Applied Optics
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Lecture 43
Circular Obstacle, Zone Plates

Hello everyone, welcome to the class.

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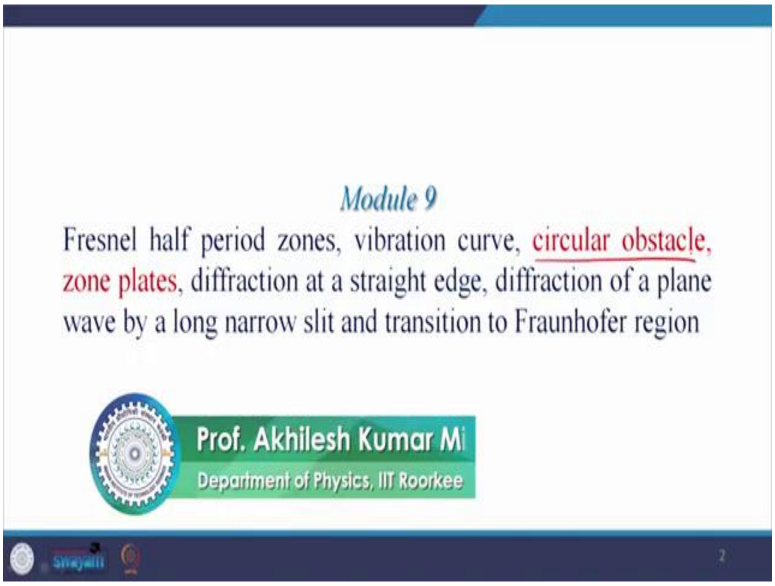

Today we will start a new topic which includes circular obstacle and after that we will talk about zone plates.

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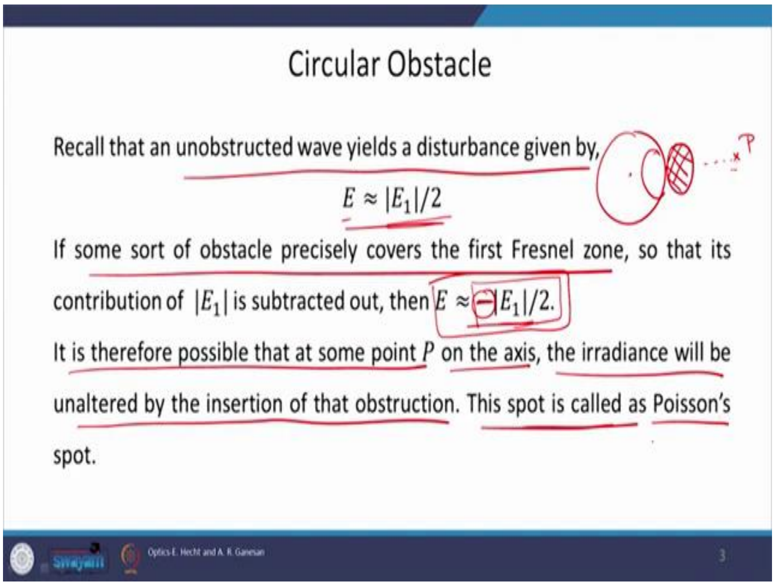

Now moving ahead with what we started in the last class. In the last class we talked about vibration curve, and thereafter, circular aperture was discussed. The pattern due to the circular aperture came out to be circular because of the circular symmetry of the aperture and we saw that there is a concentrating formation. And all this analysis were done graphically using vibrational curve.

Now what will happen if we replace this circular aperture with a circular disk or circular obstacle? Now recall that unobstructed wave yields a disturbance, which is equal to $|E_1|/2$. If we have a point source, which is emitting a spherical wave front and this is a point of observation P, then due to this point source at a point of observation P, we receive a field which is equal to $|E_1|/2$ where E_1 is the contribution from the first Fresnel zone.

Now if some sort of obstacle precisely cover the first Fresnel zone then what will happen? Suppose this is our first Fresnel zone. Now if we cover it with some obstacle, a disk is there which is covering exactly the first Fresnel zone, then what would be the disturbance at point of observation P. Now since the obstacle is exactly covering the first Fresnel zone, then the contribution from the first Fresnel zone, therefore would be deducted.

