Applied Optics
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Lecture: 42
Vibration Curve

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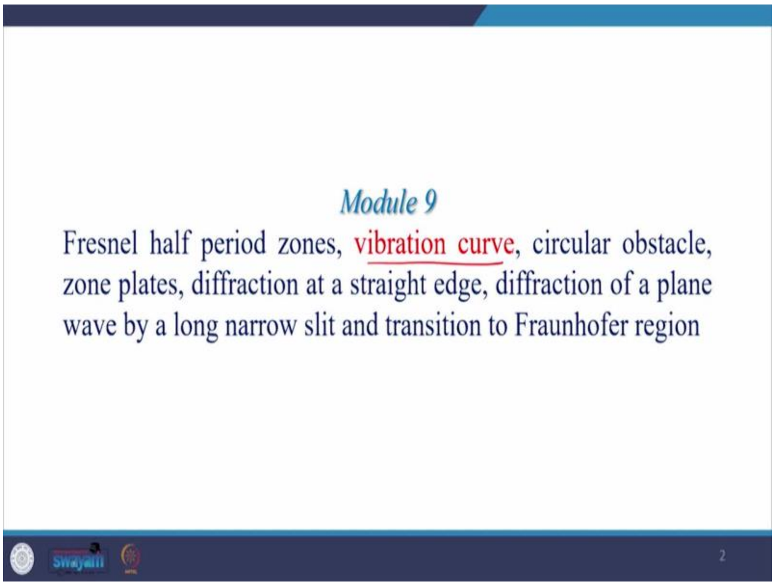

Hello everyone, welcome to my class, today we will start the new topic in Module 9 and this topic is vibration curve, this is a relatively new concept, but before moving ahead, let us revise what we started in the last class. In the last class, we started with Fresnel diffraction, wherein, we try to find the disturbance at point of observation P due to a point source.

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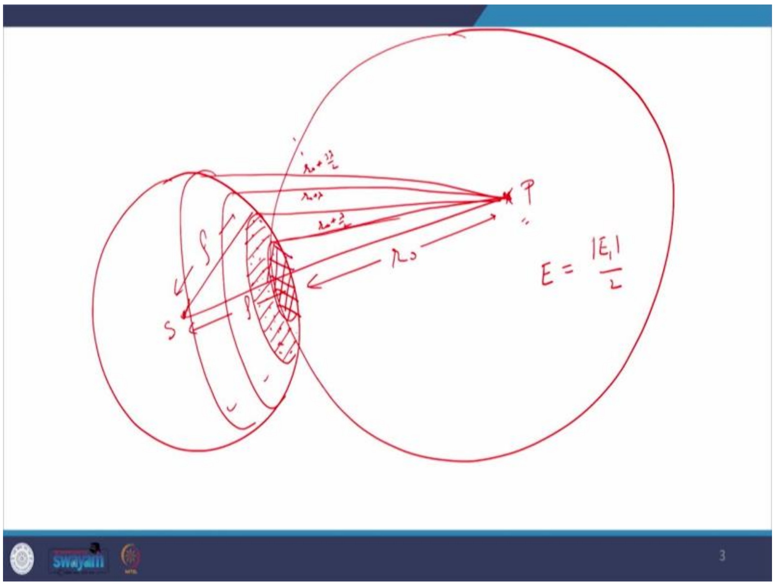

To do this what we did is that we considered a spherical wavefront of radius ρ and then the point of observation P which is at a distance $\rho + r_0$ here we measured the disturbance due to this point source. Now to do this we divided this spherical wavefront into multiple half period zone or Fresnel zone and the distance of each this zone or this annular ring from the point of observation P is such that this one is $r_0 + \lambda/2$, this is $r_0 + \lambda$, this is $r_0 + 3\lambda/2$ and so on and so forth.

Now you can see that the difference between all these distances is $\lambda/2$ the constitutive distances or the adjacent distances are differ by $\lambda/2$ and this annular rings are created by assuming a bigger sphere which is centered at P and these distances which are $r_0 + \lambda/2, r_0 + \lambda, r_0 + 3\lambda/2$, these are the radii of these sphere which is centered at point P and this is how we created all these annular rings. And then we calculated the disturbance at point P due to the point sources which are situated at one of these annular rings or annular region and then we integrated overall these annular regions and this gave us the total disturbance at point of observation P which came out to be $|E_1|/2$, where $|E_1|$ is the contribution from the central or polar region.

