Applied Optics
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Lecture 51
Analysis of Polarised Light and Optical Activity

Hello everyone, welcome back to my course, today we will talk about the next topic in module 11 which is analysis of polarized light and that would be followed by the second topic which is optical activity. In this analysis of polarized light, we will try to figure out state of polarization of a light.

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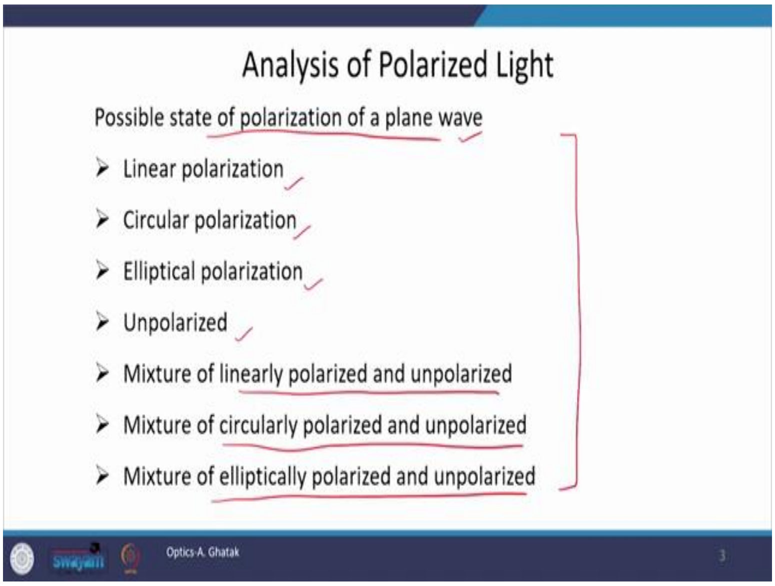

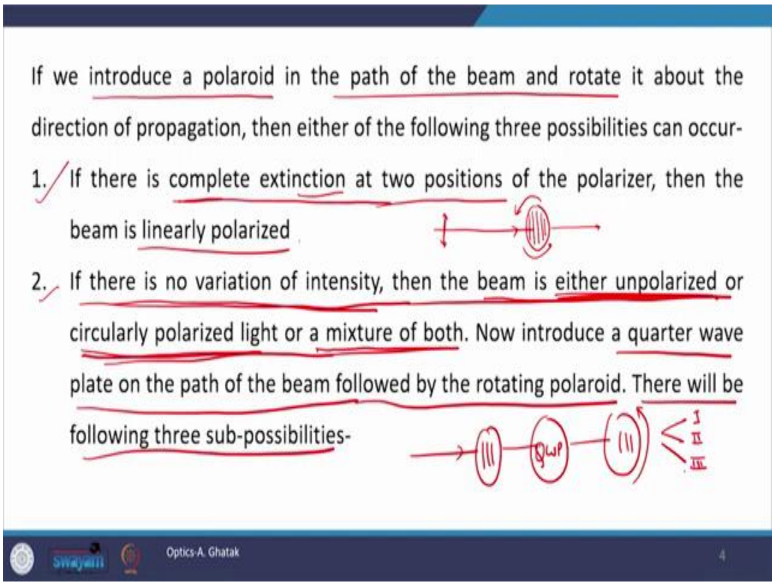

Now, the possible state of polarization of a plane wave are linear polarization, circular polarization, elliptical polarization, unpolarized light, mixture of linearly polarized and unpolarized, mixture of circularly polarized and unpolarized, mixture of elliptically polarized and unpolarized, like these are the possible state of polarization of a plane wave.

Now, suppose we are given a light whose polarization is unknown then we will have to figure out the state of polarization of this given light using birefrigerant crystal like half wave plate, quarter wave plate, polarizer, these are the tools which we will use and then we will detect or we will decide the state of polarization of this given light. Now, suppose we are given a light and then what we will do first is that we will introduce a polaroid in the path of the beam and then we will slowly rotate the polaroid or polarizer.

If you rotate the polarizer along the axis of propagation of the light then there are three possibilities which may arise and what are these possibilities, the first possibility is that there is complete extinction of intensity at two positions, we have a light which is falling on a polaroid and the polaroid is being rotated, either in clockwise direction or in anti-clockwise direction, but once you are rotating in clockwise you will have to rotate clockwise, we should not reverse it in anti-clockwise and then again go to clockwise direction it should be a uniform rotation, unidirectional rotation.

