Applied Optics
Professor Akhilesh Kumar Mishra
Department of Physics
Indian Institute of Technology, Roorkee
Lecture 46
Brewster's Law, Malus' Law

Hello everyone, welcome to the class. Today, we will start the next module which is module number 10. In this module, we will talk about Brewster's law and Malus's law. Thereafter, I will introduce you the phenomenon of double refraction, which is very interesting phenomena and then we will talk about normal and oblique incidences. We will see that what would be the path of the ray, if it is made to incident normally and obliquely in double refracting media.

And at last, we will talk about production of polarized light. Today, in this lecture, we will only talk about Brewster's law and Malus's law.

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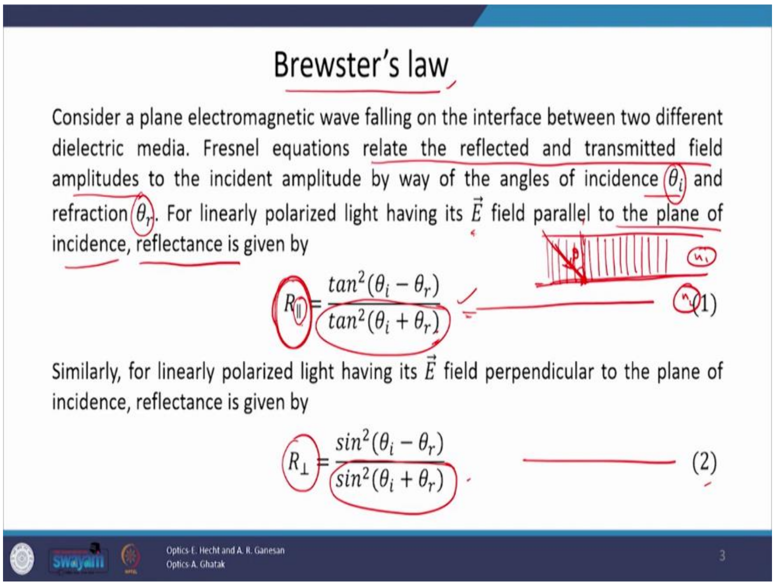

Now, the first topic which we will cover is Brewster's law. For this, consider an electromagnetic wave which is falling on the interface between the two dielectric media. Now, in electromagnetics, we talk about Fresnel equation which governs reflection and refraction in different kinds of dielectric material medium. Now, Fresnel equations relate the reflected and transmitted field amplitudes to the incident amplitude by the way of angles of incidence and refraction.

The angles of incidence is represented by θ_i, while angle of refraction is represented by θ_r. Now, suppose that the light which we made to incident on the interface is linearly polarized and the polarization is such that the field E is parallel to the plane of incidence. Now, if you are confused with the plane of incidence, then we will revisit this concept. Suppose, this is the interface and n_1 and n_2 represents the refractive indices of the two media.

Now, if a light falls on the interface at some angle θ, then the plane which have the incident light, the perpendicular and this point-point of incidence, this plane is called plane of incidence. I repeat, the perpendicular and the ray, they both would be in the plane of incidence. And of course, if the two lines are there in the plane of incidence, the point of incidence would also be there in that plane because using two lines we can generate a plane. The normal to the point and the incident ray, using these two lines we can generate a plane and this plane is called plane of incidence.

Now, if a linearly polarized light with electric field in the plane of incidence is now launched, then following Fresnel relation, we can give an expression of reflectance in such a case and the reflectance for a given polarization is given by equation number 1 which says $R_{\parallel} = tan^2(\theta_i - \theta_r)/tan^2(\theta_i + \theta_r)$, which is a symbol of parallel.