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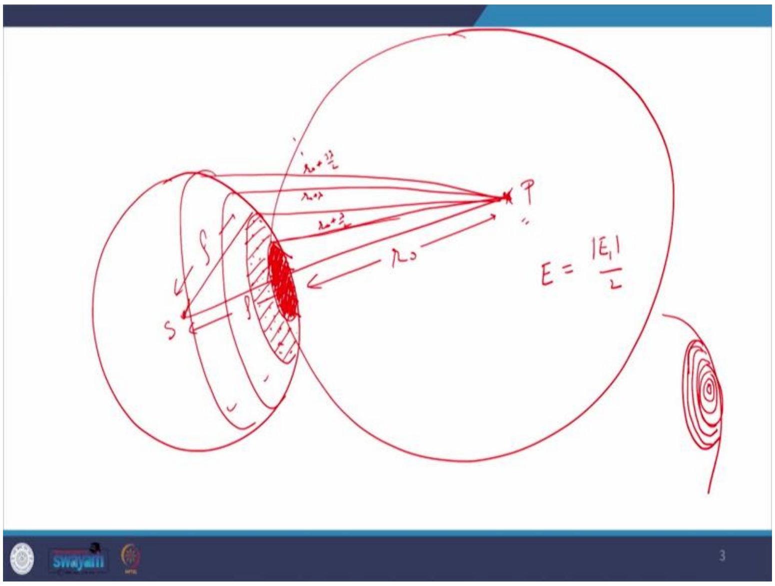

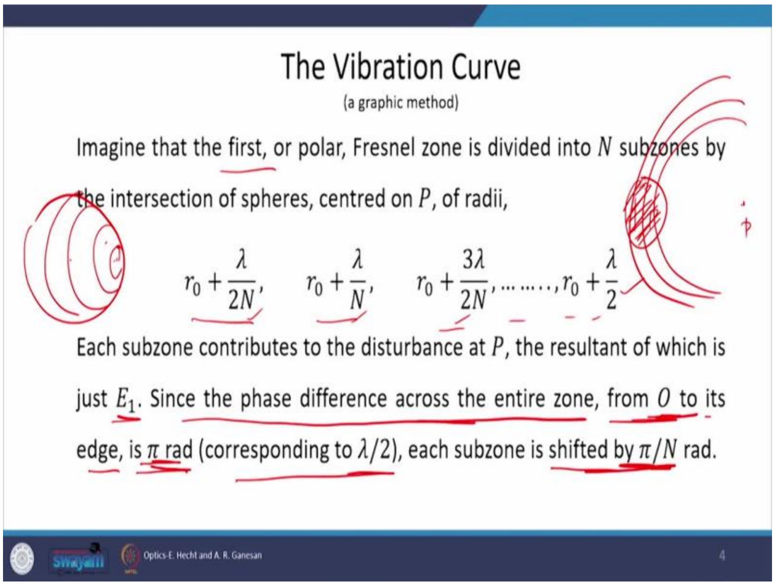

With this we will now study about or we will talk about vibration curve. This is a graphical method where we will study the diffraction pattern of mostly circularly symmetric diffracting element and this is a qualitative method. Now to start with, let us first imagine a polar Fresnel zone, polar means the first Fresnel zone which is shaded here this is called first Fresnel zone or polar Fresnel zone. Now this Fresnel zone is divided into N subzones, how to divide into it N subzones? Suppose this is our biggest sphere and this is the first zone or polar zone then we divide it into smaller strips smaller annular region. And thereby we are creating and more annular regions within the first Fresnel zone.

Now if we create these new regions then the distances of these annular region from the point of observation P are the spheres which are centered at point P will have their respective radii which are equal to $r_0 + \lambda/2N$, then $r_0 + \lambda/N$, and so on and so forth and the biggest radius would be equal to $r_0 + \lambda/2$. We are having one zone and then with centered at P we are now drawing so many spheres and it is cutting this zone into N subzones and this subzones are designated by these radii which are the radii of a sphere centered at point P.