Now what is the contribution from the first Fresnel zone? Obviously, it is E_1, therefore we will subtract E_1 from $|E_1|/2$, which is the contribution due to overall Fresnel zones. And if you perform this, if you subtract E_1 from $|E_1|/2$, then we will get $|E_1|/2$ with a minus sign. Although there is a change in the resultant disturbance in terms of sign, but irradiance at the point of observation P would be unaffected.

Then effectively what is happening is that initially we have a point source and a point of observation P and then we are observing some intensity distribution at point of observation P here due to this unobstructed point source. Now we are putting an obstacle in between; still the intensity is the same. It is counterintuitive. And it is therefore possible that at some point P, P is movable point and we are picking that particular position of P at which the obstacle is covering first Fresnel zone exactly.

Therefore, at some point P on the axis, the irradiance will be unaltered by the insertion of that obstruction or insertion of this particular disk. This spot is called Poisson's spot. It was Poisson who first proposed this problem; who first observed such a behavior in case of this circular obstacle. This is why it is called Poisson's spot.

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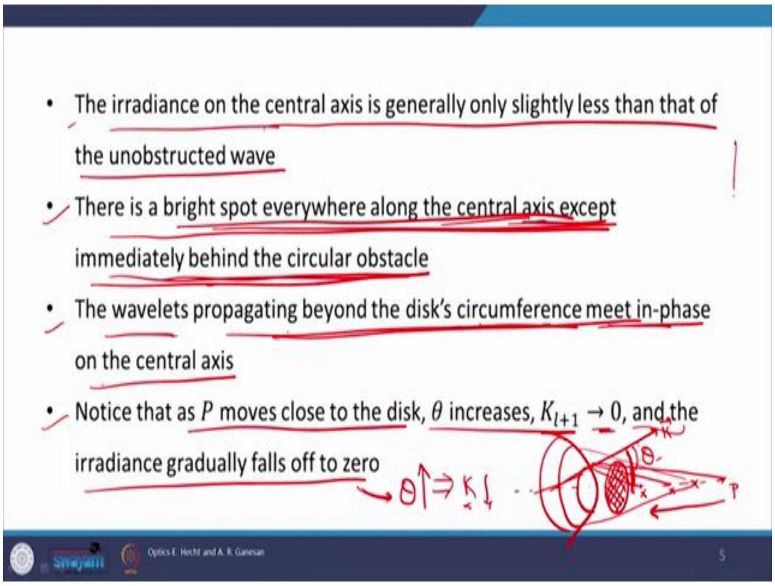

Now let us see what will happen with the insertion of circular disk? Now this is our wave front which is divided in different Fresnel zone. The first Fresnel zone is obstructed and this is our point of observation P. Now, then we get a Poisson spot on the axis. Now if the size of disk is a bit bigger, then it will cover more number of Fresnel zone. And now let us assume that the opaque obstacle or disk is covering first l zones.

The size of this disk is such that the first l zones are not visible, first l zones are not contributing at the point of observation P. Since first l zones are covered, then rest of zones will now contribute; only rest of zones will now contribute at point of observation P. What are the rest of the zones? Zones starting from counting $l+1$ till m, which is the last zone, which is here, which is in circulating point O', covering point O'.

Therefore, we will now calculate the total disturbance at point P due to all this rest of the zone. In this rest of the zone, the first zone would be E_{l+1}, because till E^{th} everything is covered. The second open zone would be E_{l+2} and so on and so forth. Now we know that the adjacent zones are out of phase, therefore we are putting minus sign here.

Now unlike the analysis for circular aperture where we assume that the only initial or lower orders zones are contributing, higher orders are not contributing; here higher orders are contributing while lower orders are not contributing. With this, since the higher orders are contributing and we know that for last zone K(θ) approaches to 0 because here θ=π, you see this angle, this is 180, π degree.

