1} and since, this distance is on left hand side of the refracting surface, we have placed a minus sign for d_{n1} while the distance of the second nodal plane from the right refracting surface is designated by d_{n2} and since it is on the right hand side therefore, this distance is a positive quantity plus sign is there.

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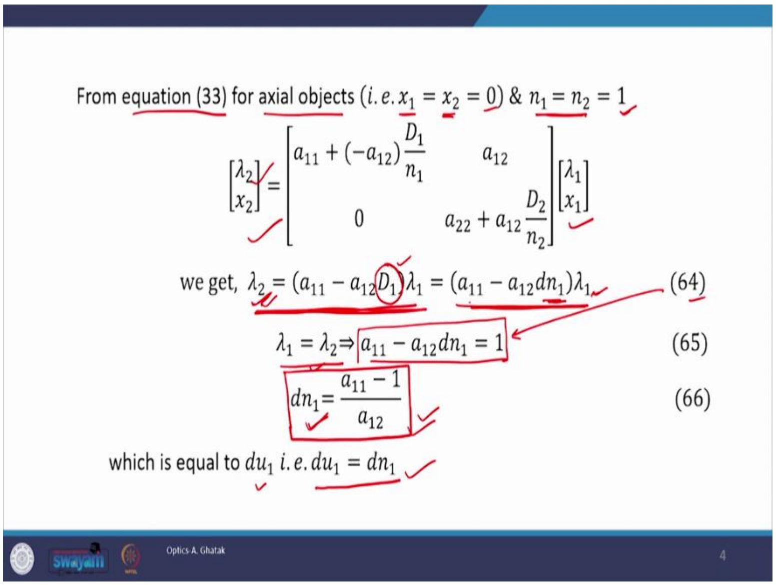

However, having understood this, we will start from question number 33 which we derived in our previous classes and this equation number 33 was derived for objects which are sitting on the axis of the system. For axial objects, we can safely substitute x_1 is equal to x_2 is equal to 0 and we also assume that on the left and right hand side of the optical system the refractive index is the same and is equal to that of the air, therefore, $N_1 = N_2 = 1$. Under these conditions, we have this matrix equation which is nothing but our equation number 33 which we derived in our previous classes.

Now, from this matrix equation, we can get an expression for λ_2, we will expand this matrix to get the expression of λ_2 and if we expand it then you see that the λ_2 has this expression. After substituting $x_1 = x_2 = 0$ and $N_1 = N_2 = 1$, we will get this expression for λ_2 which I have just expanded this matrix equation and the first term will give you the λ_2 expression. Once this λ_2

is known, we know that we see that there is a term D_1 which is appearing here in the second term on right hand side, this D_1 is nothing but d_{n1} distance of the nodal plane.

Now, in this particular case λ_2 will now be related with $a_{11} - a_{12}d_{n1}$ and then outside of the bracket we have λ_1 and this is how we get equation number 64. Now, for nodal points, $\lambda_1 = \lambda_2$, this is also valid and once we substitute $\lambda_1 = \lambda_2$ then equation 64 gives this expression from 64 we get this $a_{11} - a_{12}d_{n1} = 1$ and from here we can easily get the expression for the distance of the nodal point from the refracting surface, particularly from the left refracting surface and we see that $d_{n1} = (a_{11} - 1)/a_{12}$. Here a_{11} and a_{12} are nothing but the matrix element of 2 by 2 system matrix.

Now, this $d_{n1} = d_{u1}$, I repeat $d_{n1} = d_{u1}$ and what is d_{u1}? d_{u1} is the distance of the unit plane, d_{n1} is the distance of the nodal plane from the refracting surface and d_{u1} is the distance of the unit plane from the refracting surface and in case where $n_1 = n_2 = 1$,, $d_{n1} = d_{u1}$ this is what we see because we have already derived the expression for d_{n1} and d_{u1} have the same expression but d_{n1} is equal to therefore, d_{u1}.