Now each subzone contribute to the disturbance at P because now each subzones will have lot many point sources and these points sources will emit some fields which will reach at P and then the resultant would be calculated at point of observation P. Now we know that, if we take into account all the subzones then we will get a total field which would be equal to E_1 because the first zone is contributing E_1 disturbance at point of observation P and we also know that the adjacent zones are out of phase, they are out of phase by π and since the phase differences across the entire zone from point O to its edge is π radian. Because we know that when we have this original Fresnel zone, then each zone has their own phase and adjacent zone differ by

π. Therefore, if we go from center to the periphery, the phase difference is π radian, which correspond to a path difference of $\lambda/2$.

And therefore, what we can assume is that, each sub annular ring or each subzone which are now created from the first zone each will be sifted by π/N radian, the whole zone has a π radian shift and since we are dividing this one zone into N subzone, therefore, the contribution from each subzone would be π/N radian, each subzone will therefore be shifted by π/N radian.

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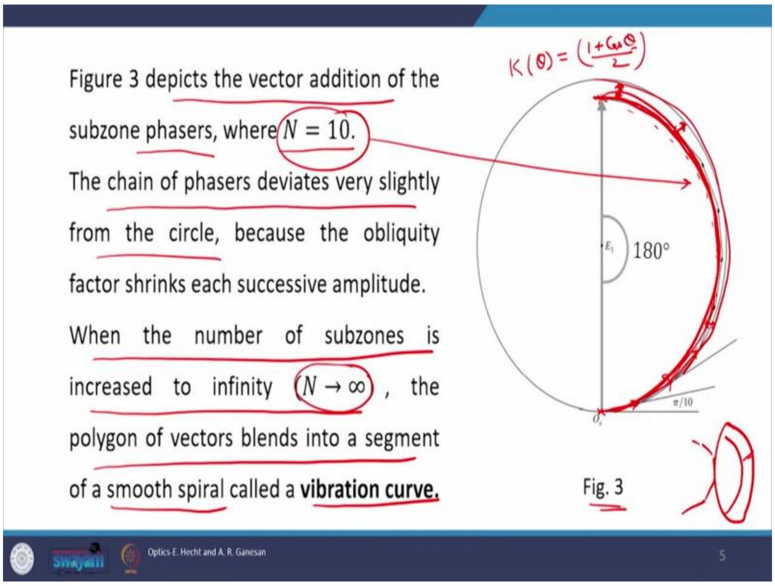

Now in this figure we depict a vector addition of subzones phasers. We know that in each subzone there are points sources and they are contributing some field at point of observation P and this field are vector quantity therefore, we can replace the contribution from each of these subzones by a vector field and therefore, we can adopt the phaser addition. Now say that this first zone is divided into N into 10 subzones therefore, we will have 10 such fields which would be added vectorially at the point of observation P.

Now you can see that in this figure, we start with some point OS and then the contribution from the first subzones is depicted by this arrow, the next contribution is this arrow, the next comes like this and this way we reach at this point. Here we added all the 10 electric field vectors which owes their origin in 10 different subzones. Now you see that the chain of phasers deviates very slightly from the circle, this is the circle and you see that the vector addition give us a curve which is almost looking like a circle but there is a slight deviation. And this deviation owes its origin in obliquity factor, because, we know that obliquity factor is the θ dependent parameter, the obliquity factor $K(\theta) = (1 + cos\theta)/2$.

Now as we move up as we go into a higher subzone, the angle θ will increase and with increase of θ, the $K(\theta)$ will go down. Therefore, the contribution from successive higher number of subzones will reduce down therefore, as you go up in subzone number, the deviation from the circle will increase, here you see the deviation is smaller while the deviation is larger here and even larger here at this point.

And when the number of subzones is increased to infinity here, we assume that the number of subzones is 10 and therefore, you are seeing some discreteness here, but if the number of subzone is increased to infinity, the polygon of vector blends into a segment of a smooth spiral. Therefore, instead of this discrete addition you will get a very smooth spiral, and this is spiral we name it as vibration curve and since it is a graphical therefore this method is called graphical method and this a spiral is called vibration curve which represents the resultant field or the field variation at a point of observation P due to different zones and subzones.

