Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology, Roorkee
Lecture 05
Refraction from Double Interface

Hello everyone, welcome to my class. In the last class, we talked about refraction from a single spherical surface.

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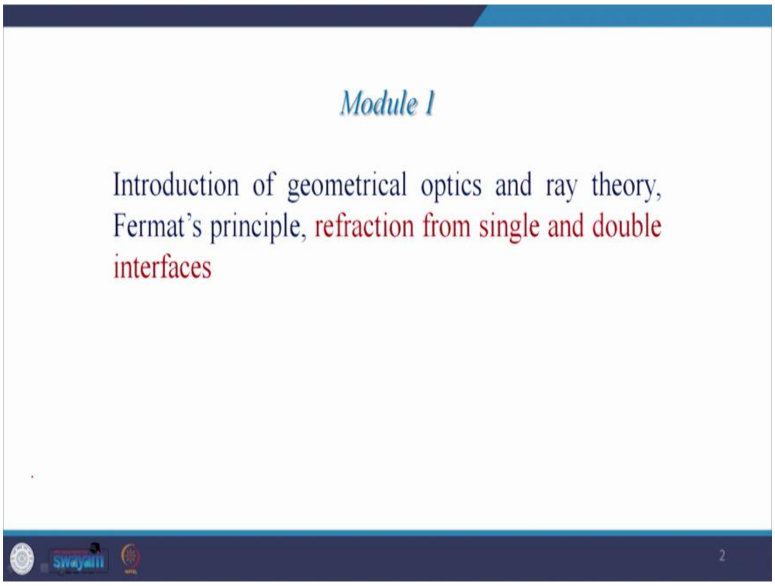

In today's class, we will talk about reflection from a single spherical interface. And thereafter, we will talk about refraction from a double spherical interfaces.

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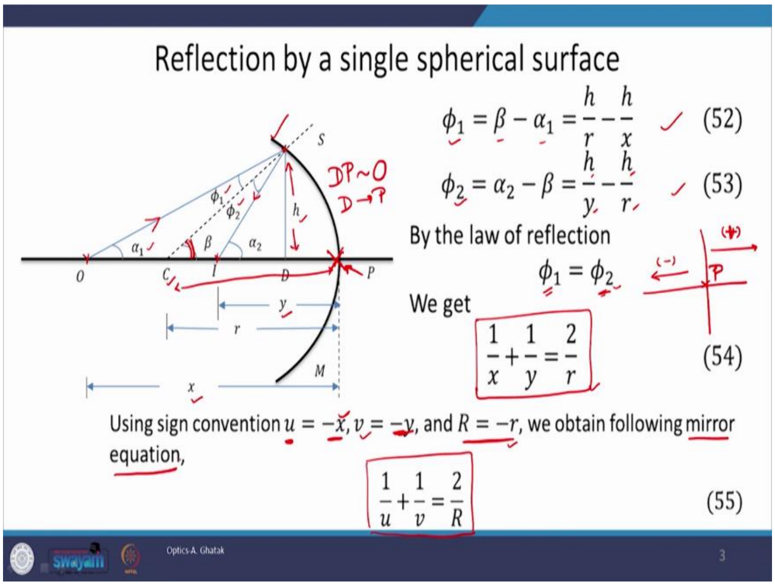

So, let us, begin. Let us, first talk about reflection by a single spherical surface. Here in this picture, this curved surface represents mirror and our object is situated at point O and our rays start from point O it goes to the surface of this reflecting mirror and then after getting reflected, it falls at point I, where it forms the image.

As discussed in the previous class C is the centre of spherical mirror for this particular case. object O is situated at a distance x from this point, which we name as point P and the image is formed at a distance y from this point P and the radius of curvature of this spherical surface is r.

The point at which the ray starting from object O strikes is designated by S and the height SD is small h. The angles are also represented in this figure, the angle of incidence is φ_1, angle of reflection is φ_2, and now, using the geometry, we can say that $\varphi_1 = \beta - \alpha_1$.

Now, as discussed in the previous class, since, we are in the paraxial regime, paraxial means we are considering only the rays which are very close to the axis of the system. What do I mean by saying very close to the axis of the system means, these rays makes very small angle with the horizontal axis of the symmetry and once these angles are very small then what we can write is that tan $\beta = \beta$ and tan$\alpha_1 = \alpha_1$, tan$\varphi_1 = \varphi_1$ and in this particular case we can write β =h/r, β is this angle and then h is this height and r is the radius of curvature.

Now, since we are in the paraxial regime the distance DP, it would be approximately equal to 0, what do I mean by 0 is, D would approach to P it would be very closely situated to P and therefore, this distance DP can be neglected and in this particular case these relations will hold good $\varphi_1 = h/r \ - h/x$ while $\varphi_2 = h/y - h/r$. Now, we know what is law of reflection. The law of reflection says that angle of incidence is equal to angle of reflection. Therefore, $\varphi_1 = \varphi_2$ and if we substitute for φ_1 and φ_2 in this relation, we will get 1/x +1/y =2/r. This is what we get for spherical reflecting surface.