$R_{\parallel}$ represents that polarization is parallel to the plane of incidence. Similarly, suppose we have a linearly polarized light, whose electric field is perpendicular to the plane of incidence. In that particular case, from Fresnel relation, we can again derive the expression for reflectance, for this perpendicular polarization, $R_{\perp}$ and this is given by equation number 2, which reads as $R_{\perp} = sin^2(\theta_i - \theta_r)/sin^2(\theta_i + \theta_r)$.

These two equations are very important. This represent reflectance for a light which is polarized in plane of incidence and reflectance for a light which is polarized perpendicular to the plane of incidence. Now, in equation 1 and 2, you see that these are the expressions for reflectance for a light whose electric field is parallel to the plane of incidence and equation 2 is expression of reflectance for a light whose polarization is perpendicular to the plane of incidence.

Now the denominator in equation 2 is a function of sine. You see in the denominator we have $sin^2(\theta_i + \theta_r)$ and $sin\theta$ varies from 0 to 1. Therefore, $R_{\perp}$ can never be equal to 0, it can never go to 0. While in equation 1, we see that the denominator $tan^2(\theta_i + \theta_r)$. It is a tan, there in the denominator and which can assume ∞ value also.

When $\theta_i + \theta_r = 90$, then tan of 90 which is equal to ∞ and then in that particular case, $R_{\parallel}$ is equal to 0 it means if we launch a light at the interface between two dielectric media at an angle, which is defined in such a way that $\theta_i + \theta_r = 90$, then there would be no reflectance

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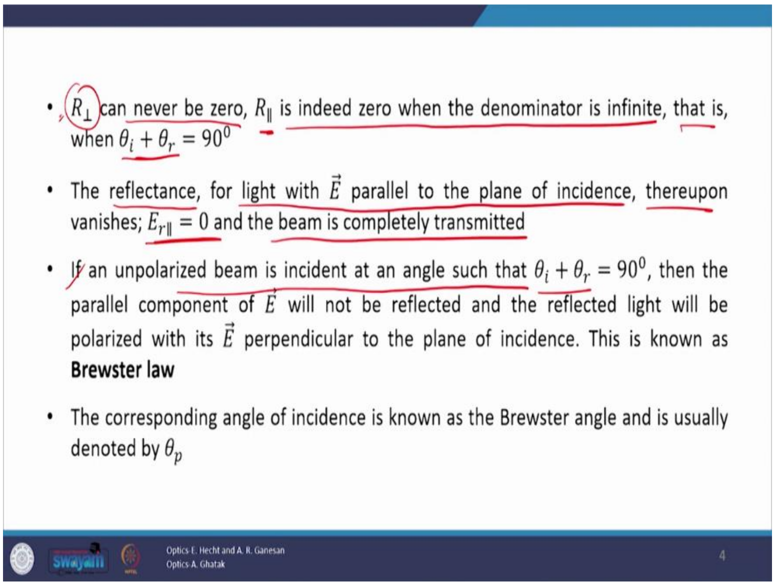

Therefore, $R_\perp$ can never be 0 while $R_\parallel$ is indeed 0, when the denominator is ∞. That is, when $\theta_i + \theta_r = 90$ degree and tan 90 is ∞. Therefore, equation 1 can go to 0, $R_\parallel$ can be 0. The reflectance for light with E parallel to the plane of incidence thereupon vanishes. Because $E_{r\parallel} = 0$ and therefore, the beam is completely transmitted. We are launching a light at a certain angle of incidence and which is having a polarization which is parallel to the plane of incidence. In this particular case, there would be no reflectance. There would not be any reflected light and therefore, what will happen is that all the light will go into the second medium, all the light would be transmitted.

Now, the third point is that if an unpolarized beam is incident at an angle such that $\theta_i + \theta_r = 90$ degree, then the parallel component of E will not be reflected and reflected light would be polarized with its field, E field perpendicular to the plane of incidence and this is known as Brewster's law. I repeat. If we launch unpolarized light at an angle such that $\theta_i + \theta_r = 90$ degree, then in this case, the parallel component of field will not be reflected, it would be transmitted totally and therefore, in the reflected arm, we will have a light whose polarization, whose E field is perpendicular to the plane of incidence and this is exactly the Brewster's law.