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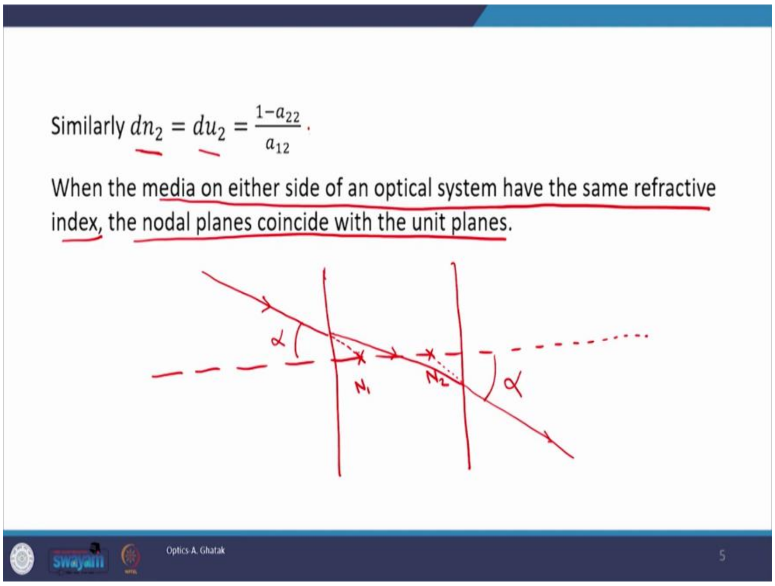

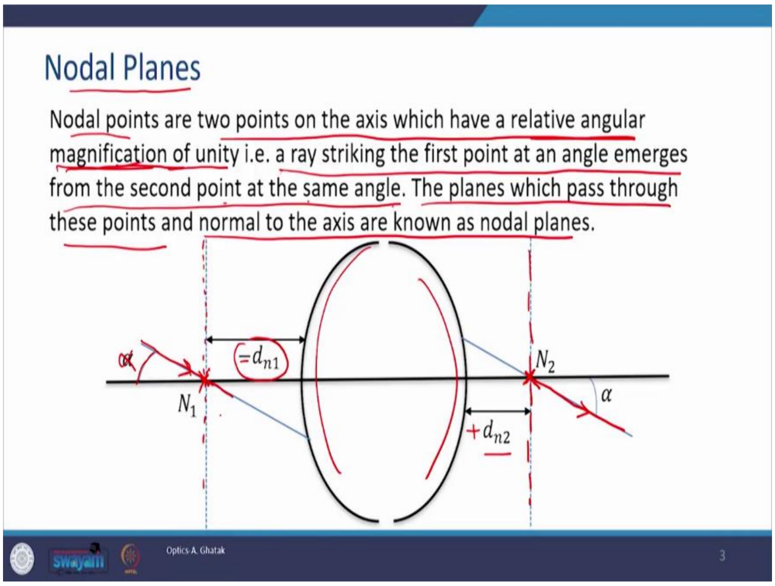

Similarly, we can get the expression for d_{n2} and we will see that the $d_{n2} = d_{u2}$ which is equal to $(1 - a_{22})/a_{12}$, which means that when the medium on the either side of the optical system have the same refractive index, the nodal planes coincide with the unit planes. If the medium on the left hand side and on the right hand side of the optical system is the same then nodal plane coincides with the unit planes. Now, in this slide the points N_1 and N_2 which are your nodal points and the plane which is passing through N_1 and N_2 which are your nodal planes, they are outside of the optical system, but it may also happen that they reside inside the optical system.

Suppose this is the boundary of the optical system and this is our axis and suppose this N_1 is here and N_2 is here, then what will happen? A ray which is directed towards N_1, it will emerge from N_2 but within the medium there would be some refraction, sorry and within the medium we can join these two lines here, what we see that a ray which appears to be directed towards N_1 it appears to be emerging from N_2 in this particular case where N_1 and N_2 both are within the optical system, this is what exactly happens. Now, you can see that this angle which we assumed to be α is equal to this angle, angle of emergence.

Therefore, irrespective of whether N_1 and N_2 are inside the optical system or outside the optical system, they preserve the property of unit angular magnification and as long as the media outside the optical system is same medium on the right hand side and the medium on the left hand side of the optical system. We will get coincidence of nodal planes and unit planes, nodal planes will fall exactly at the same position where unit planes appear.