Therefore, last zone does not contribute. The contribution from E_m now approaches to 0 therefore. Then we will repeat the previous procedure of evaluating the series, what we did before and with that we get the resultant disturbance at point of observation P, almost equal to $|E_{l+1}|/2$. When there was no obstacle the resultant disturbance at P was $|E_1|/2$, when l is covered then the first uncovered Fresnel zone…

Now from equation number 36, you see, it is now behaving like first zone. What I mean to say is that when there was no obstacle, the field was $|E_1|/2$. Now if till lth zones are covered we are getting similar expression with only change that E_1 is replaced by E_{l+1}. E_{l+1} is the first unobscured zone, first exposed zone. And if there is not great difference between E_{l+1} and E_1, then this resultant distribution would again be almost equal to this E, which is equal to $|E_1|^2/2$.

This is what is written here the irradiance on the central axis is generally only slightly less than that of the unobstructed wave. Because all the Fresnel zone are of equal area, they are almost equally contributing to the resultant disturbance at point of observation P, therefore, there is a little bit difference, there would be only little bit of difference if you cover a portion of this spherical wave front with some obstacle.

The second point, which is very important, there is a bright spot everywhere along the central axis except immediately behind the circular obstacle, contrary to the case of circular aperture. In case of circular aperture, when we moved along the axis we saw maxima and minima, but here since only few initial Fresnel zones are covered while rest are open, we will see bright spot everywhere along the central axis.

But just behind the obstacle we will see a darkness? Why? Now this is a part of the spherical wave front and these are the Fresnel zone, let us draw the axis here, this is our point of observation P and suppose that this is the obstacle. Now say this is the direction of wave vector k, now if this is the direction of wave vector k. From here we can measure the angles θ for the point of observation P.

If P is here the angle is θ here, if P is say here, then this angle will increase. If P is here then this angle would be even larger. Now you see that as you move P or the point of observation close to the obstacle the θ increases and we know that as θ increases K decreases; the obliquity factor decreases. This is wave vector k and this is obliquity factor K; here do not get confused.

Now obliquity factor now it goes down with increase in θ and this is why, if you are very close to the obstacle there would not be any intensity because θ is very big, almost equal to 90 and therefore, no field would be there, no intensity would be there. If you are exactly behind the circular obstacle, zero intensity, zero irradiance.

Now the wavelets propagating beyond the disk's circumference meet in phase on the central axis. The first part, this part, correspond to this point also, the wavelets propagating beyond this circumference meet in phase on the central axis, whatever wavelets are propagating here, from here and from here, they would, of course, be in phase.

Now notice that as P moves close to the disk θ increases and therefore $K_{l+1} \rightarrow 0$ and the irradiance gradually falls up to 0, this is what is explained here. I try to quickly cover these points but these are very much clear, we have already discussed too much on vibration curve.

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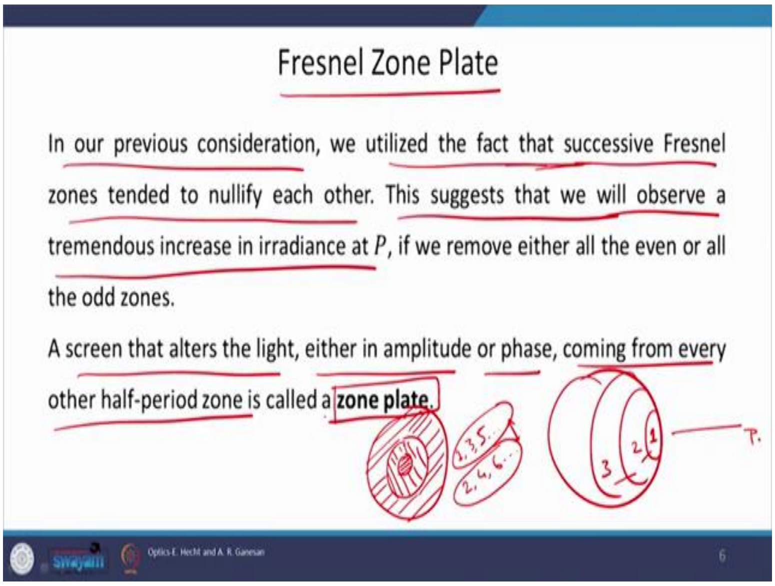

Now the next topic in today's lecture is Fresnel zone plate. Now in our previous consideration we utilize the fact that successive Fresnel zone tended to nullify each other. If you remember this spherical wave front and there we assume that the first nullifies two and two nullifies three because they are out of phase by 180 degree. Now this suggests that we will observe a tremendous increase in irradiance at point of observation P, if we remove either all even or all odd zones.