So, if we rotate the polaroid then what may happen is that at two positions of the polaroid the intensity may completely go to zero, we may observe a complete extinction of intensity. Now, this extinction of intensity says that beam is linearly polarized, why say this is the polaroid whose pass axis is vertically oriented, now say this beam of light is falling normally on the polaroid and this incident beam is linearly polarized with its state of polarization.

The incident beam is linearly polarized with its direction of polarization oriented along vertical, now if you rotate the polarizer say we are rotating the polarizer this way then what will happen is that whenever the pass axis of the polarizer is vertically oriented we will see maximum intensity and the this vertical orientation will occur twice in a full rotation, first is this and the second is this, when the polaroid is rotated by 180 degree in this case also the pass axis would be in vertical direction and then the incident light would be completely allowed to pass through.

Therefore, in one rotation of polaroid for linearly polarized light there would be two positions of the polaroid where we will get zero intensity at the output, and what are these two positions when the polaroid is oriented in a cross position, the pass axis is in the horizontal direction while the vibration direction is vertically oriented.

Similarly, if you again rotate the polarization by 180 degree then this would be the situation pass axis is again horizontal and the direction of vibration is vertical, in these two situations we will not receive any light. While in this situation and in this situation we will receive complete brightness, full light is there, full intensity is on the other side of the Polaroid.

It means if there is a complete darkness in a full rotation of a polaroid then the incident beam is a linearly polarized beam and this is how we detect a linearly polarized light using polarizer. Now, what is the second possibility, the second possibility is if there is no variation of the intensity there is a light which is falling on the polaroid and we are rotating the polaroid but at the output there is no variation in intensity. It means that there is a circular symmetry in the state of polarization of the light, because it is independent on the rotation of the polaroid axis.

Since the output intensity is independent of the orientation of the polaroid or rotation of the polaroid then the incident beam is either unpolarized because in unpolarized light the electric field may orient in any random direction, therefore if you pass unpolarized light through a polaroid, the output intensity would be independent of the orientation of the polaroid here therefore the first possibility is that either beam is unpolarized and the second possibility is that the beam is circularly polarized, because in both the cases if it is unpolarized all the orientation of the electric field is possible.

If it is circularly polarized still all the orientation of the electric field is possible, the only difference between unpolarized and circularly polarized light is that the capability of predicting the orientation of the electric field which polarization gives us is there in a circularly polarized light.

If we know that the electric field is vibrating in vertical direction at a time t=0 then we can predict what would be the state of polarization or orientation of this electric field vector or D vector at a later time, because if it is circularly polarized it will go like this on a helical path, it will start from here and then it will move like this therefore we can predict although it is changing its orientation but it is a predictable while in unpolarized light sometimes electric field would be like this, another instant this, next instant, next instant, next instant, it is completely random.

Therefore, with the unpolarized light as well as with the circularly polarized light, the output or the other end of the polaroid will see no variation in the intensity with the rotation of the polaroid and the other possibility is that the incoming light is a mixer of both unpolarized light and circularly polarized light. I repeat if there is no variation of intensity then the beam is either unpolarized or circularly polarized or a mixture of both.

Now, how to distinguish between these three cases, to again decide what exactly is the state of polarization what people do is that they introduce quarter wave plate on the path of the beam that would be followed by again a rotating polaroid, there is a rotating polaroid then after rotating polaroid we received a light whose intensity is independent of the rotation of the polaroid or orientation of the polaroid then people introduce a quarter wave plate and again a rotating Polaroid.

Now, with this there are again three sub possibilities, we launched a light then we introduced polaroid then after this it is here quarter wave plate and again a polaroid here. It is rotating polaroid again after quarter wave plate, in this situation there are three possibilities again what are these three possibilities, let us go through them one by one.