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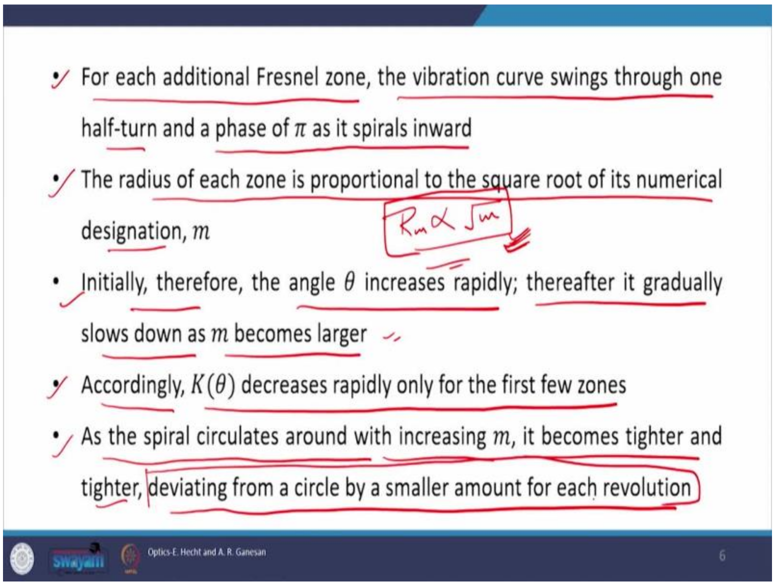

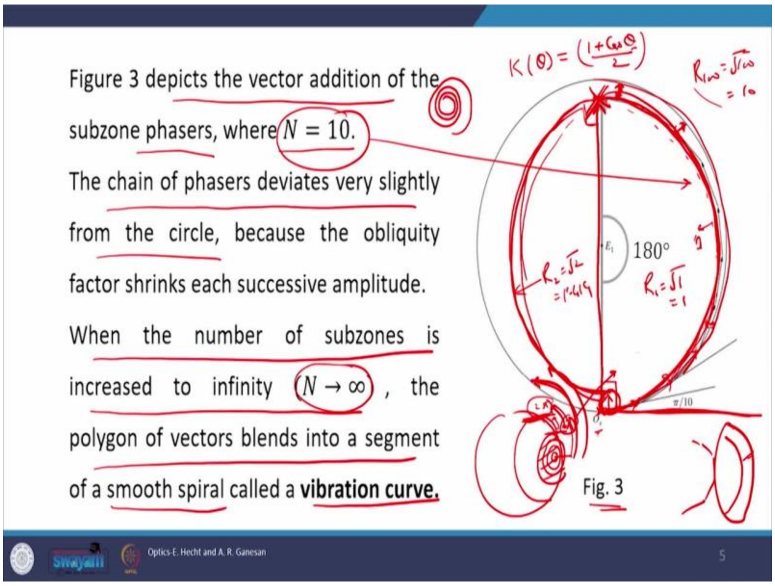

Now we can now derive a few points from this analysis and these points are listed here, the first point is that for each additional Fresnel zone the vibration curve swings through one half turn and a phase of π as it spirals inward. This point can be understood here in this figure, the first subzone is out of phase with respect to the second subzone and if you move from the center to the periphery of the first zone, first zone is this zone here we are talking now about this zone and this is our second zone.

Now the here the phase is π and here this there would be again a π phase difference therefore, phase would be 2π. Now this is our horizontal axis and with respect to this axis we measure angle. Now you see that we divided the first Fresnel zone in subzone and then we added all the contribution and then ultimately we reached to this point, this is our ultimate point which we reached after adding all subzones in first Fresnel zone and this is what said here and this phase is π, it means is that everything is following properly.

Now if we go to next Fresnel zone, the next Fresnel zone is this edge of the next Fresnel zone is again phase shifted by π degree. It means if we go from here to here, the phase difference would be π, but if we go from center to here, that phase difference would be 2π from center to this extremity phase difference is 2π, but from the edge of first Fresnel zone to the edge of second Fresnel zone the phase difference is π. Therefore, if we create a vibration curve for the second Fresnel zone, then we will again get a spiral and it will come to almost the original position, you see that if you include the second Fresnel zone this curve completes full circle it means the rotation is by 360 degree. And therefore, from center the overall deviation is 2π, but if you compare it with respect to the edge of first Fresnel zone, then you again see there is a π phase shift.

Now you see that the vibration curve due to the second Fresnel zone it is not touching the OS point, the point where from the first vibration curves started, why? Because again the obliquity factor is reducing the contributions from the higher order zones as we increase θ the contribution reduces, the contribution from successive zones reduces and this is why there is a difference here, you see a gap here at this point. Now the second point is that the radius of each zone is proportional to the square root of its numerical designation. It means the radius of m^{th} vibration curve would be proportional to $\sqrt{m}$. I will derive it at the end of today's lecture, but for now, assume that this is true.

