Applied Optics
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Lecture 53
Plane Wave Propagation in Anisotropic Media – II

Hello everyone, welcome to the class. Today, we will start with where we stopped in the last class, we were studying Plane Wave Propagation in Anisotropic Media. In the last class, we derived determinant from the coefficient matrix, and we said that coefficient matrix must be equated to 0 and from that equation, we should get two values of n_w^2.

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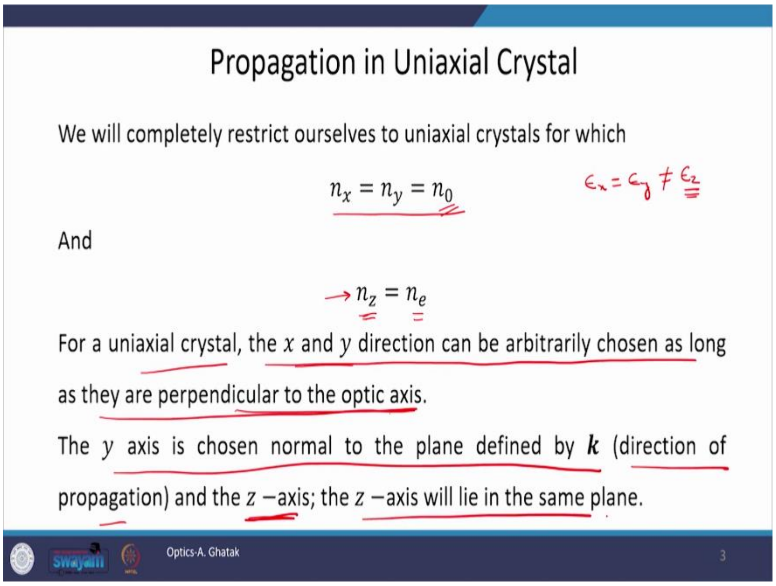

Now, we will consider a case of uniaxial crystal here, where in the refractive index along x and y axis are equal and we assume that they are equal to n_0, which represents the o wave refractive index and along z axis n_z, which is the refractive index along z axis is equal to n_e. Now, n_z represents the refractive index along with z axis we and we assume that it is equal to n_e, the e wave refractive index.

Now, for a uniaxial crystal x and y direction can be arbitrarily chosen as long as they are perpendicular to the optic axis, because we know that in uniaxial crystal $\epsilon_x = \epsilon_y \neq \epsilon_z$, it means, ϵ_z that is the permittivity along z axis. Now, the z axis is an axis which is unique, therefore, the x axis and y axis they would be identical and this is why the x and y direction can be arbitrarily chosen and they must be perpendicular to the optic axis and we choose optic axis along z axis. The y axis is chosen normal to the plane defined by k that is direction of propagation and the z axis and z axis will lie in the same plane of course.

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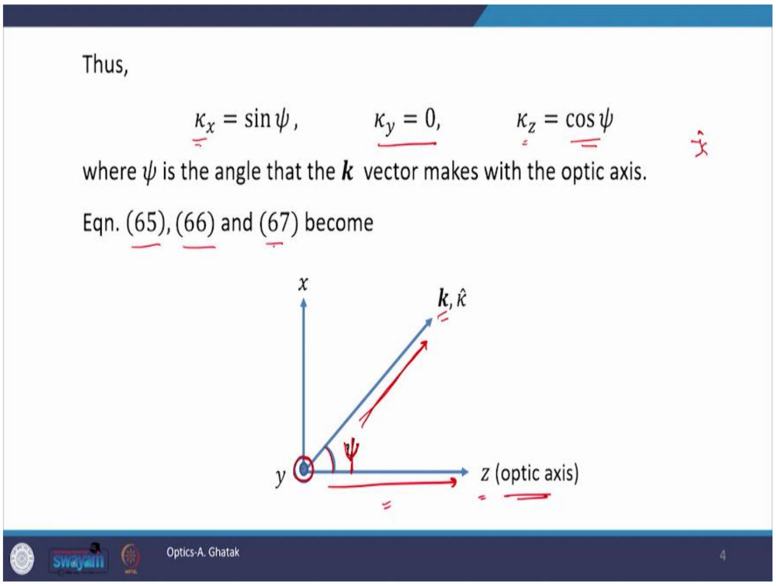

And which is given by this figure here this is the direction of z axis, this is the direction of k axis and y axis is perpendicular to the plane, which plane? a plane which is made by z axis or optical axis and the direction of propagation which is k. Now, since k is direction of wave and say it is at an angle ψ with respect to optic axis or z axis then $\kappa_x = sin\psi$, $\kappa_y = 0$ of course, because y is perpendicular to vector k direction and $\kappa_z = cos\psi$.