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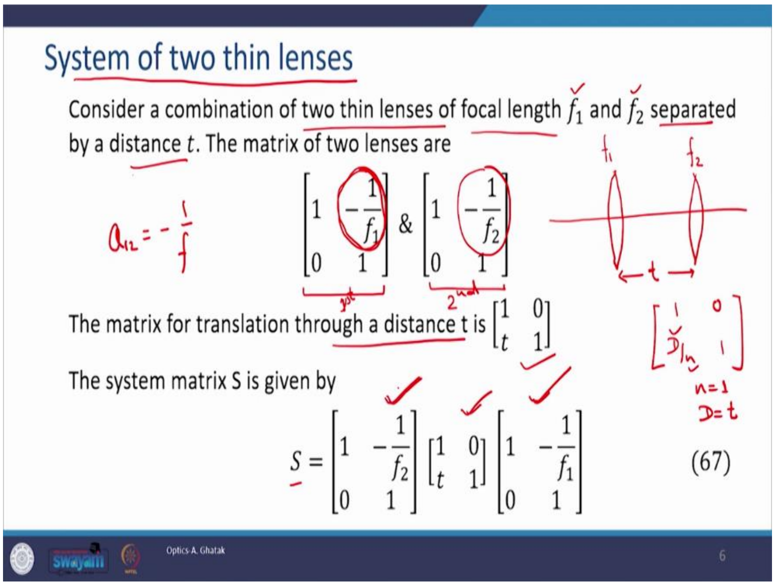

Once this is understood, we will move to the next topic which wherein we will analyze a system of two thin lenses. Here in this case we will consider two thin lenses, thin lenses have their usual definition which we have already talked about and these two thin lenses are separated by certain distance. These lenses have their own focal length which may be different and they are separated by certain distance and we assume in this case that this distance is t. We can draw the figure schematically here. Suppose this is our one of the lens and this is our second lens and the separation between the two lenses is t and the focal length of first lens is f_1 while the focal length of second lens is f_2.

Now we will analyze this system using matrix method, the matrix formalism and try to get an expression for the focal length of the combined system, the combination of the two lenses. Now these things are described here, we are considering two thin lenses whose focal lengths are given by f_1 and f_2 and they are separated by your distance t. Now, these are the two lenses therefore what would lenses do is that they refract the rays. Therefore, we can write the refraction matrix for first lens and for second lens here, this is the refraction matrix for first lens and this is the refraction matrix for second lens. This term represents a_{12}, this is the second term in the first row.

And the second term in first row which is a_{12}, we know that this is equal to 1/f. Therefore for the first lens we can write, we can replace f by f_1 and we will have $-1/f_1$ while for the second lens we have $-1/f_2$. Once we have matrixes for lens one and left lens two, what is left is the matrix for translation of the ray between the two lenses, we know that the two lenses are separated by distance t therefore ray which is falling on lens one, after getting refracted through lens one it will travel through the distance t and to represent this translation we again use the translation matrix which is again a 2 by 2 matrix and how to write this translation matrix this translation we know that translation matrix is represented by this expression, where D is the distance, n is the refractive index. And since these two lenses are kept in air the refractive index n would be equal to 1 and D here is equal to t, the separation between the two lenses therefore, the translation matrix in this case would be this 1, 0, t, 1.

Once we have matrices for two lenses and the translation between the two lenses we can write the system matrix for whole system. Therefore, the system matrix would be the matrix multiplication of the three matrices. This is the matrix for first lens, this is for translation and this is the matrix for second lens.

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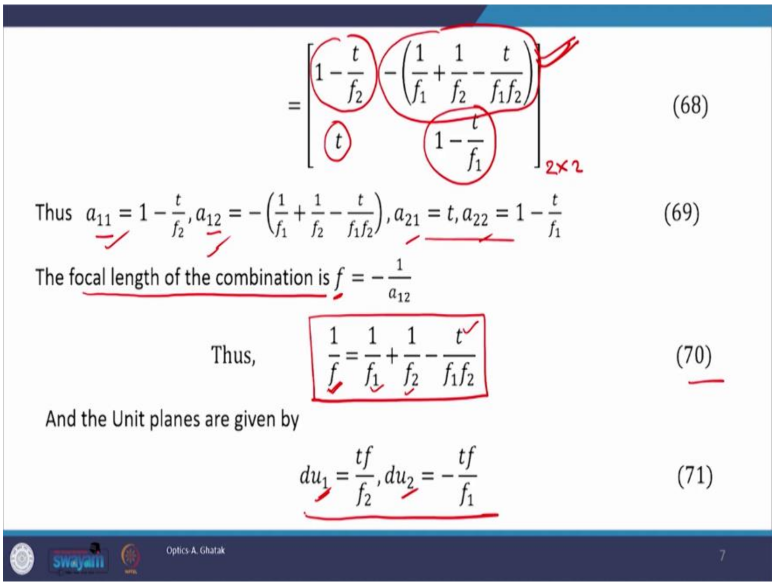