And the corresponding angle of incidence is known as Brewster's angle. θ_i, this particular θ_i, it is called Brewster angle and is usually denoted by θ_p.

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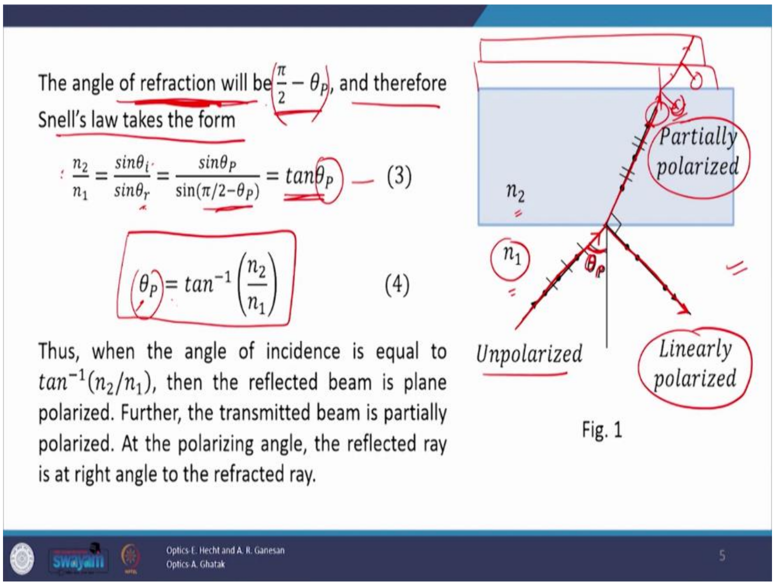

Now, let us apply Snell's law here. Since $\theta_i + \theta_r = 90$ degree and when the Brewster law is satisfied, then $\theta_i = \theta_p$. From there, the angle of refraction would be given by $\pi/2 - \theta_p$. I repeat. In this particular case, the angle of refraction would be $\pi/2 - \theta_p$ and therefore, the snell's law takes the following form, $n_2/n_1 = sin\theta_i/sin\theta_r$, but $sin\theta_r = \pi/2 - \theta_p$, which is nothing but $cos\theta_p$ and therefore, on the right hand side of equation 3, we will have $tan\theta_p$.

And from here, we can calculate the value of θ_p which would be equal to $tan^{-1}(n_2/n_1)$ and θ_p is Brewster angle. Thus, the angle of incidence is equal to $tan^{-1}(n_2/n_1)$. And when the angle of incidence is equal to θ_p, the reflected beam is plane polarized and further the transmitted beam is partially polarized. At the polarizing angle the reflected ray is at right angle to the refracted ray. And this is shown here in this figure schematically.

In this figure, we have two media of refractive indices n_1 and n_2 respectively and an unpolarized light is launched from medium of refractive index n_1 into medium of refractive index, n_2 and at the interface if the angle of incidence is equal to θ_p, the reflected light will have only field component for which the polarization is perpendicular to the plane of the paper, while the transmitted light will be partially polarized, because it will majorly have field components which are parallel to the plane of incidence.

And therefore, if we launched an unpolarized light in the reflected arm, we will have a linearly polarized light, whose field component R, perpendicular to the plane of the paper are perpendicular to the plane of incidence because the fields which are in the plane of incidence are not allowed to get reflected. And therefore, all these field components will reflect in this transmitted arm and this transmitted part or this transmitted light would be partially polarized.

We can again filter the polarization from this transmitted light by putting more layers of material and arranging them in such a way that this light also falls at this θ_p angle, Brewster angle and then there will be multiple reflection and transmission and each reflected light would be linearly polarized and a transmitted light would be filtered, it would get more and more polarized. And this is all about Brewster angle.