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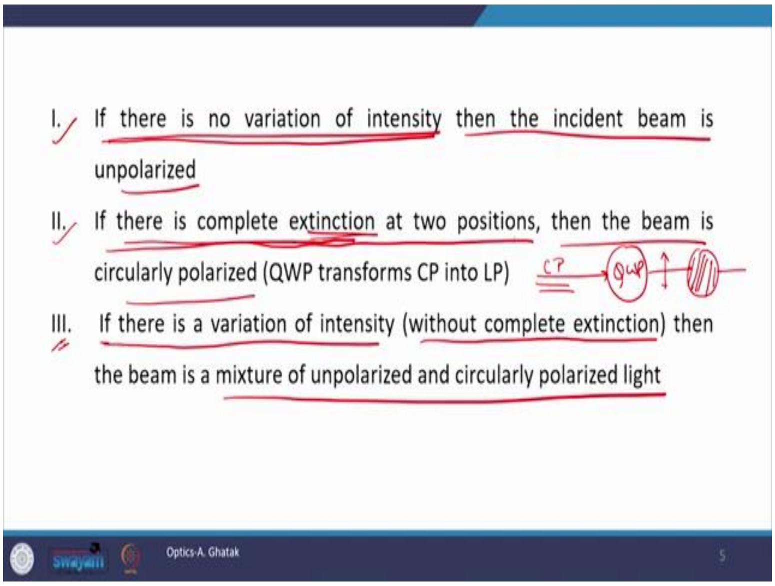

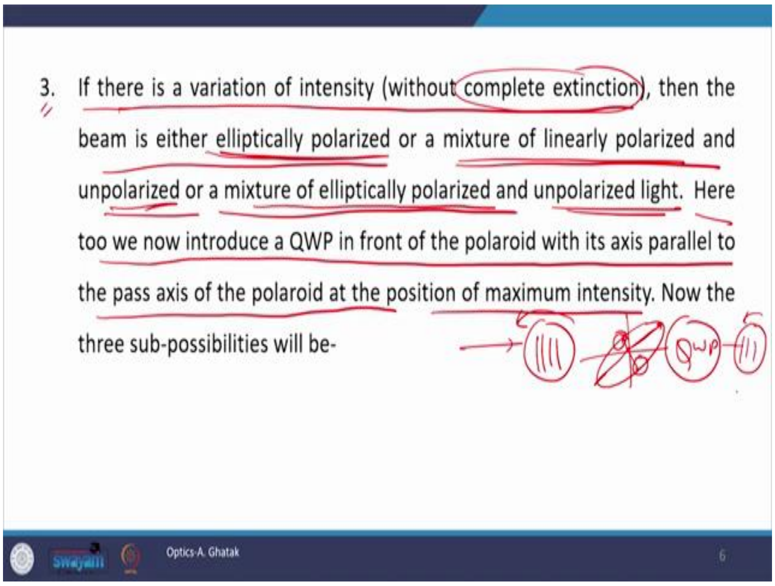

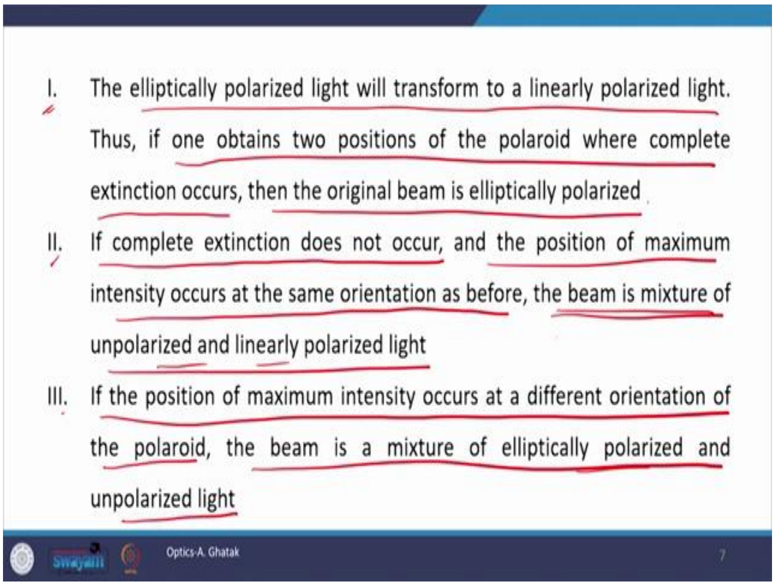

The first possibility is that if there is no variation of intensity then the incident beam is unpolarized, the light which is coming out of the first polaroid it is then passed through a quarter wave plate and then the light which came out of the quarter wave plate is again passed through a rotating polaroid and if there is no variation again in the intensity then the beam is said to be unpolarized and which is very much obvious because unpolarized light it does not changes with anything whatever you just apply in its path, the intensity of the unpolarized polarized right here of course when you put a polaroid in the path of unpolarized light, it will get polarized linearly polarized but since the received intensity is independent of the rotation of the polaroid in this situation will not alter, even after the second polaroid you are seeing that the intensity is unaltered.