It says that for the first Fresnel zone the radius would be R_1 would be $\sqrt{1}$, for the second R_2 would be $\sqrt{2}$, for the 100^{th} R_m would be $\sqrt{100}$, which would be equal to 10, here it would be equal to 1, here will be equal to 1.414. It means that as you increase as you move up with the zone number, this curve will shrink, they will come closer and closer as you increase m or as you increase the zone number. Therefore, initially the angle θ increases rapidly and thereafter it gradually slows down as m becomes larger, which is very much obvious because of this relation this is not a linear relation it is a parabolic relation.

Now the 4^{th} point is accordingly K(θ) decreases rapidly only for first few zones, for first few zones K(θ) decreases rapidly but if we go to the higher order zones then the reduction in K(θ) is smaller, the rate gets slowed down. And the last point is that as the spiral circulates around with increasing m it becomes tighter and tighter and this is what I said a minute before, deviating from a circle by a smaller amount for each revolution if you increase m, the circulations become tighter. Initially it started from here and then it goes like this and then you see a denser vibration curve here around the center. And the one point which is to be noted is that the deviation from the ideal circle becomes less if you go higher in m, the variation will be more close to a circle they would be almost mimicking a circular pattern.

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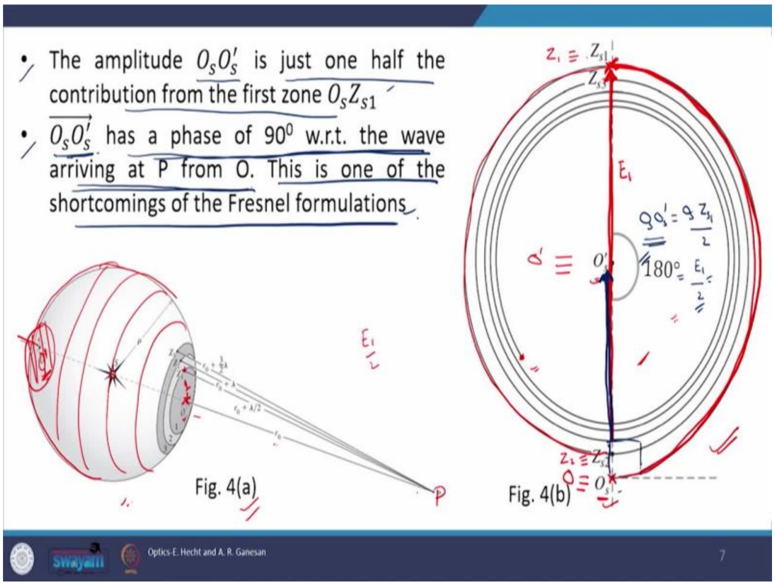

Now with this introduction, let us see how do this vibration curve look like and this is a schematic diagram which shows the features of vibration curve. You see here that OS is a point and OS is nothing but it correspond to point O which we consider that the center of the first Fresnel zone, O is the center of first Fresnel zone, P is the point of observation and r_0 is the separation between O and P and this is the spherical wavefront which is generated from point source S. Now if you extend the line joining O and P then it will come out of this sphere from point O', do make it a point, point O' is sitting exactly behind point O, the O and O' are on the same diagonal, they are diagonally opposite points.

Now the O corresponds to O_s in the vibration curve. Similarly, O' corresponds to O_s', O' correspond to O_s' and O correspond to O_s in vibration curve. The first zone is this outer line outer curve and phase is π, the second zone is this. Now Z_{s1}, this point on the first Fresnel zone the extremity here where the first vibration curve in, this is sitting here, the Z_1 on the Fresnel zone correspond to Z_{s1} here. Similarly point Z_2 correspond to Z_{s2} here and so on and so forth.

Now if there is no obstruction in the last lecture, we calculated that a point source will contribute a total field which would be equal to $E_1/2$ at the point of observation P and what is E_1? E_1 is the contribution from the first zone and which is clearly depicted in the vibration curve 2. See that contribution from the first zone is this vector, if you draw a vector then this is the contribution from the first zone, the first zone starts from here we are adding up all the vectors and then we are ultimately getting to point Z_{s1} and the resultant the vector addition would be O_{s1}, Z_{s1}, this line would be the resultant of all this vector addition, this is nothing but

E_1, this is the contribution from the first Fresnel zone. And in this figure O_{s1} and O_s' represents O and O' of the actual figure.