Just to remind you, what are these κ_x, κ_y and κ_z, they are the components of unit vector $\hat{\kappa}$. κ_x, κ_y, κ_z are components of unit vector $\hat{\kappa}$, unit vector $\hat{\kappa}$ is along wave vector k direction. Now, from the three-component equation which we derived in our last class.

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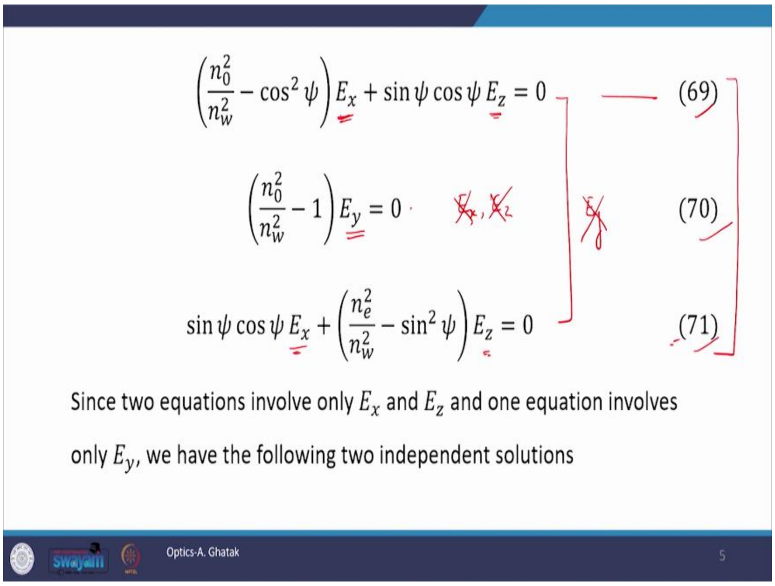

Now, they take these form, the new modified form of these equations are given by equation number 69, 70 and 71 respectively. Now, you see here in first equation this equation number 69 which has E_x and E_z term and equation number 71 also has this E_x and E_z term. And both these equations they do not have any E_y term, E_y dependencies absent in equation number 69 and 71. While in equation number 70, we do have E_y but this equation does not have E_x and E_z. Now, since two equations involve only E_x and E_z and one equation involves only E_y, we have the following two independent solutions.

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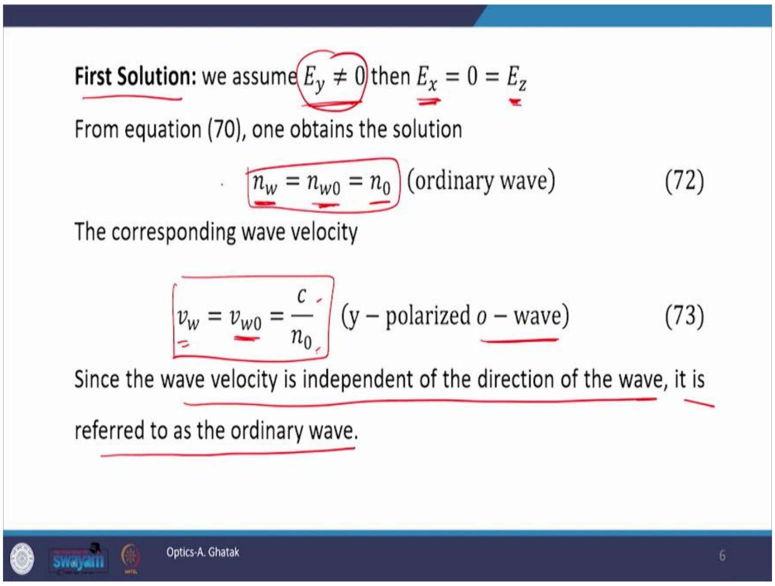