Now, let me clarify this case, this was the first polaroid and after this we saw that intensity is independent of θ, it does not depend upon or let me write like this I is not a function of θ it is independent of θ and θ is the angle of rotation here when we found this then we introduce this quarter wave plate here, we replace this with the quarter wave plate and then after quarter wave plate rotating polaroid was introduced.

Now, the second possibility if there is a complete extinction at two positions then the beam is circularly polarized, why is it so? Because quarter wave plate transform circularly polarized light into linearly polarized light, the beam is launched and we have here quarter wave plate. Say its a circularly polarized light then quarter wave plate will convert into a linearly polarized light and if you put a polarizer with a certain orientation of pass axis and rotate it then you will get intensity minima at two positions of θ, or at two positions of polaroid because polaroid is continuously rotating and at two positions of the polaroid, the intensity is going down to 0. Then it confirms that the incident beam is circularly polarized, the extinction is happening at two positions of the Polaroid.

The third case, the first is that there is no variation of the intensity, second case is that there is a two complete extinction as two position, the third point is that or third possibility is that if there is a variation of intensity without complete extinction the intensity is not completely going down to 0, but still there is a variation in the intensity then the beam will be made up of unpolarized light as well as circularly polarized light. It would be a mixture of unpolarized light and circularly polarized light. Now, these three are the sub possibilities of case number two which we discussed in the previous slide now let us go to the case number three.

Now, in this case number three the light is launched on a Polaroid, and the polaroid is rotating now if there is a variation of intensity without complete extinction then the beam is either elliptically polarized or a mixture of linearly polarized and unpolarized or a mixture of elliptically polarized and unpolarized and this is very much obvious, because the intensity is not going to 0, it is reducing down but not down to 0.

Then light will not be linearly polarized of course since intensity is varying therefore it is not circularly polarized then possibilities that it may be elliptically polarized, because if you rotate the ellipse then here intensity is maximum it reduces here at semi minor axis then again it increases here, the intensity is fluctuating its periodically varying but it is not going down to 0. Therefore, the one possibility is that the incoming light is either elliptically polarized, the second possibility is mixture of linearly polarized and unpolarized this will also produce the same effect and the third possibility is a mixture of elliptically polarized and unpolarized, equally probable.

Here too, we now introduce quarter wave plate in front of the polaroid with its axis parallel to the pass axis of the polaroid at the position of maximum intensity, here too people introduce quarter wave plate and then again, a polaroid which rotates. Here again we have three sub possibilities what are these possibilities, the first one is the elliptically polarized light will transform to a linearly polarized light. After passing through a quarter wave plate here the elliptically polarized light will transform into a linearly polarized light, thus if one obtains two positions of polaroid where complete extinction occurs then the original beam is elliptically polarized.

The second possibility if complete extension does not occur and position of maximum intensity occur at the same orientation as before the beam is a mixture of unpolarized and linearly polarized light, which is very much clear. The third possibility here is that if the position of maximum intensity occurs at different orientation of the Polaroid, several positions of the polaroid the beam is a mixture of elliptically polarized and unpolarized light.

Now, these are the sub possibilities which may arise and with this these observations, we may predict the state of polarization of the light, we can exactly tell the state of polarization of the light or the component of the different polarized light out of which the original light is made up of.