What it says is that suppose this is the front view of zone, this is your first zone, this is your second zone, this is again your third zone; then from this figure you see that first, third, fifth and odd zones are in phase, while second, fourth, sixth and all this even zones are also in phase, but these odd and even numbers, they are out of phase. Then if we somehow cover either odd number of zone or even number of zone then the total irradiance at the point of observation P, it should increase tremendously, it should heavily increase.

Now therefore, people devised such a screen and a screen that alters the light either in amplitude or phase coming from every other half-period zone is called zone plate. A plate that covers either even number of half-period zone or odd number of half-period zone is called zone plate.

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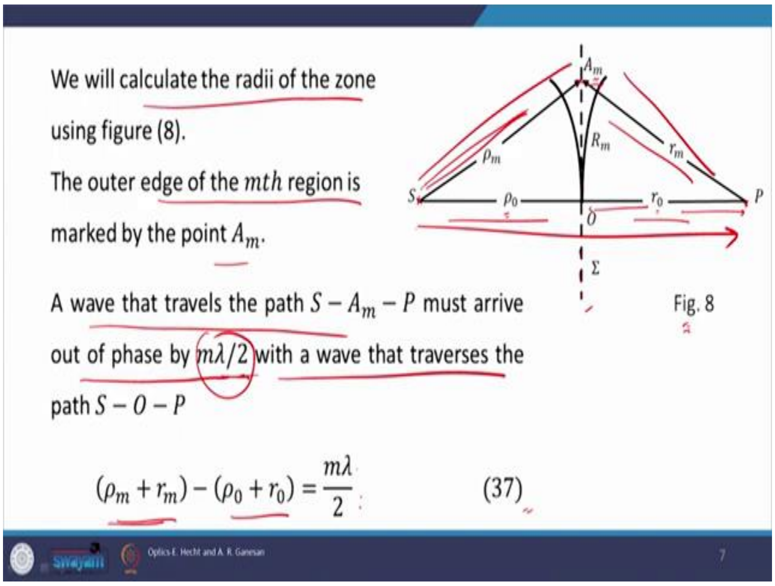

Now let us analyze it, suppose this vertical dash line represent one of such zone plate, a point source S is here which is at a distance ρ_0 from the zone plate and a point of observation P is here on the other side of the zone plate which is at a distance r_0. This distance is r_0. Now we will calculate the radii of zone using this figure.

Now say A_m is the outer edge of the mth region. Now a wave that travels this path $S - A_m - P$ must arrive out of phase by mλ/2 with a wave that travels this straight line path. This is because of the property of this point A_m, because the path length difference would be mλ/2, half integral multiple of the wavelength.

With this let us look on the geometry, let us look on the figure and from figure we can derive this relation here, $(\rho_m + r_m) - (\rho_0 + r_0)$, since this $S - A_m - P$ is a longer distance and this path $S - A_m - P$ is larger than by $S - O - P$ by mλ/2 therefore the difference between these 2 paths must be equal to mλ/2.

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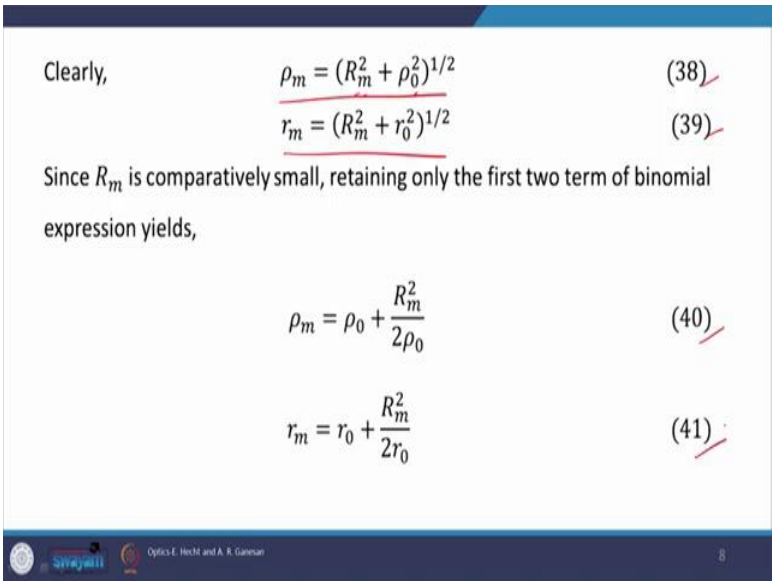