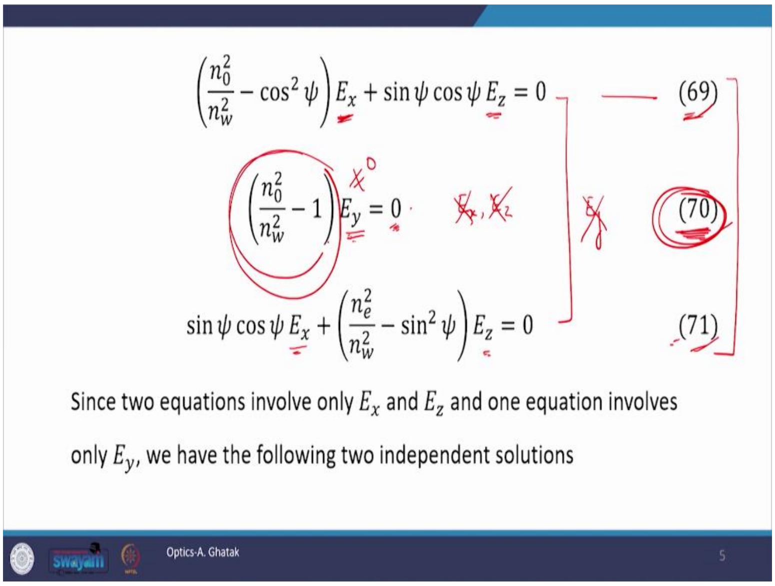

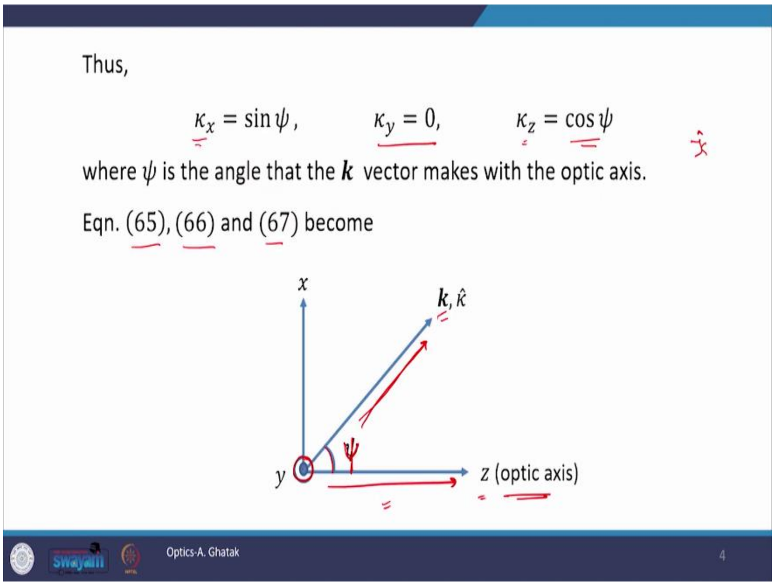

And what are those two independent solutions? The first solution is solution wherein we assume that the y component of the field is nonzero while the x and z component of the field is 0. With this E_y is not equal to 0 and E_x and E_z both have 0. Now, since both E_x and E_z is equal to 0 then we will we cannot use 69 and 71, what is left with is equation number 70.

Since E_y is not equal to 0 then this term must be equal to 0, because E_y is not equal to 0 and on the right-hand side we have 0. Therefore, in equation 1, this coefficient must be equal to 0 and from here we can write $n_w = n_{w0} = n_o$. This quickly comes from equation number 70, and $n_w = n_0$ and we assume that it is n_{w0}.

Now, once the value of refractive index is known, we can easily calculate the wave velocity in such a way, the wave velocity would be given by c/n_0 again we represent is represented as

v_{w0}, but do remember all these things are value valid only for $E_y \neq 0$ and E_x and E_z is equal to 0. It means electric field is pointing only along y direction, it means the wave is y polarized then if the wave is y polarized then we have ordinary refractive index. y polarized wave in such a situation is called o-wave, do see here in this figure.

Since, it is y polarized wave, the direction of E is perpendicular to k as well as optic axis. Now, since the wave velocity is independent of the direction of the wave it is referred to as ordinary wave you see there is no dependence of ψ, the velocity is independent of the direction of propagation or refractive index is independent of the direction, it is a constant quantity irrespective of the direction it is the same.