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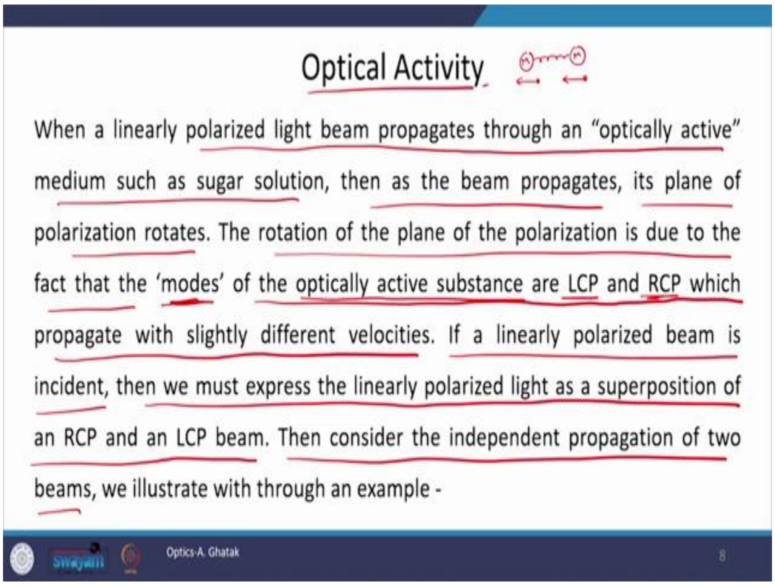

Now, we move to the next topic which is optical activity, when a linearly polarized light beam propagates through an optically active medium such as sugar solution, then as the beam propagates, its plane of polarization rotates. The rotation of plane of polarization is due to the fact that modes of the optical active substance are left circularly polarized and right circularly polarized which propagates with slightly different velocities.

Now, this says that suppose we have a linearly polarized light which is oscillating and in which the field is vibrating in some direction now if this linearly polarized light is passed through a sugar solution then what will happen is that this direction of vibration will rotate with propagation inside the sugar solution and materials which exhibit such a property is called optical active material or optical active substance, they have a tendency to rotate the state of polarization.

Now, since they are rotating therefore we may say that probably they have some property of circular polarization here because rotation is always associated with circle, now the rotation of a plane of polarization is due to the fact that modes of the optical active substance are left circularly polarized and right circularly polarized.

Here modes means possible solutions or the allowed states which system support and suppose we have a dipole and we want a dipole to vibrate, let us consider we have two masses which are attached with a spring then what are the possible ways in which the two masses can vibrate?

The first possibility I would like to draw here, these are the two masses and this is the spring, the first possibility is that they may vibrate, we may take these two masses stretch them and leave them then they will start doing this vibration here, they will vibrate in this direction simultaneously, they will do like this, this is called one of the mode of this system which consists of two mass and one spring.

Similarly if we take one mass only and stretch it and leave it then what will happen one mass will start vibrating rapidly, but slowly its energy will get transferred through this spring, the part of energy get transferred to the second mass and this mass will also slowly start to vibrate and the situation will come when they both synchronously will vibrate the earlier situation and this happens because this type of vibration is a mode of the system and we may also say like you must have solved different type of differential equations, the solutions to the differential equation may also be called mode of the differential equation. This is allowed states or allowed solutions of the system, now I repeat in optically active substances left circular polarization and right circular polarizations are modes.

Now, whatever light you launched in such optical active substance it will split the incoming polarization into right circularly polarized light and left circularly polarized light, because we already know that any polarization can be expressed in terms of two orthogonally polarized light. Any circular, elliptical polarized light can be expressed as two orthogonally polarized linear polarizations or two orthogonally polarized circular polarizations and what are two orthogonally polarized circular polarization?

Left circular polarization and right circular polarization, they are orthogonal, therefore using left and right circular polarization, we can express any type of polarization, there are the basis of the system. Now, therefore if a linearly polarized beam is incident, then this linear polarization it can be expressed in terms of terms of LCP and RCP and then we must express the linearly polarized light as superposition of RCP and LCP. Therefore, consider the independent propagation of two beams and this independent propagation it can be illustrated through an example.