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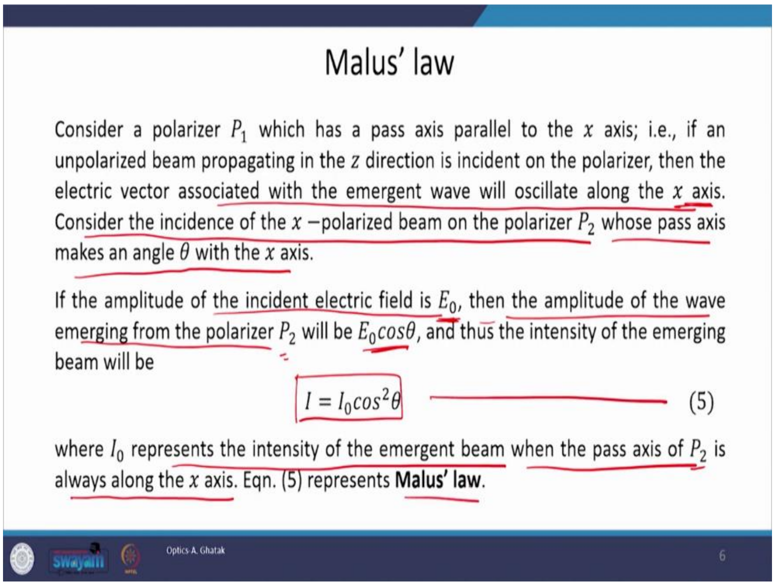

Now, we will start another topic which is law of Malus or Malus's law. Now, suppose, you launch unpolarized light on a polarizer, on a device which is called polarizer. When unpolarized light is made to incident on the polarizer, then what does polarizer do is that it makes this unpolarized light, it converts this unpolarized light into a linearly polarized light, how does it do so? It has some arrangement of polymers which allows only a certain polarization to pass through.

It has pass axis which is oriented in a certain direction. Say, the pass axis is oriented along my fingers. Therefore, it will allow, this particular polarizer will allow only polarizations which are oscillating in a vertical direction. The field component which is oscillating this way, is only allowed to pass through this polarizer. And therefore, if we launch an unpolarized light, then only component which are oscillating along this pass axis of the polarizer, pass axis means the axis along which the light is allowed to pass.

Now, consider a polarizer P which has a pass axis parallel to x axis and say x axis is vertically up and if an unpolarized beam propagating in the z direction, z direction we assume as the direction of propagation, is incident on the polarizer, then the electric vector associated with the emergent wave will oscillate along x axis, that is pass axis of the polarizer. Now, consider the incidence of x polarized on the polarizer P_2, which is the next polarizer, which is coaxially placed with the first polarizer, whose pass axis makes an angle θ with the x axis.

Now, we are considering two polarizers and the pass axis of first polarizer is along x axis while the pass axis of the second polarizer is rotated, it is making an angle θ with the vertical and they are coaxially placed. Now, as I explained before, if we launch an unpolarized light, the polarizer will allow only those field components which are oscillating in vertical direction for this particular polarizer and rest of the field components would be stopped.

And the present case we are considering two polarizers which are rotated by θ degree. Now, if the amplitude of the incident electric field is E_0, then the amplitude of the wave emerging from the polarizer 2 would be $E_0 cos\theta$. And therefore, if you want to calculate the intensity of the emerging band, then this would be given by this relation is equal to $I = I_0 cos^2\theta$, where I_0 represents the intensity of the emergent beam when the pass axis of P_2 is always along x axis. And this equation, equation number 5 is called Malus's law.