Now, after multiplication we get this 2 by 2 matrix which is our system matrix and we see that in this system matrix we have four terms here, there are two columns and two rows. The first term is a_{11}, second term is a_{12}, the third term is a_{21} and the fourth term is a_{22} which are written here, $a_{11} = 1 - t/f_2$, $a_{22} = -1/f_1 + 1/f_2 - t/f_1 f_2$ and so on. Now, $a_{12} = -1/f$, f is the focal length of the combination as per the derivations we did in our previous classes, we know that this term will be equal to $-1/f$.

Therefore, we introduce a focal length of the combination and this focal length would be equal to minus of inverse of a_{12} and therefore, get this formula. The focal length of the combination is, in equation number 70, the focal length of the combination is related to the focal length of the two lenses and the separation between the two lenses. Now, once the expression for the focal length of the combination is known, we can also write down the expression for the distances of unit planes. These are the expression for the distances of the unit planes which is equal to tf/f_2, for d_{u1} while for d_{u2} it would be $-tf/f_1$.

Here what we did is that we just substituted in the usual expression of the distance of unit plane which we derived in our previous classes. We just substituted these a's into that expression and this gave us the final expression of d_{u1} and d_{u2} for the combination of the two lenses and thus we saw that using matrices we can very easily calculate the expression of the combination of lenses and also can calculate the expression for the unit planes, the distances of the unit planes. Now, we will take example on this.

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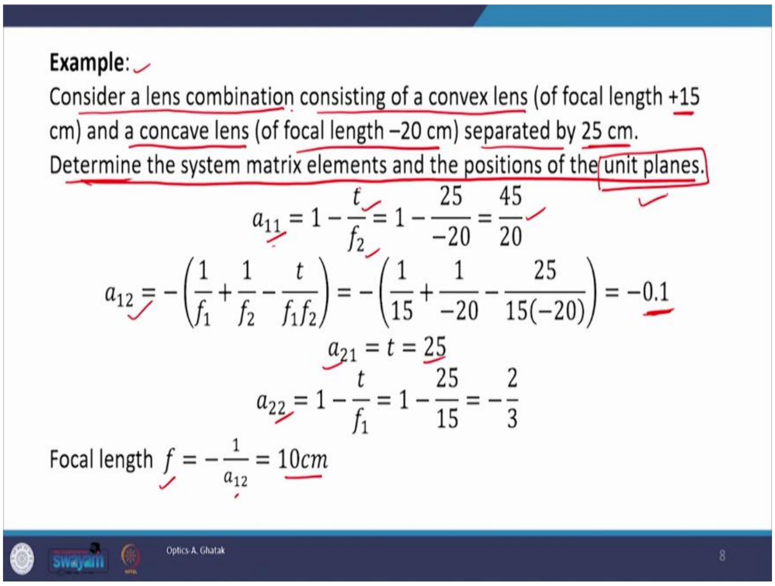