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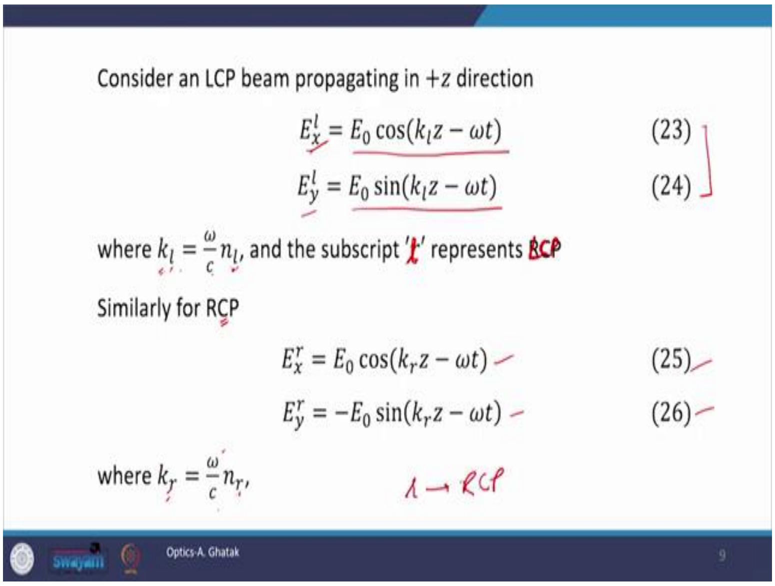

n_r and n_l are the refractive index corresponding to the RCP and LCP beams respectively.

If we assume the simultaneous propagation of two beams then the x and y component of the resultant fields would be given by

$$E_x = E_0[\cos(k_r z - \omega t) + \cos(k_l z - \omega t)] \qquad (27)$$

or

$$E_x = 2E_0 \cos\left[\frac{1}{2}(k_l - k_r)z\right]\cos[\omega t - \theta(z)] \qquad (28)$$

Similarly,

$$E_y = 2E_0 \sin\left[\frac{1}{2}(k_l - k_r)z\right]\cos[\omega t - \theta(z)] \qquad (29)$$

where $\theta(z) = \frac{1}{2}(k_l + k_r)z$

Thus the resultant wave is always linearly polarized with the plane of polarization rotating with z. If the direction of the oscillating electric vector makes an angle ϕ with the x axis (vertical direction), then

$$\phi(z) = \frac{1}{2}(k_l - k_r)z \qquad (30)$$

Consider an LCP beam propagating in $+z$ direction

$$E_x^l = E_0 \cos(k_l z - \omega t) \qquad (23)$$

$$E_y^l = E_0 \sin(k_l z - \omega t) \qquad (24)$$

where $k_l = \frac{\omega}{c} n_l$, and the subscript '$l$' represents LCP

Similarly for RCP

$$E_x^r = E_0 \cos(k_r z - \omega t) \qquad (25)$$

$$E_y^r = -E_0 \sin(k_r z - \omega t) \qquad (26)$$

where $k_r = \frac{\omega}{c} n_r$,

What is this example, this is given here, consider LCP, left circularly polarized beam propagating in plus z direction and LCP can be expressed through equation number 23 and 24, where x component is expressed as $E_0 cos(k_1 z - \omega t)$ and y component is expressed as $E_0 sin(k_1 z - \omega t)$, where k_1 is wave vector whose expression is omega by c/n_l, the subscript l represent LCP.

Similarly, for RCP, we have these two expressions the x and y component of RCP light is where beam is given by equation number 25 and 26, here $k_r = (\omega/c)n_r$ where r represents RCP. Here n_r and n_l are refractive indices that correspond to RCP and LCP beams respectively. Now, we consider that both LCP and RCP beams are propagating simultaneously in this optically active medium, since they are simultaneously propagating, they will superimpose.

Now, since we are representing LCP as well as RCP in its component form, the resultant x component of the field would be vector sum of x component of RCP and x component of LCP, this is what is done here in equation number 27. We added the x component of RCP and LCP to get E_x and you see that in equation number 27 in this bracket, we have $cos\ c + cosd$ like term, on simplification we get on equation number 28.

Here θ is nothing but $(k_l + k_r)z/2$ and similarly we can get E_y component which we received by adding the y component of LCP and RCP beams, in both the expression in expression 28 and 29 $\theta(z)$ is expressed by $(k_l + k_r)z/2$. Now, you see that the equation 28 and 29 we have cosine term here and we say that the resultant wave is always linearly polarized with the plane of polarization rotating with z, we have this resultant expression of E_x and E_y and from here we clearly see that this resultant is linearly polarized beam and its plane of polarization is rotating with z.