Now, we will implement the sign conventions. What are our sign conventions? We studied them in our last class and the sign conventions are, let suppose we have origin at point P, then all the distances left to P would be in negative, all the distances right to P would be in positive. We will say that these distances are positive and the distances which are left to P they are said to be negative. Now from the figure x, y and r, the all three are on the left side of point P here this is point P.

Since these three parameters are on the left hand side of P. Therefore, they all will be negative. And as per the sign convention we represent the object distance by u and therefore, u would be minus of x. Since x is on the left-hand side of P therefore, u would be minus of x.

Similarly, v would be minus of y and the radius of curvature which is represented by R it would be minus of r. Now, substituting this back into equation number 54 we will end up to this relation, which is 1/u+1/v=2/R. This relation is called this expression is called mirror equation.

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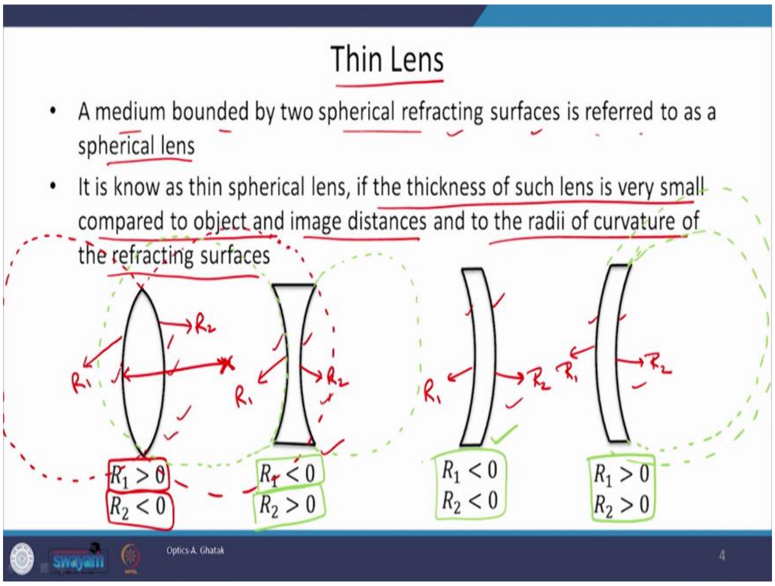

Now, we will again start talking about refraction. Now, we have already studied refraction from a single spherical surface. Now, we will move towards two spherical surface, double spherical surface. Now, as soon as we start talking about double spherical surface something which is called lens comes to the existence. Using double spherical surface, we can bound a medium and this bounded medium is called lens. This is what exactly defined here. A medium bounded by two spherical refracting surfaces is referred to as a spherical lens.

Now, here you can see different pictures of different kinds of spherical lenses and these lenses as you can see, they are bounded by two spherical surfaces here these are the two surfaces in, this figure these are the two surfaces in this figure, these are the two and so on, with they have their own respective radius. Now, the topic is thin lens, when to call a lens thin? The word thin is a relative word and therefore, to say something thin or thick we must have a reference thickness.

We must compare it with something standard. Now, the definition here is that, the definition of thinness here is that, if the thickness of a lens is very small compared to the object and image distances and to the radii of curvature of the refracting surfaces, then only a lens is said to be a thin lens.

I repeat, if the thickness of the lens is such that it is very small as compared to the other related thicknesses or other related distances. What are the other related thicknesses or distances in the case of a lens system, the height of a lens, the distance of an object, the distance of an image, the radii of curvature of the refracting surfaces out of which the lens is formed.

If these distances are much larger than the thickness of the lens, then we may call a lens to be a thin lens. Now, these things you can find in a book Optics by Ajoy Ghatak for covering most of the geometrical optics. I am following the book by Professor Ajoy Ghatak. Now you can see here is a lens which is formed by two refracting surfaces and since there are two refracting surfaces involved there would be two radii of curvature.

The first surface has radii R_1 while the second surface has radii R_2. Similarly, here in the second case, this is the radii of the first surface, this is the radius of the second surface. Similarly, this is R_1 and this is R_2. Now, let us go to the sign of R_1 and R_2. Now, in the first figure, you see that this first curvature which is of radius R_1, it is a part of a big sphere, you can draw a sphere like this. This sphere is extending towards the right of the lens.

Now, the origin or centre of this sphere would be somewhere here. Now, this radius would be in this direction, isn't it! This radius would be on the right-hand side of the centre of the lens, since R_1 is on the right-hand side, the associated sphere is on the right-hand side of the lens system. We will call R_1 to be positive, that is why it is written here R_1 is greater than 0.

And now, for the second spherical surface, if you draw the corresponding sphere and you will see that this is sphere is on the left-hand side of the centre of the lens. And therefore, R_2 would be negative. The distances which are on the right-hand side as per sign convention would be positive and the distances which are on the left-hand side as per the sign convention will be negative and we will follow the same approach here.