With this from the figure we can also calculate the expression for ρ_m and r_m where r_m is the distance between point A_m and O. Since R_m is small and r_0 is very large we will resort to binomial expression and therefore the equation number 38 and 39 reduces respectively to equation number 40 and 41. Now let us substitute equation number 40 and 41 back to equation number 37.

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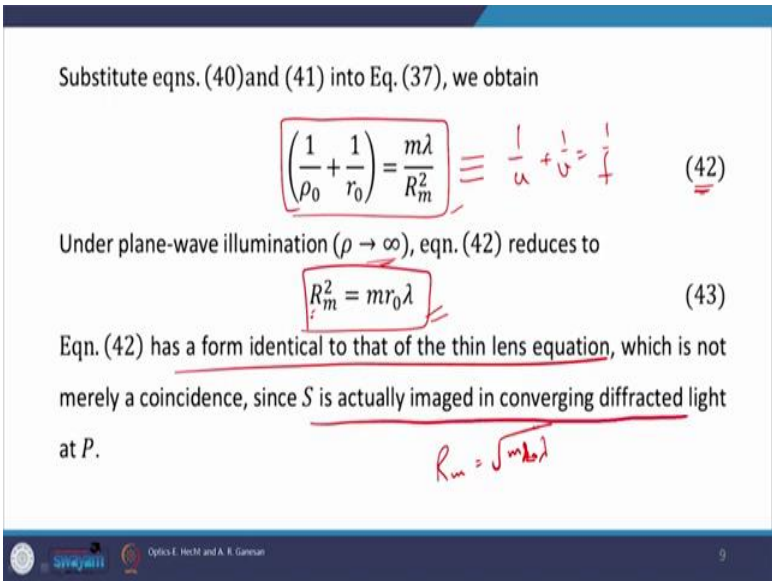

With this substitution we get this expression. Now suppose instead of point source the zone plate is being illuminated by plane wave. For plane wave ρ would be tending to infinity and therefore this equation reduces to this equation $R_m^2 = mr_0\lambda$.

And this tells the radius of the zone plate and this tells that if we want to block the m[th] zone then this radius would be $\sqrt{mr_0\lambda}$.

Now equation number 42 has a form identical to that of a thin lens equation, its $1/u + 1/v = 1/f$. This is the resemblance. This resemble with that relation. And which is not a coincidence, because S is actually imaged in converging diffracted light at P.

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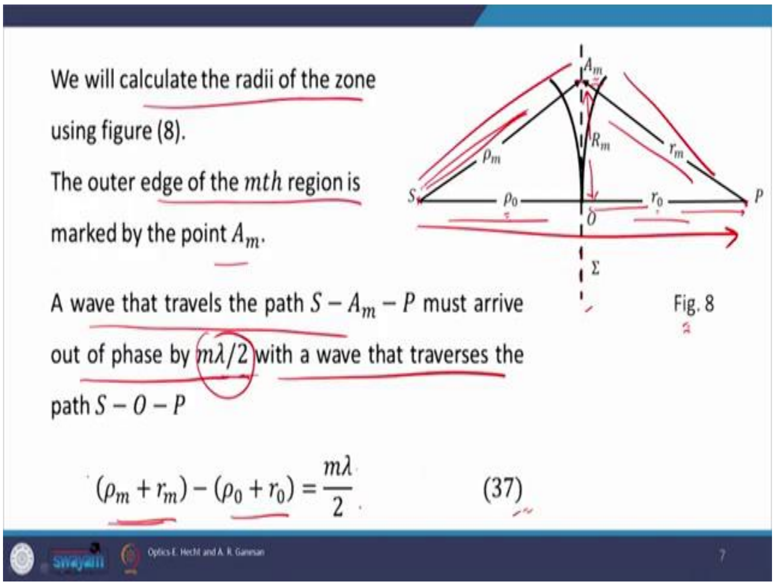

Here this zone plate is actually imaging point source S to the point P, therefore this zone plate is actually working as a lens. A lens which is transmitting at certain part and it is opaque at certain parts.