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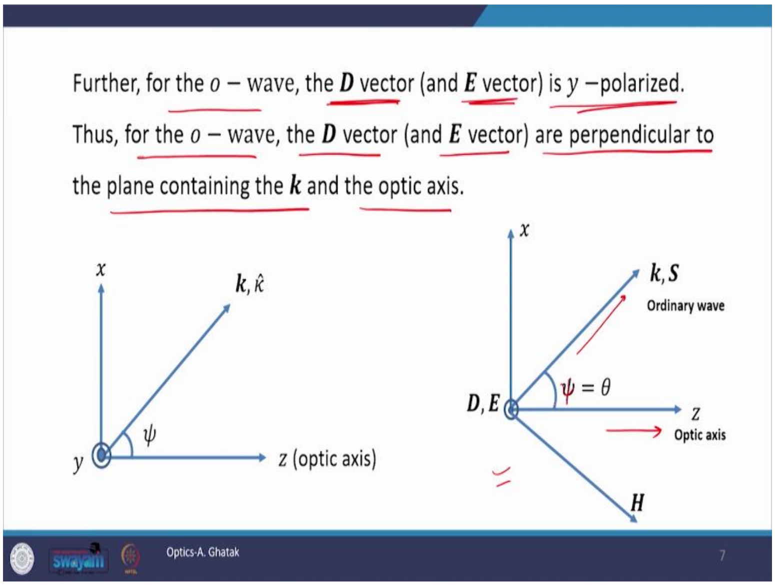

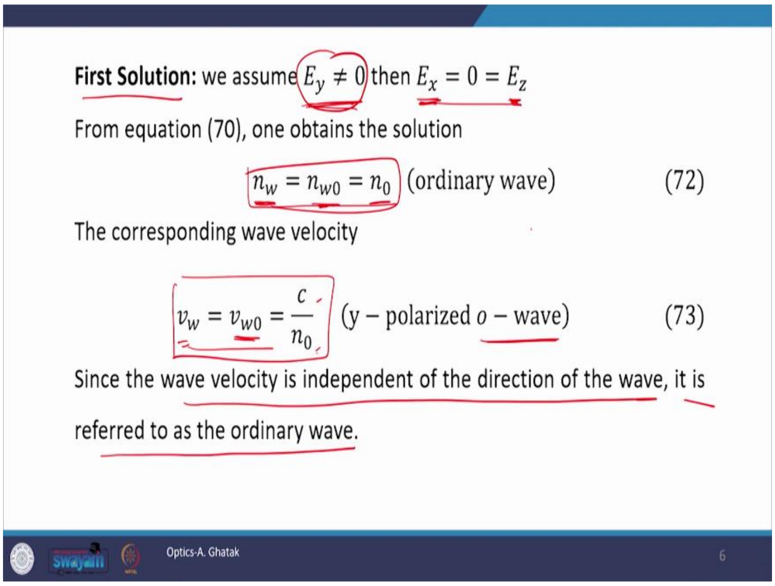

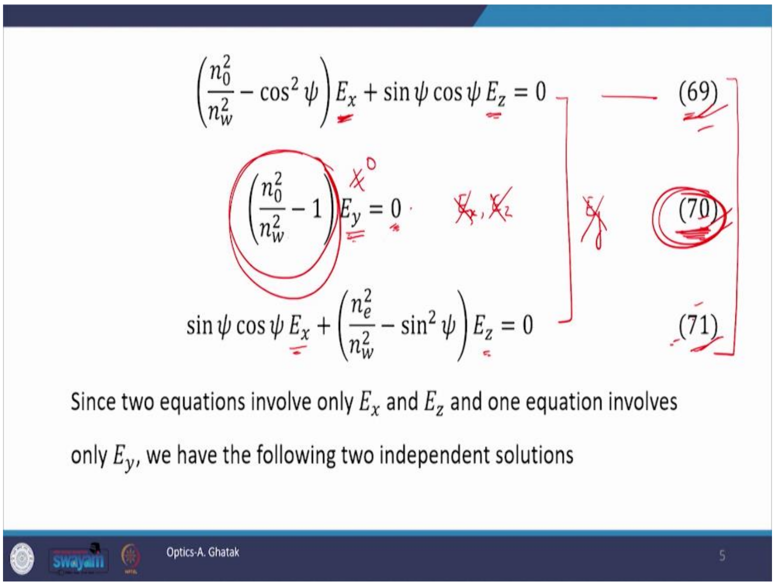

Now, further for the o-wave, D vector and E vector is y polarized, because the D is in the direction of E. Why D is in the direction of E? let me remind you that these were questions 69, 70 and 71, these were the three equations which were written in the component form and these are nothing but it $D = \epsilon E$ relation, but since we are dealing with anisotropic crystal, epsilon is a tensor and therefore, why we wrote $D = \epsilon E$ in component form and this gave us these three equations, but now, we are solving for a case where in E_y is not equal to 0 and E_x and E_z is equal to 0.