Now, in the second case, let us see what are the signs? Now, in the second case, we will again draw the sphere. Now, R_1 is the radius of curvature of the first spherical surface now in the green colour, I am drawing the related sphere, you see that this is sphere appear on the left-hand side of the lens, left hand side means negative and therefore, you can see R_1 is negative here.

Similarly, the second sphere will appear on the right-hand side of the lens and therefore, R_2 would be positive. Similarly, for this lens, for this particular lens, you see that the first sphere is on the right-hand side of the lens and also the second sphere is on the right-hand side of the lens, both spheres are on the right-hand side of the lens.

Therefore, in this particular case, both R_1 and R_2 are positive. While in the third case both R_1 and R_2 are negative. Now, once we know what is the thin lens and how their radii of curvature are calculated or once they are given whether they are positive or negative, once these things are understood, then we can safely move towards the analysis of thin lens.

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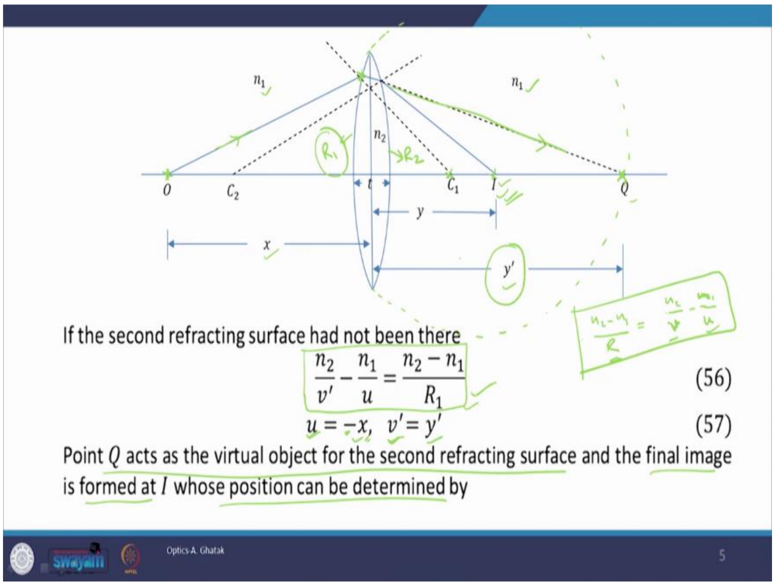

Now, here you can see that a thin lens is given and we want to analyse these thin layers. We want to know how this thin lens form an image. Previously, we have studied the image formation from a single spherical surface. Now, we are going to study the image formation from 2 spherical surfaces and these 2 spherical surfaces are placed in such a way that it formed a lens.

And what are these 2 spherical surfaces? This is the first surface with radius of curvature R_1 and this is the second surface with radius of curvature R_2. R_1 is the radius of curvature of the first surface and R_2 is the radius of curvature of the second surface. And these 2 surfaces they joined together and they confine material of refractive index n_2 between them.

Therefore, the refractive index of the lens is n_2 and this lens is kept in a medium of refractive index n_1. Both on left and right-hand side of this lens we have same medium and therefore, refractive index on both left- and right-hand side of this lens is n_1. The axis of this lens is given is this OQ line and at point O we have a point object.

I repeat again here we are doing all these analysis in paraxial regime where we only consider the rays which are close to the axis of symmetry, which are close to the axis of the system and they make very small angle with the horizontal axis, this symmetry axis. Now, since the point object is situated at point O and then the ray which emanates from this point will travel in this direction and it will fall at this point on the first surface of the lens.

Now, for the time being we will neglect the second surface of the lens and we will only consider that the lens has only the left surface, a single surface. And we will now implement our previous knowledge of image drawing in a single spherical surface case and let us see what do we get.

Now, we are just considering this single surface then this sphere will go like this. This sphere has centre C_1, C_1 is the centre of this sphere and the light after reflection now will go, let us suppose that it goes to point Q. The light is start at point O and then it falls at the first interface of the spherical surface and due to the refraction at this point of incident on the spherical surface the light rays follows this path and it falls at point Q.

Now, this point Q will work as a virtual object first for the second spherical surface with radius of curvature R_2. Now, we will treat point Q as a virtual object for R_2 surface, the second surface and then we will again implement the formula which we derived in our previous classes and then for this virtual object Q of final image I would be formed at this point, this is how we will move ahead.

Now, let us start doing it step by step. The first step is that we will exercise this formula which we derived in our previous classes. Now, object O is situated at a distance x from the lens and image Q is being formed at a distance y'. We know that the distance of object is represented by u therefore, we will write u is equal to minus x and since Q is image which is working as a virtual object for the second interface and instead of writing v we will write v' for the distance of Q from the lens.

Therefore, y' which is the distance of Q from the lens. We will replace this y' with v' and since y' is on the right hand side of the lens system, we will say that it is positive distance and x is on the left hand side of the lens and therefore, we will call x as negative or negative sign is there before x.

Now, we will apply the formula which we derived earlier was $\frac{n_2-n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$. This was the formula which we derived in our previous class. We will use this formula here. In this formula u is minus x which is given here and v is v' which is y', and what is R? R is the radius of curvature for the first interface. And what is radius of curvature of the first interface? The radius of curvature of the first interface is R_1.