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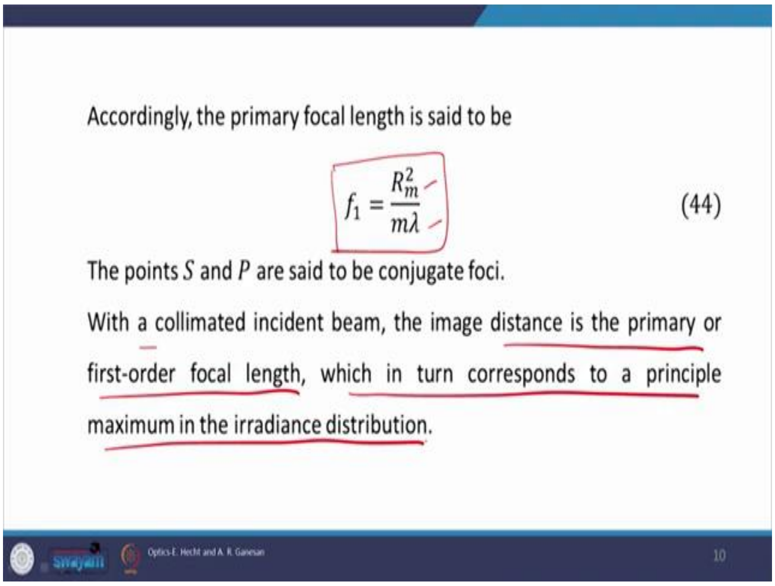

Now with this lens let us study now the property of the lens, let us with this equivalence what we see is that the focal length of this lens would be equal to $R_m^2/m\lambda$, which is coming from this relation $f_1 = R_m^2/m\lambda$. This would be the focal length of this lens which is constructed by this zone plate.

And point S and P are said to be conjugate foci because they will represent the focal point of the lens, left and right focal point of this lens. Now if we launch a collimated beam of light, then what will happen is that all these beams will get imaged to the point P. The image distance is the primary or first order focal length, which in turn correspond to a principal maximum in the irradiance distribution. Because whenever the light focuses maxima get created there, therefore we call it a principal maximum of the irradiance distribution.

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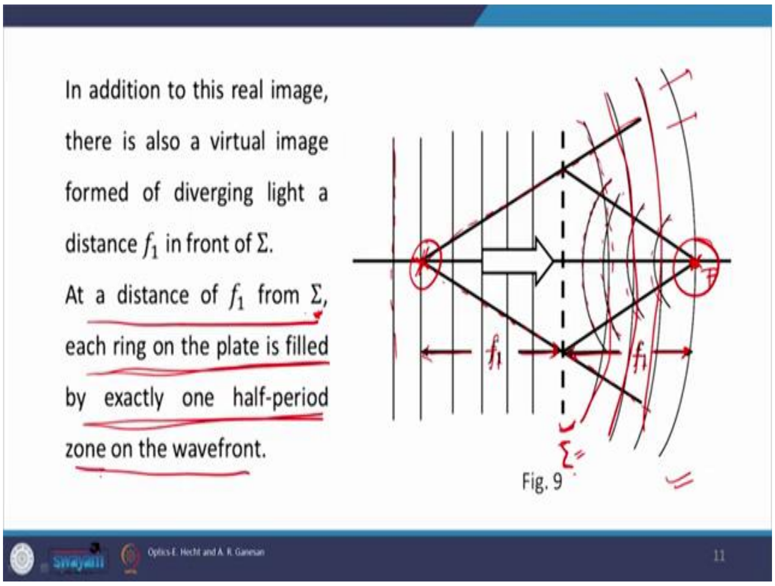