Now, with this we got the refractive index is a constant quantity therefore, velocity is also a constant which is direction independent. Therefore, the scalar relation between D and E now becomes valid, $D = \epsilon E$ and ϵ is a scalar quantity and therefore, D points along E. And therefore, for o-wave, D vector and E vector booth because they are pointing in the same direction are perpendicular to plane containing k and optic axis. The wave vector k and optical axis forms a plane and D and E, they both are perpendicular to this plane and which is shown here in this figure.

So, z is the direction of optic axis k is wave vector propagation direction, the orientation ψ=θ here and both D and E are along y, and they are perpendicular to the plane of this paper and this plane of paper is made by optic axis and the ordinary wave propagation vector. If the wave is o-wave, then we draw this figure. You see that H is along y, here it is coming out of the paper.

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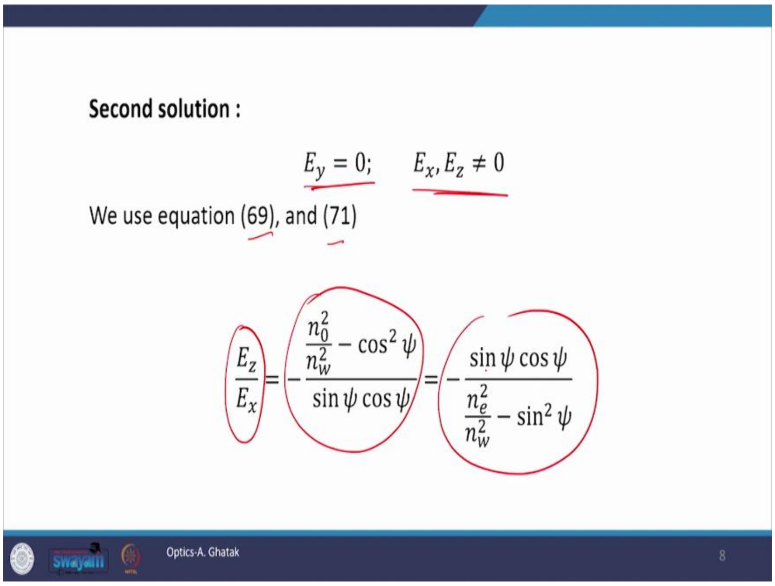

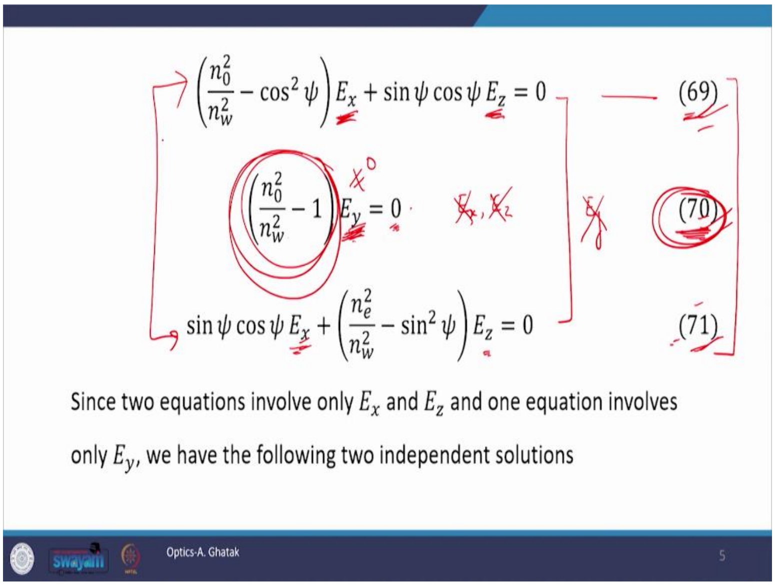

Now, this was all about first solution where E_y is not equal to 0 while other two components were 0. Now, let us go pick the second solution which says that E_y is equal to 0 while E_x and E_z is not equal to 0. Let us go back to the three component equations. Now, in this equation. Now, since E_y is equal to 0, then this term will not be equal to 0 while E_x and E_z is not equal to 0. Therefore, this time we will just look into these two equations the 69 and 71 equation and from this equation 69 and 71 let us write the ratio between E_z and E_x. From 69 we get this value of the ratio and from 71 we get this expression for E_z/E_x.