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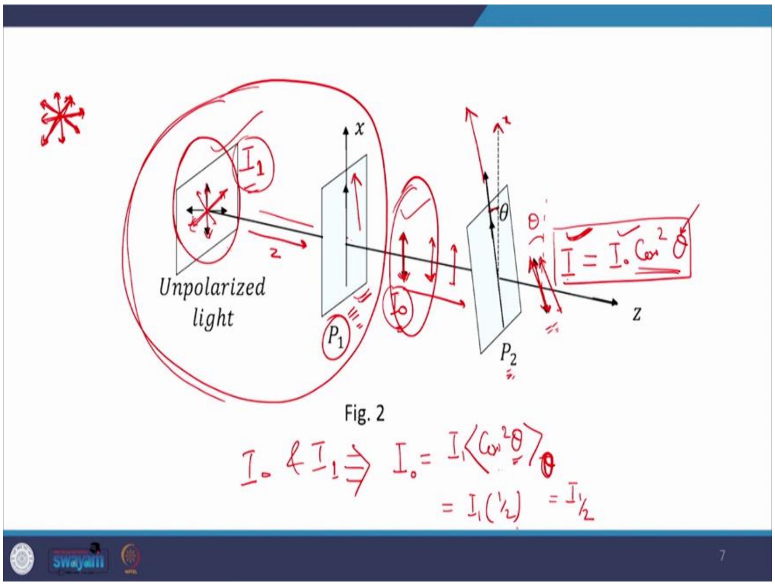

Now, we will try to understand this concept schematically through these figures. Now, we know that in unpolarized light, the electric field can randomly orient itself in any direction and therefore unpolarized light is schematically represented by this star type figure. This is double headed arrows and which represents the direction of oscillation of the field and since the field can orient in any direction, this star shape field symbol is chosen to represent unpolarized light.

Now, suppose this is unpolarized light which is being launched on this polarization P_1 and the pass axis of the polarizer is along x axis and the direction of propagation is along z axis. As I said before the polarizer only allows those fields whose direction of vibration is along the pass axis of the polarizer. Therefore, in this figure the field components which will pass through this polarizer are those which oscillates vertically. Therefore, only this vertical oscillating field will come out from polarizer P_1.

And we see here that we are launching an unpolarized light and after polarizer P_1, we are getting a linearly polarized light because the light which is coming through P_1, it has a field component which is only oscillating in the vertical direction along x axis and this is the property of a linearly polarized light or a plane polarized light. Therefore, this device is called polarizer because it is polarizing the unpolarized light.

Now, suppose, this linearly polarized light, say of intensity I_0 is passed through another polarizer P_2 and say the pass axis of the second polarizer is rotated by angle θ from the x axis**.** Now, this is our x axis and this is now the pass axis of the polarizer P_2. Now, in this case, the light component which are allowed to pass would be those whose field of vibration is tilted by θ degree with respect to the vertical.

The law of Malus's here says, the intensity which we receive here would be equal to $I_0 cos^2\theta$. Now, here if I ask you that if the unpolarized light has intensity I_1, then what would be the relation between I_1 and I_0? Can we find out? Or the reframed question is that, can we find out the relation between intensity or irradiance I_0 and I_1 using Malus's law and the answer is of course, yes. But how?

Because, Malus's law says that if the incident intensity is I_0, then the output intensity I would be related to I_0 through this term, $cos^2\theta$, where θ is the angle of rotation of the polarization. θ is the angle between the polarizer's pass axis but here we do not have two polarizers. But do remember, this θ is not only the angle between the two pass axis of the polarizer.

θ is the angle between the electric field oscillation direction. The electric field before polarizer P_2 is oscillating along x axis, while the electric field after polarizer P_2 is oscillating along an axis which is oriented at angle θ with respect to the vertical direction with respect to x axis. This vertical axis is at angle θ with respect to the vibration of final field. Now here the problem at hand is that we need to develop a relation between I_0 and I_1. I_1 is the intensity of unpolarized light and in unpolarized light, all the orientation of field is allowed, while I_0 is the intensity of light which is linearly polarized and the field in this linearly polarized light is oscillating only in vertical direction along x axis.