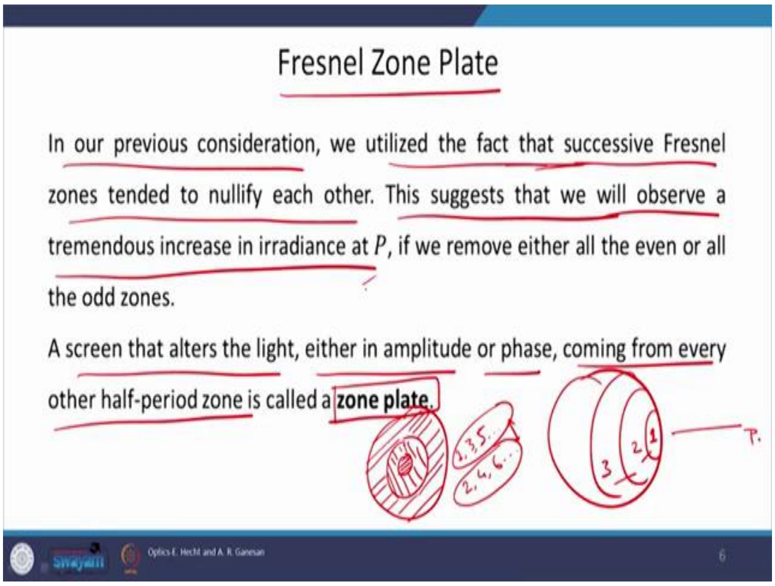

And this is clearly depicted in this figure. This vertical lines represent plane wave which is illuminating this zone plate and a part of the light after passing through this zone plate get focused at this point, which is point of observation P and this distance is called f_1, the focal length. And this get focuses because due to the property of the zone plate the plane wave get converted to a spherical wave, the wave front curvature is modified here.

Now it is, you see that the curvature is modified in such a way that with propagation it converges to point of observation P, but alongside this wave front also produces a different curvature, which is shown here with this line. This is opposite curvature. Now if you extend this back, then you see that it appears that these wave fronts are coming from a point source situated at this point.

And this point is also at a distance f_1 from this zone plate. This is called virtual image and this is called real image. Now at a distance of f_1 from this zone plates, this zone plate plane is designated by capital sigma. Now at a distance f_1 from this zone plate, each ring of the plate is filled by exactly one half-period zone on the wave front.

Now if you look on the zone plate from point P then you see that that each ring on the plate is filled by exactly one half-period zone and which is how the zone plates is defined here in this slide. It says that "A screen that alters the light either in amplitude or phase, coming from every other half-period zone." The alternate zones are now modified. And this happens only if we are at a point P or at the point where the virtual image is being formed. If we are at a f_1 distance then only there is a correct obscuring of the alternate zones.

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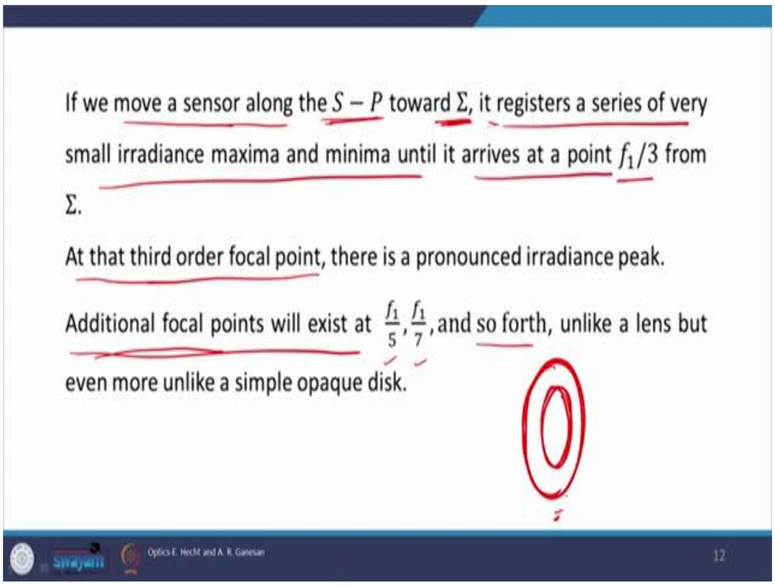