Now, after this substitution we will get equation number 56. As I stated before point Q acts as a virtual object for the second refracting surface and the final image is formed at I here at this point. And whose position can be determined by what? We will again use this formula for the second interface treating Q as a object, and what is the distance of object from the lens? It is y' or v'. And then let us see what do we get.

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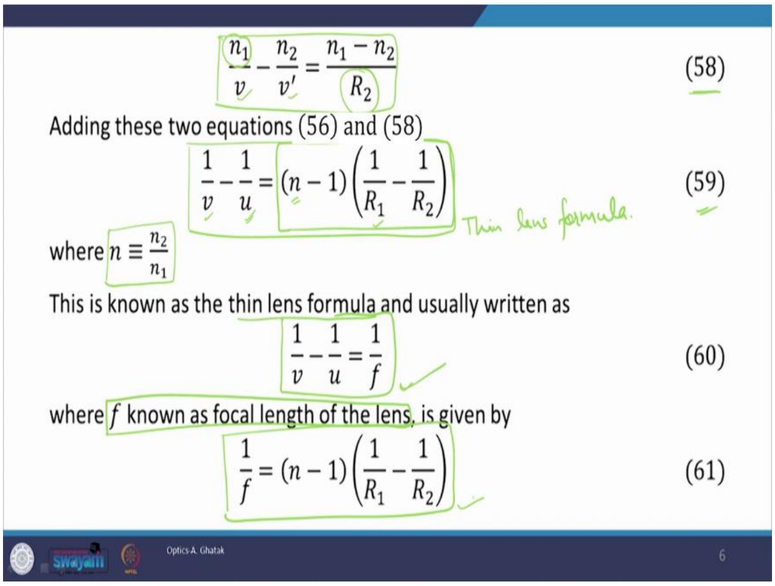

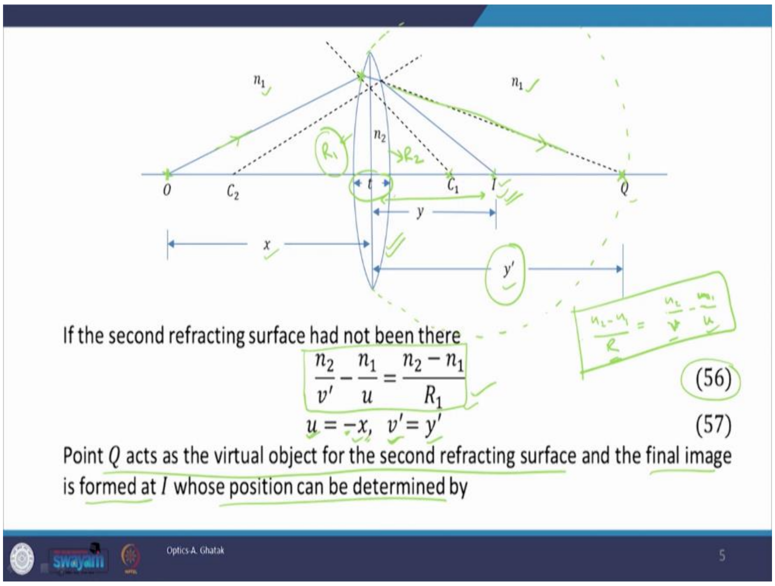

Now, here the same formula is used here, but the parameters are changed. Now, instead of n_2 we are writing n_1 because the second medium has the refractive index n_1. what is v? v is this distance and v' is the distance of the object which is virtual here. And R_2 is the radius of curvature of the second interface.

Second is spherical interface which is this spherical part. And now what to do? Now, we will add equation number 56 and 58. The addition will give this relation and where small n is given by n_2/n_1 it is the ratio of n_2 to n_1, n_2 is the refractive index of the lens material and n_1 is the refractive index after outside medium.

R_1 is the radius of curvature of the left spherical surface and R_2 is the radius of curvature of the right spherical surface. And what are u and v? u is the distance of the object from the lens and v is the distance of the final image I from the lens. This formula is known as thin lens formula.

Now, usually instead of writing this big right-hand side expression what people do is that they replace this right-hand side in equation number 59 by 1/f and therefore, expression number 59 reduces to expression 60 which is this $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$.. This is our final formula where 1/ f is expressed by equation 61 which is nothing but right-hand side of equation 59. And f is known as focal length of the lens. f is known as focal length of the lens.

Now, you see that while measuring the distances we did not take into account the thickness t of the lens. Why? Because t is assumed to be very small and therefore, it can be safely neglected and therefore, it did not appear in our formulation and therefore, we got equation 60 as a final form of lens formula and focal length is given by expression number 61.