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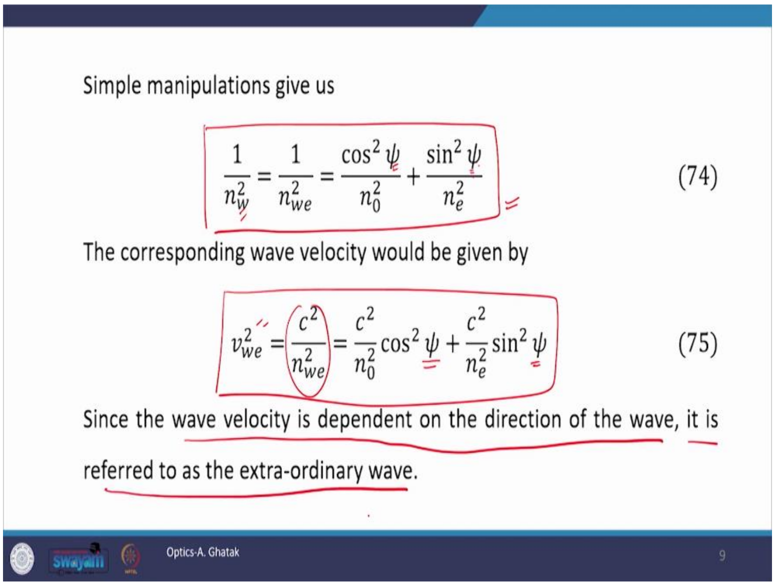

After a little bit of mathematics, we get the expression for n_w^2 and this is the expression which we saw in our first few classes of introduction to double refraction, there I showed you the velocity of E wave is given by this expression. Then once the refractive index is known, we can easily write the expression for the velocity using this relation. And you now see that the both the velocity and the refractive index are direction dependent, they depend upon the angular wave vector with the optic axis. Now, since the wave velocity is dependent on the direction of the wave, it is referred to as extraordinary wave or e wave.

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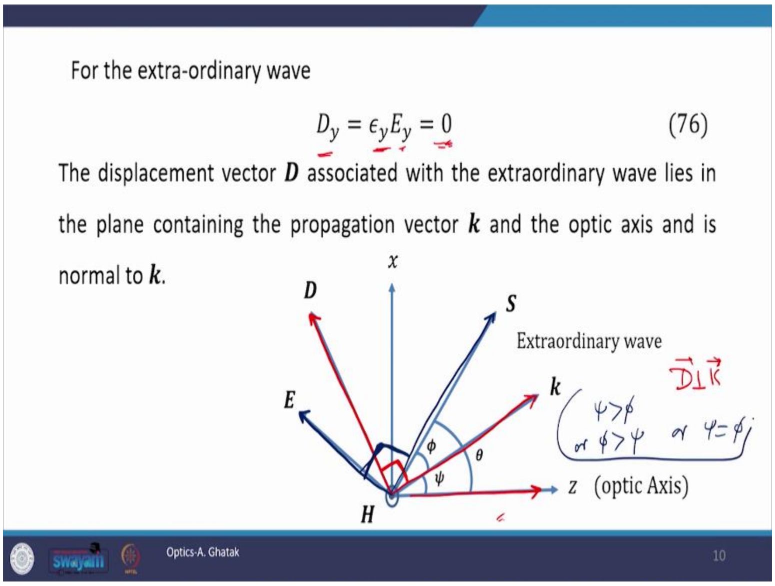

Now, for extra ordinary wave since E_y is equal to 0, $D_y = \epsilon_y E_y = 0$, this would be equal to 0, the displacement vector D associated with extraordinary wave lies in the plane containing propagation vector k and the optic axis and is normal to k. Therefore, since y component is y component of the displacement vector is 0, means displacement vector must be in xz plane, which is the plane of the paper and is shown here, this is the direction of optic axis and if say k is oriented angle ψ with the optical axis then D which is perpendicular to k would be in this direction and this angle would be 90-degree D would be perpendicular to k this is what we also seen while solving the equation in the last class in the last lecture.