If the direction of the oscillating electric vector makes an angle φ with the x axis means the vertical axis here, the direction of propagation is z and x is vertical axis, of course y is perpendicular to the plane of the paper, now if the direction of oscillating electric vector makes an angle φ with the x axis that is vertical direction then $\varphi(z) = (k_l$-$k_r)z/2$, which is this angle. How to calculate this, just divide 29 by 28 this will give this angle φ.

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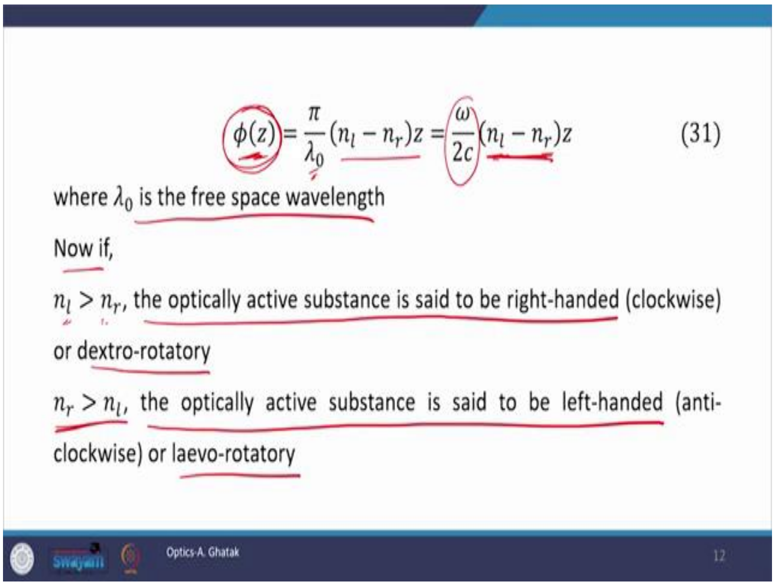

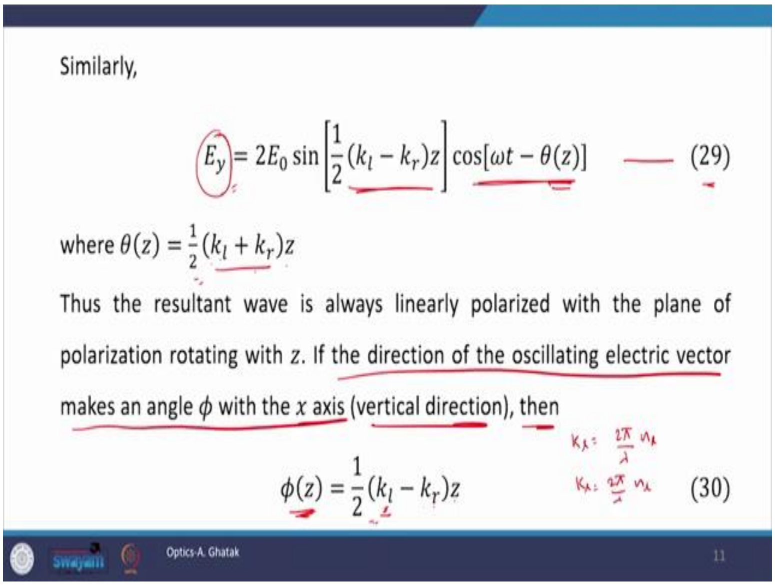

Now, once the expression of φ is known, let us expand k_l and k_r and what are k_l is nothing but $2\pi/\lambda \times n_l$ and similarly $k_r = 2\pi/\lambda \times n_l$. Now, instead of writing λ or λ_0 here it is expressed in frequencies, λ_0 is nothing but it is a free space wavelength.

Now, you see that due to difference in refractive indices as seen by LCP and RCP beam after propagation in an optical active medium, the beam receive a phase difference φ, if n_l is larger than n_r then this φ is positive and the optical active substance is said to be right handed or dextro rotatory and this is what our sugar solution is.

It means the state of polarization it will rotate in a clockwise fashion if it pass through a detro-rotatory solution and if n_r is larger than n_l then φ is negative, then the optical active substance is said to be left handed or laevo-rotator. In this case the plane of polarization will rotate in anti-clockwise fashion. This is all for today, thank you for joining me; see you all in the next lecture.