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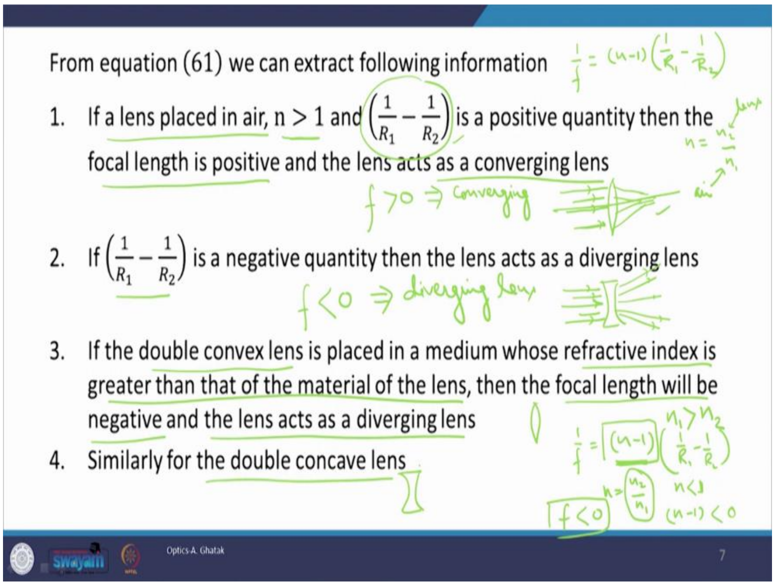

Now, from this final expression 61, we can extract few useful information and what are these pieces of information. Let me write this formula here $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.This is the formula which we derived. Now, if a lens is placed in air, what is n? n is n_2/n_1 and n_1 is the refractive index of air and that is n_2 is refractive index of the lens material.

Now, if the lens is placed in air, therefore, n_2/n_1 will always be larger than 1 and this is what it is written here, small n is larger than 1 and 1 if $\frac{1}{R_1} - \frac{1}{R_2}$ is also a positive quantity if these 2 conditions are met, what are these 2 condition? n is larger than 1 and $\frac{1}{R_1} - \frac{1}{R_2}$ also larger than 0.

If this is positive quantity then the focal length will definitely be positive and if focal length is positive, then the lens will work as a converging lens. If f is greater than 0 then lens will act as a converging lens. We have already talked about converging lens. What is converging lens? If you shine a parallel beam of light or parallel beam of rays on a lens system and if these rays converge to a point on the axis of the system and this lens is called converging lens. The lens system bends the rays towards the axis of the lens and such a lens is called converging lens.

Now, if we have a lens which is kept in the air, but if $\frac{1}{R_1} - \frac{1}{R_2}$ is negative quantity, then f would be less than 0 and if f is less than 0 and such a lens is called diverging lens. What is a diverging lens? If you launch a parallel beam of light, then what will happen? These rays will diverge away. This will not converge to a point on lens axis. These rays will spread away and the lens which does so is called a diverging lens.

Third point is that that if a double convex lens, double convex lens means this lens here if double convex lenses placed in a medium whose refractive index is greater than that of the material of the lens, what does it say? It says that if $n_1 > n_2$, if lenses placed in a medium whose refractive index is larger than that of the material with which our lens is made.

And lenses usually made out of glass and here what we are considering is that the outside mediums refractive index is larger than that of the glass. If this is the case then the focal length will be negative and the lens acts as a diverging lens. Why? Because in this formula on the right hand side we get this relation and here the first term is n$-$ 1. Where n is equal to n_2/n_1.

If $n_1 > n_2$ then n would be less than 1, and if n is less than 1 then n$-$1 would be less than 0 and therefore, f would be less than 0, f would be a negative quantity. It means that the lens which previously was working as a converging lens, a lens which previously was kept in air, which initially was working as a converging lens, if we put this lens in a medium whose refractive index is larger than that of the medium of the lens then the same converging lens would behave as a diverging lens.

It means it is the relative refractive index which plays a major role in deciding whether a lens would be working as a converging lens or a diverging lens. I repeat we have a lens and we put this lens in air and this lens is made up of a glass then of course this lens would work as a converging lens.

But the same lens same structure, if you put this glass lens into a medium whose refractive index is larger than that of the glass then the same double convex lens will behave as a diverging lens. And similarly, the same concept will hold for a double concave lens. Double concave lens means this type of lens. Hope this is understood.

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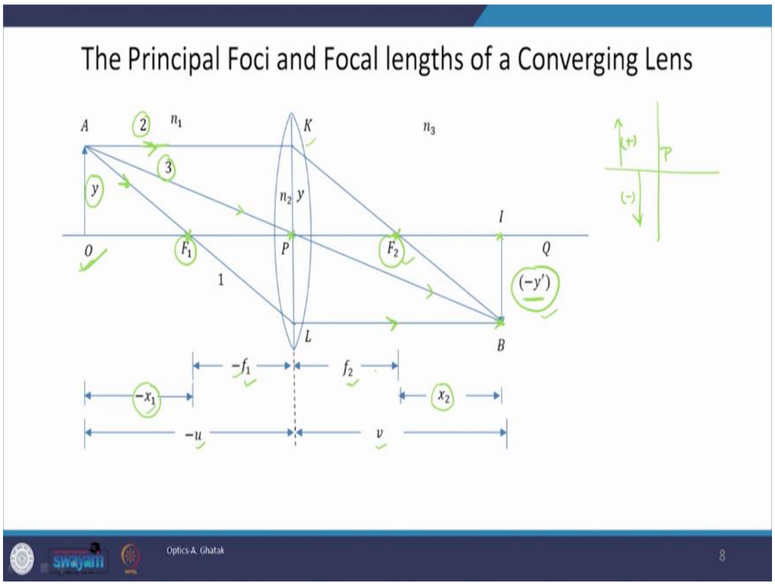

Now, let us move to the definitions of principle foci and focal length of converging lens. Now, lenses given here and an object of height y is kept at a point O on the left hand side of the lens. Till now, we were considering our point object now, we are not considering a point object we are considering a point which has a certain height.