Similarly, say this is the direction of pointing vector then E field electric field would be in this direction and this angle would be 90 degrees now. Pointing vector will make an angle of 90 degree with the E field direction and wave vector k will make an angle of 90 degree with the displacement vector direction. Now, whether ψ would be larger than φ, or φ would be larger than ψ or ψ would be equal to φ, all these depend upon the material properties.

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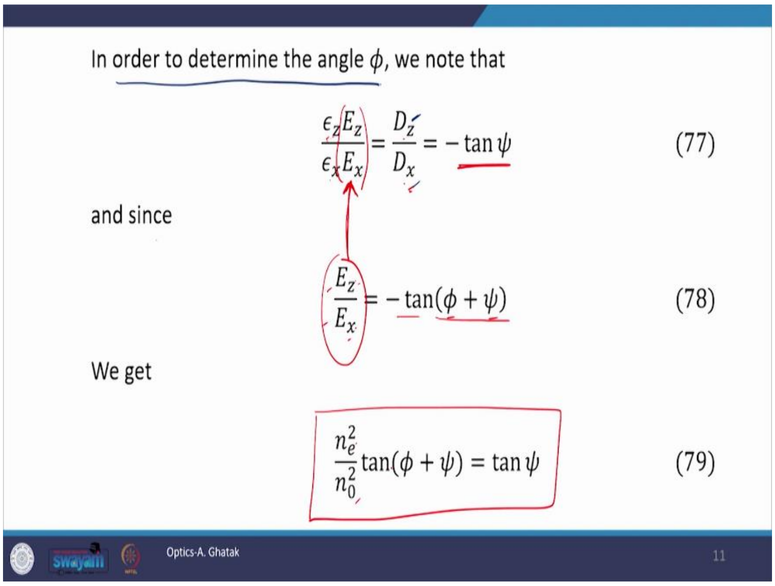

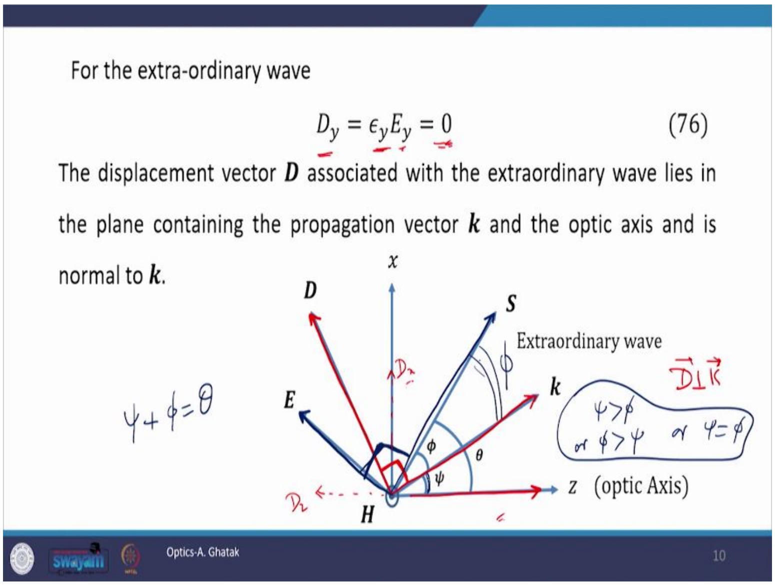

Example we will treat one case now, in order to determine the angle φ, φ is this angle, angle between S and k is φ. φ is the angle of orientation of k with respect to optic axis and ψ+$\varphi = \theta$, these are given now, we want to know the relative orientation between S and k which is φ.

How to get φ, let us see what is D_z by D_x from the figure the z component of D would be in this direction and x component would be in this direction. Therefore, D_z/D_x, D_z is in negative direction therefore, $D_z/D_x = -tan\psi$. Because D is perpendicular to k.

Therefore, $tan\psi = D_z/D_x$ and we know that $D = \epsilon E$ therefore $D_z = \epsilon_z E_z$ and $D_x = \epsilon_x E_x$, and similarly from figure E_z/E_x which is E is pointing in this direction therefore, E_z/E_x, since E is perpendicular to S, S we will not therefore, take this angle θ into account therefore, $E_z/E_x = -tan\theta$ and $\theta = \psi + \varphi$.