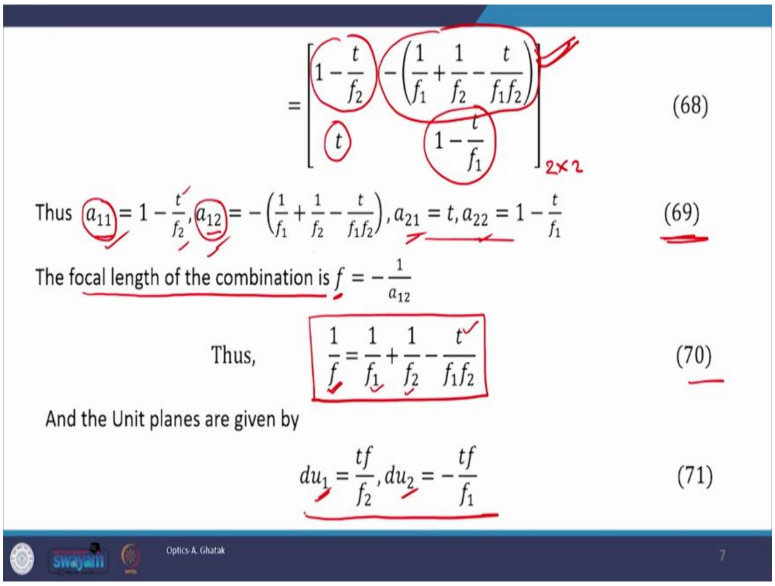

And the statement of this example or the problem statement is, consider a lens combination consisting of a convex lens and the focal length of the convex lenses plus 15 centimeter and the second lens is concave lens and the focal length of the concave lens is -20 centimeter and these two lenses are separated by 25 centimeter, we have a combination of two lenses, one lens is a convex lens whose focal length is plus 15 centimeter while the second lens is a concave lens whose focal length is -20 centimeter, the focal length of convex lenses are always positive as the focal length of concave lenses are negative.

And these two lenses are separated by 25 centimeter in the given combination, what is being asked? Determine the system matrix elements and the position of the unit plane, this is what is being asked. Now, we will have to first determine the system matrix elements, what would be the system matrix element, we will go back to our previous slide we know that system matrix elements are given by a_{11}, a_{12}, a_{21} and a_{22} whose expressions are given here by equation number 69.

We will substitute for t, f_2, f_1 and so on in equation number 69 and this will give us the matrix element, the required matrix element and therefore a_{11} here t is 25 centimeter, f_2 is given which is -20 if you substitute for these two parameters you get the value of a_{11}. Similarly, the a_{12} would be equal to -0.1, a_{21} would be equal to 25, $a_{22} = -2/3$. Once all the elements all the matrix elements are known, we can easily calculate the focal length of the system, how, just take the inverse of the a_{12} term and put a minus sign before it and which is exactly what is done here, the focal length f would be equal to minus half a_{12}.

And what is a_{12}? $a_{12} = -0.1$, if you substitute minus 0.1 in place of a_{12} then you will get 10 centimeter and therefore, focal length of the system of lenses is 10 centimeter, what is next which is being asked? The next thing which is being asked in the question is the position of unit planes.

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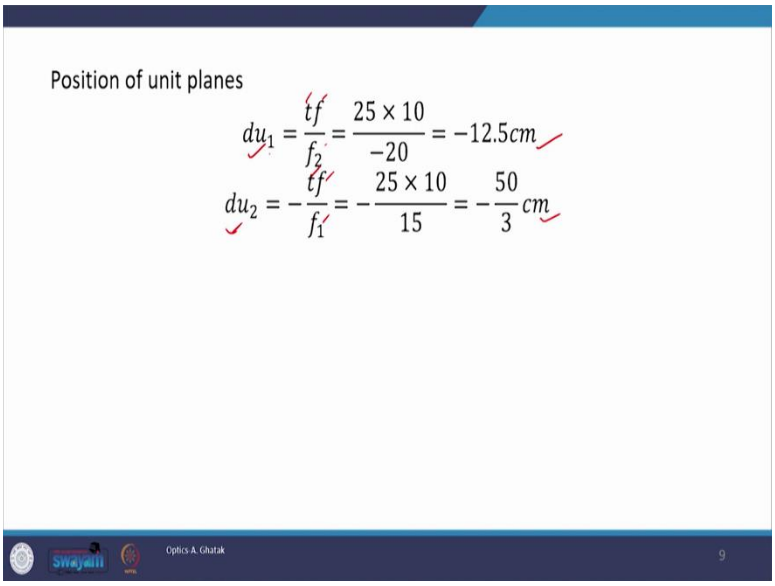

How to get the position of unit plane, we will use the formula which is derived earlier, d_{u1} which is the position of the first unit plane which is given by tf/f_2, just substitute for tf and f_2, you get a position of first unit plane, substitute again for tf and f_1 and this will give you the position of the second unit plane. This is how we can easily calculate the position of unit plane, matrix elements, focal length, very easy.

Therefore, we can see that this matrix formalism makes our life very easier. It removes all the complexity which is related with the ray tracing and the focal length finding, image formation and all. Now, this is all for this class. And thank you for listening me and we will see in next class.