Now, in this particular case the height of the object is AO which is equal to y unit. Now, for this object, the lens images this object and a image is formed here at point I. And you can see in the figure that this image has a certain height which is equal to BI. And since the image is inverted, the object was upright it was above the axis of symmetry.

Now, the image is inverted, it is formed in a downward position. The height of the image therefore, would be measured in minus. You see that the height is minus y'. The height of the object is y' which is positive quantity since it is upright and the height of the image it is minus y'. Why minus? because our sign convention say is that if we have object which is upright then its height would be measured in plus while image is inverted than its height would be measure in minus. We will put a minus sign before the height. Now, to form an image usually what people do that they pass one ray with a point F_1 which is called first principle focus of the lens.

What is the property of first principle focus of a lens? If a ray pass through first principle focus of a lens in case of double convex lens then after refraction after suffering refraction through the lens this ray will be parallel to the axis of the lens. Here you see that this first ray is pass through point F_1 which is first principle focus and then after getting refracted it became parallel to the axis of the lens. Now, apart from this ray to form a image, we also draw a ray which is parallel to the axis of the system, the lens axis in our case. Which is designated by ray 2 you can see 2 is written here the ray 2 is parallel to the axis of the system on object plane and after getting refracted and this point is called second principle focus and it is represented by F_2

And these two array meets at point B, we also pass a third ray which passes through the centre of the lens and it goes undeviated.. If the lens is thin and we are in the paraxial regime, then we may assume that there is no deviation in the ray path, here in this particular ray path. All these three rays meet at point B and then we draw perpendicular from B to the axis of the lens which meets the axis of the lens at point I and this BI is the image of object AO.

Now, this first focal point F_1 is at a distance f_1 from the lens. Second focal point F_2 is at a distance f_2 from the lens and the object from first focal point is supposed to be situated at distance x_1 while the image from second focal point is assumed to be situated at a distance x_2. Now, the distances on the left-hand side of this lens are measured in negative while the distances is on the right-hand side of this lens is measured in past. Therefore, you can see that f_2, x_2 are positive while f_1 and x_1 are negative.

The distance of an object from the lens is u since it is on the left-hand side it is minus u and the distance of the image from the lens is v. Now I repeat f_1 and f_2 are first and second focal length, F_1 and F_2 are principal foci's and x_1 and x_2 are distance of object and image from first focal point and second focal point respectively.

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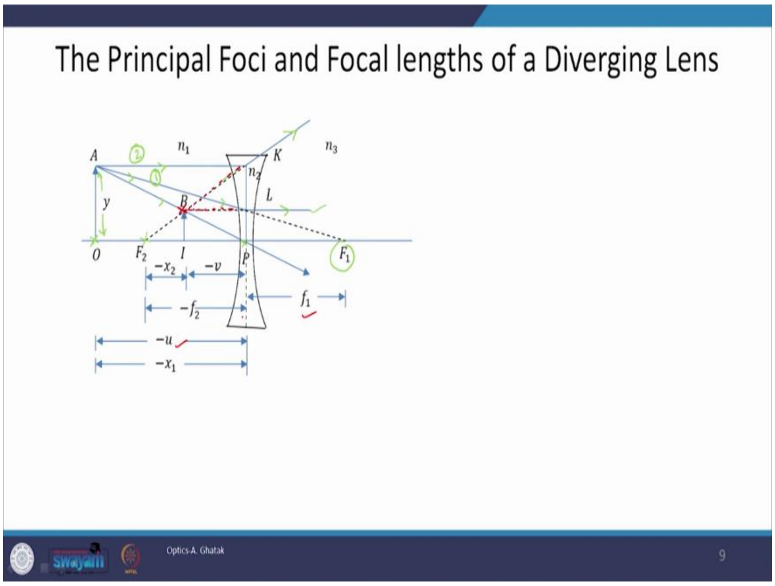