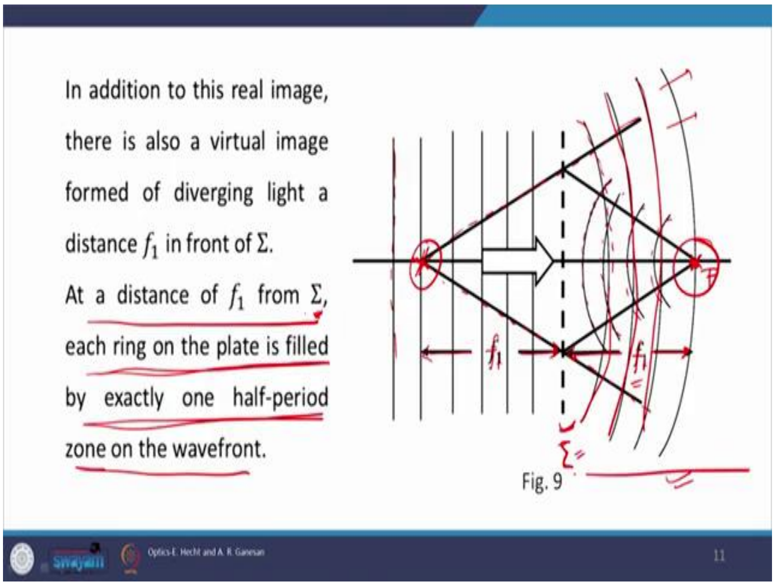

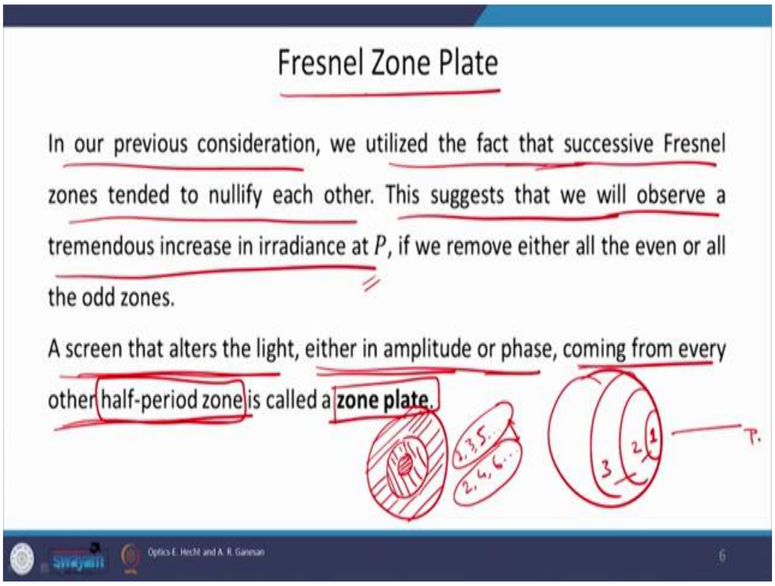

Now if we move the point of observation or the sensor along $S-P$, it means along this line, this is S which is source, now if we move it along $S-P$ towards the plane of the zone plate, then we see that there are several maxima and minima appears, if we move along the axis we see several maxima and minima and therefore the detector are the, our sensor it registers a series of very small irradiance maxima and minima until it arrives at a point which is $f_1/3$ from the zone plate.

And that third order focal point, $f_1/3$ is called third order focal point. At that third order focal point there would be a pronounced irradiance peak, the maxima value there would be the highest. It means that since there are several maxima, therefore it corresponds to several focal points. And therefore, additional focal points will exist at $f_1/5$, $f_1/7$ and so forth, so on and so forth.

It means this Fresnel zones or this Fresnel zone plate, although it seems that they are behaving like a lens, but they are not behaving like a usual lens, because they exhibit several maxima and minima or several focal length. And they are not behaving like a simply opaque disk, but if you want to construct a lens using this Fresnel half zones, then most of the time people use metallic strips.

And these metallic strips has to be somehow suspended in air. Now with this metallic strip and with this metallic strip you can image radiance starting from X-ray to visible to gamma and radio. Therefore, the bandwidth over which this modified lens work is much larger or much wider than visible spectrum lenses. And this is all for today, thank you for joining me. See you in the next class.