Then how to calculate θ here in this case? In this case, we will again implement this Malus's law, which says, $I_0 = I_1 cos^2\theta$, but $cos\theta$ here is the angle between x axis and the direction of oscillation of field in unpolarized light. But we know that in unpolarized light, field changes its direction of oscillation, its orientation is random. Therefore, in unpolarized light, the direction of oscillation is a time dependent function.

Therefore, here we will take time average of $cos^2\theta$. And if you think of this, then time average of $cos^2\theta$ is average of $cos^2\theta$ with θ. Or let me reframe it, the average of $cos^2\theta$, we will take, because things are time dependent in unpolarized light. The electric field is changing in orientation randomly. Therefore, all possible values of θ are allowed. All values of θ are allowed in this case. Therefore, we will have to take average over θ.

Instead of writing time I am now writing θ, we are taking average over θ. And if you take average of $cos^2\theta$ then you will get half. Therefore, the relation between I_0 and I_1 would be this. $I_0 = I_1/2$.

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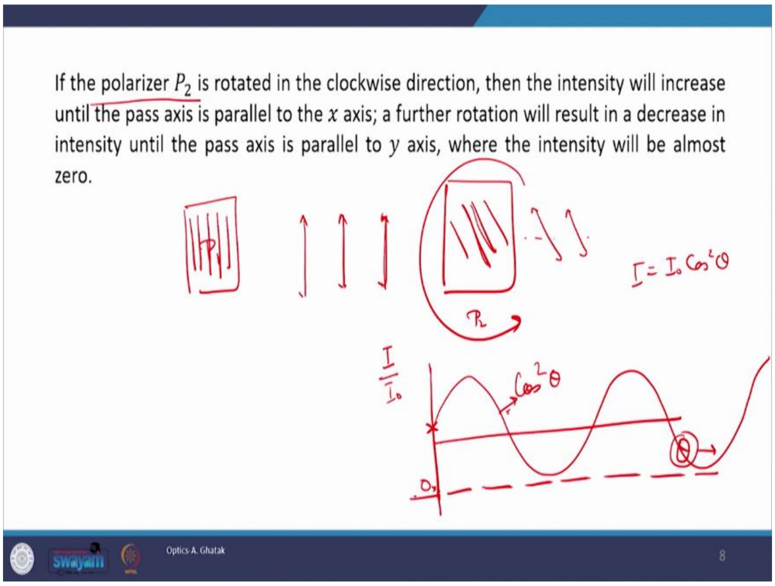

Now, if the polarizer P_2 is now rotated in clockwise direction, then intensity will increase until the pass axis of the polarizer is parallel to x axis. A further rotation will result in a decrease in intensity until the pass axis is parallel to y axis, where the intensity will be almost 0. And this can be understood easily, because after polarizer P_1, this is P_1, we have this linearly polarized light and this is the pass axis of polarizer P_1.

Now this, we have a polarizer P_2 here, whose pass axis is oriented in this direction say and therefore, here at the output we are getting this type of polarization. Now, if you rotate this polarizer P_2 then what will happen is, its pass axis will rotate along with this polarizer and when this pass axis become parallel to the x axis, we will see maximum intensity and this happens twice in 360 rotation. Therefore, at the output if you plot the relative intensity with respect to the angle of rotation, then you will get the sinusoidal type pattern.

The intensity will be function of θ and it will follow the sinusoidal variation because we are plotting effectively $cos^2\theta$ function. And this is not 0. The 0 is somewhere here. This is the 0 because it is a $cos^2\theta$ function. Because $I = I_0 cos^2\theta$. So, this is $cos^2\theta$ function. Now, this start value will depend upon this angle. The relative orientation of the pass axis of the polarizer P_2 and direction of oscillation of the linearly polarized light, which is falling on the polarizer P_2.

Hope this is understood. And this is all for today. Thank you for being with me. See you in the next class.