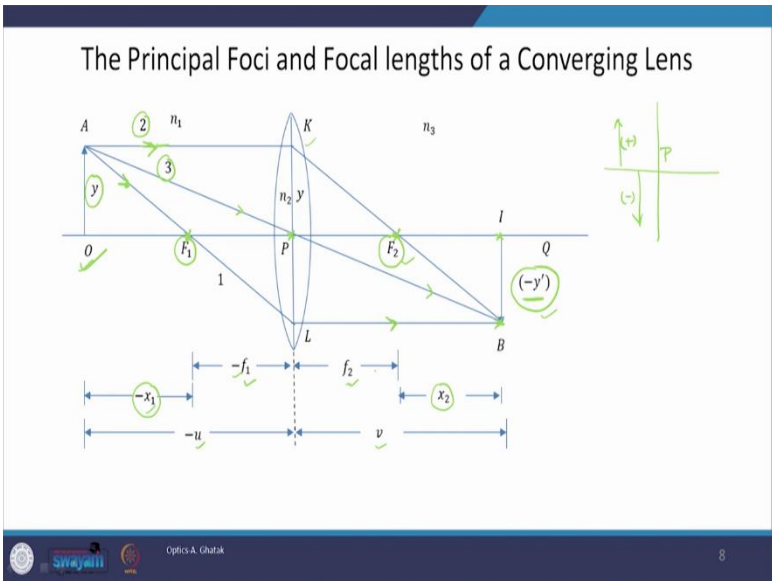

A similar ray diagram can be drawn for a double concave lens which is shown here in this figure. Now, here the object is situated at point O it has a height y. Now, we will again draw three rays. The first ray will pass through the first focal point. The first focal point in double convex lens resides on the right-hand side of the lens which is here. And it will start from point A and will pass through the centre but since the medium of the lens is different here after refraction, it will be parallel to the axis of the system.

This is similar to what we have seen here. A ray which is passing through F_1 becomes parallel to the axis of the lens system. The same concept is also applicable here. A ray which is passing to F_1 but here it is not passing to it which appears to pass through F_1, here we will call a ray which appears to pass through F_1 will become parallel after refraction.

Now, let us again go to the previous figure ray number 2, which is parallel after refraction will pass through the second focal point. Let us, go to the next slide this is the ray number 2, the uppermost ray after refraction will appear to come from second focal point. It appears to coming from because it is getting tilted outward. It is diverging now.

And the third ray which is passing to the centre of this lens it will go and deviated, if we are in the paraxial regime and lens is thin enough then this ray will go and deviated. And then we will see where does these all three rays meet. We see that the ray number 1 and ray number 2 they are not meeting here because they are going in different direction but if you extend them back, let us choose different colour.

If you extend these rays back then you see that all these three meet at point B and then you draw perpendicular from point B to the axis of the system and then you will get IB as the image of object OA. Here too, we can define distances that first focal length and then the second focal length and then the object distance image distance and so on and so forth.

This is how the focal length the first and second principal foci’s are defined for both converging and diverging lens and this is how image formation are done. We usually take three rays. First goes through are appeared to go through F first focal point. The second ray which is parallel to the lens axis, it either goes to the second focal point or appears to emerge out of the second focal point and third ray which pass undeviated through the lens it must pass to the centre of the lens. If the third ray which is pointing towards the centre of the lens it will pass through un-deviated and these three when they cut together and when they cross each other's paths or appears to cross each other’s paths. That point of cross section will define the image, the height of the image in particular ray, this how the image is formed.

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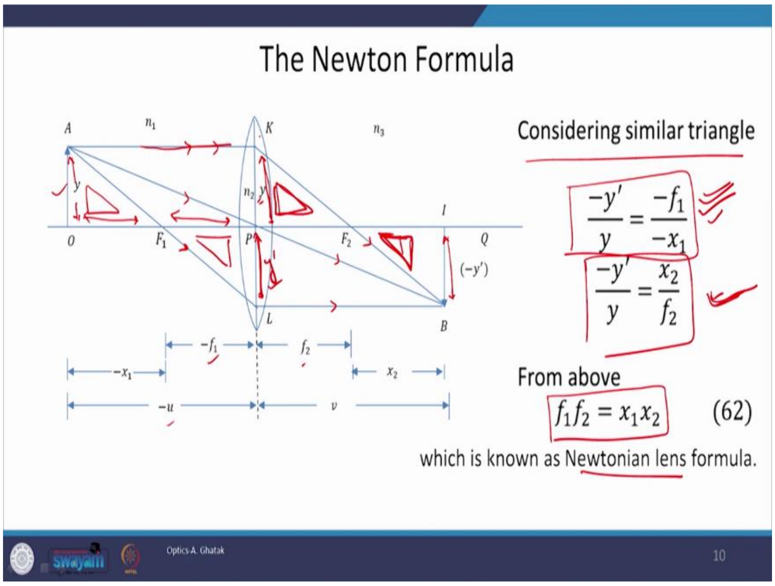

Once we know these things, then considering similar triangles, we can write these 2 formulas: $-\frac{y'}{y} = \frac{-f_1}{-x_1}$. What are these two triangles? Now see here the AO is your object and the BI is your image, these are the three rays which forms this image. Here the height would again be equal to y because the AK, this first parallel this array is parallel to this axis on the system therefore, this would be y here.

Now f_1 and f_1 are the focal length and the different distances are shown here. And then if you consider these two triangles these which are similar triangle and apply the rule then you will get this formula. Similarly, you can get this. This distance would be y' sorry this is y and this is y prime and using the property of similar triangle y'/y this would be equal to this distance by this distance, this is what is given here in this first formula.