Therefore, $E_z/E_x = -tan(\psi + \varphi)$. Now, let us substitute the expression of E_z/E_x into equation number 77. The expression of E_z/E_x from 78 is now substituted in equation number 77 and this gives this relation. We are n_e^2 comes in the expression of ϵ_z and n_0^2 comes in the expression of ϵ_x.

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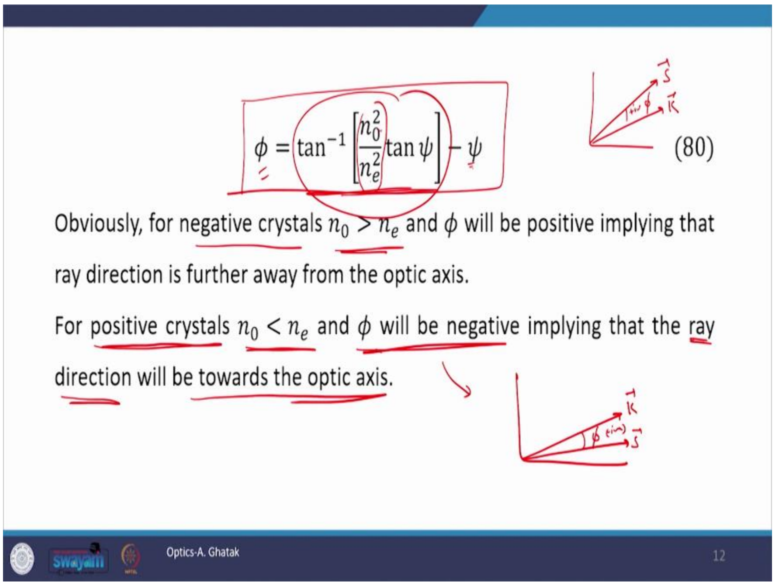

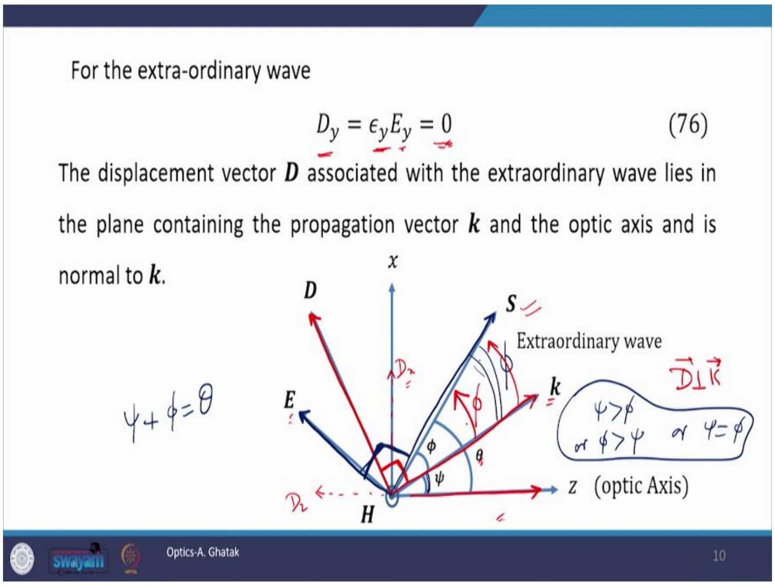

Now, from here from equation number 79 if you just simplify it you will get the expression of φ. So, now, for negative crystal we know that n_o is larger than n_e, and if n_o is larger than n_e then this term would be larger than ψ and this quantity on the right-hand side of equation number 80 therefore, will be positive and it says that the ray direction is away from the optic axis.

If φ is positive, it means S would be away from the optic axis it would look the position of S would be in this direction from k, because φ is positive and what is φ, φ is orientation of point D vector S with respect to k direction and positive φ means φ is in this direction. And this happens in a crystal for which n_o is larger than n_e that is for negative crystal, we will have k in this direction and S in this direction, this is positive φ.

For positive crystal, n_o would be less than n_e and φ therefore, since n_o is less than n_e this quantity would be smaller than ψ and therefore, φ would be negative and it means that ray direction means direction of pointing vector it will be towards the optic axis, it means in this particular case, if k is pointing in this direction an S would be pointing here this would be the φ which is negative.

With this I end my lecture and this also finishes the second last module that is module number 11. From next lecture onwards, we will start module number 12, which is totally dedicated on the applications of whatever we have studied till now. Thank you for joining me. See you in the next class.