Similarly, in this triangle too, we can do this the second two triangles in which we will apply the similar triangle rule to these two, this is the second two triangles and from these two triangles, we will get this expression. In these two expression the left hand side are the same. Therefore, from above we will get $f_1 f_2 = x_1 x_2$. This particular formula is called Newtonian lens formula or Newton formula.

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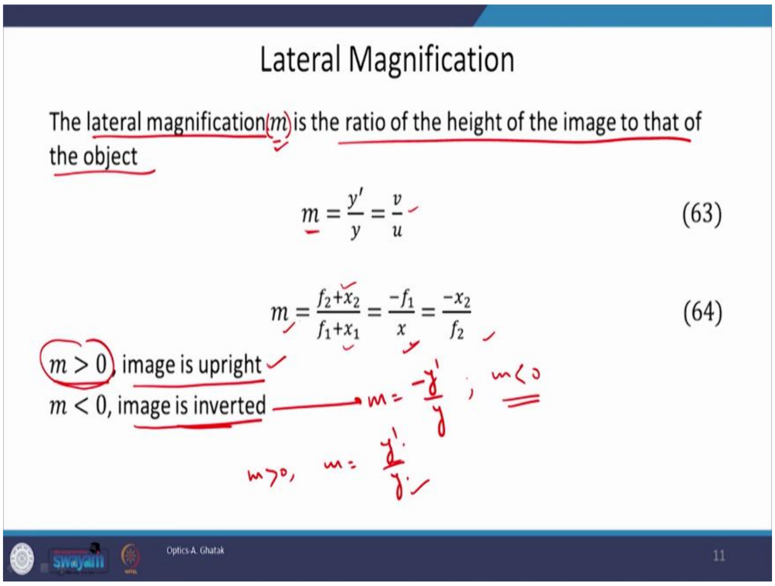

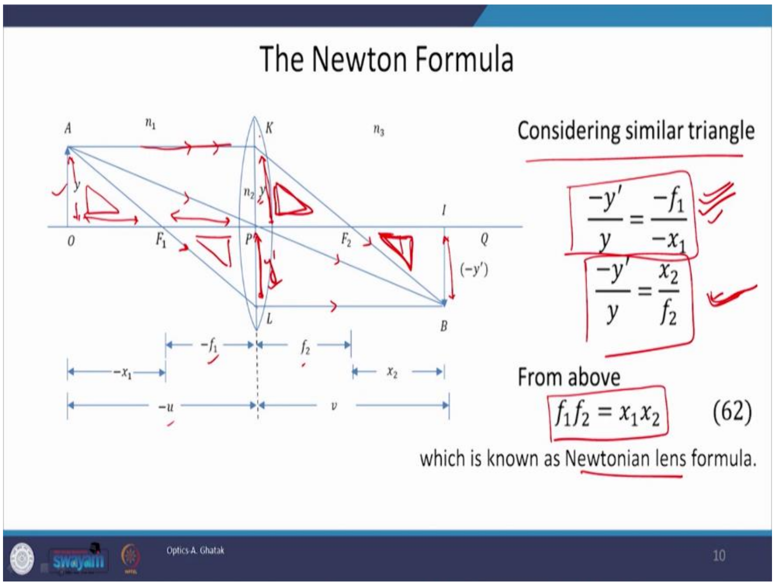

Now, once these things are known, there is one more important definition which is left which is called lateral magnification. Till here, we have talked about the distance of object from the lens, distance of image from the lens, distance of object from the first focal point, distance of image from the second focal point so on and so forth.

But we did not talk about the relation between height of object and height of image that is considered in this definition in lateral magnification. Lateral magnification m, m is the representation for the lateral magnification. The lateral magnification is represented by m and it defines the ratio of the height of the image to that of the object. In this figure in the previous slide, height of the image was y' and height of the object was y. Therefore, y'/y will define lateral magnification and from the geometry you can easily write that y'/y=v/u.

Now, again using the geometry we can write m=$(f_2 + x_2)/(f_1 + x_1)$ and that is again equal to $-f_1/x$ and that is again equal to $-x_2/f_2$. There are these are the different ways of calculating lateral magnification. Now, if m is greater than 0, the image is upright. What it says is that m is equal to y'/y and if you see this figure, then you see that IB is inverted, y' is negative therefore, you put minus y' and therefore, m would be less than 0. It would be a negative quantity.

And if m is less than 0, it clearly says that image is inverted. Now what if m is greater than 0? m is greater than 0 means, m is equal to y'/y and they both are positive, y' is positive as well as y is positive. When will this happen? When both object and image when they both are upright, then only this will happen.

And it means that when m is greater than 0, image is upright. This is very important definitions and it tells the lateral magnification tells how big the image is as compared to the object and whether it is upright or inverted. The magnitude of m tells the relative size of the image with respect to the object and the sign of m, whether m is positive or negative. This will tell you whether the image is upright or it is inverted. And thank you. This is all for today, and we will meet you in the next class. Thank you for your patience.