Now if you keep drawing the zones then the last zones, say m^{th} zones, is covering O' point, O' would be center of the last zone, say the last zone is m^{th} zone. Now we know that K (θ) is 0 for the last zone, because K is $(1 + cos\theta)/2$ and θ is π degree there and therefore, the last zone will not contribute to the vibration curve and therefore, the ending point of a vibration curve will never touch O_s' here in this figure since the last zone is not contributing. Therefore, before reaching O_s' the vibration curve must end and this is what exactly is happening here it is not touching O_s'.

Now if we want to calculate the total disturbance at point P due to all zones which is shown in figure 4(a) then this would be equal to this vector, you will have to just join O_s with O_s' , this vector is the resultant due to the point source situated at certain distance from point P. Now you can see from this figure that this blue vector is just half of the red vector, what I mean to say is that O_s O_s'= O_s $Z_{s1}/2$ which is nothing, it means the resultant which is O_s , O_s' the resultant would be O_s $Z_{s1}/2$ and O_s $Z_{s1} = E_1$ is even, the field the contribution from the first zone by 2 and this is what we derived in our last class.

You can see now there is one to one correspondence, for deriving this we had to resort to a very complex mathematics which involve lot many terms. And now using this graphical method you can clearly see that you can predict the resultant field at a point of observation P just by looking at these figures. Now the 2 important points which can be derived through this analysis are listed here, the amplitude O_sO_s' is just one half the contribution from the first zone which is very much clear and well explained and O_sO_s' has a phase of 90 degree with respect to the wave arriving at P from O, which is also clear this is 90 degree.

But, in the last class, we saw that if you write the expression of disturbance at point of observation P due to a point source which is at a certain distance from P then we get some amplitude and then $sin\omega t$ and some phase part, but if we do it using a Fresnel way, then we found that instead of getting $sin\theta$ we were getting $cos\theta$, cos into some phase term. And there we said that if we assume that the secondary wavelets are out of phase by 90 degree with respect to the primary one then there is a one to one correspondence.

But here too what you see is that the O_sO_s' has a phase of 90 degree with respect to the wave arriving at P from O and this 90 degree here. And but this is not exactly the case when you do it directly and this is one of the shortcomings of the Fresnel formulation that introduces a phase shift of 90 degree between primary and secondary waves.

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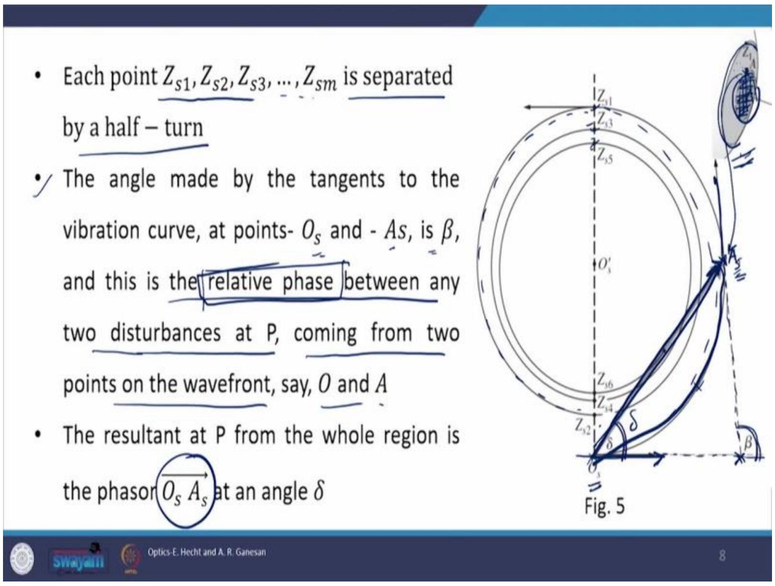

Now each points Z_{s1}, Z_{s2}, Z_{s3} and Z_{sm}, which are shown here these points is separated by half turn, which is very much clearer you have to just move one half turn here you reach Z_{s1}, then again move half turn here you reached at to Z_{s2}, keep rotating by half turn and you reach next Z_{s1} and there is also a phase shift of 90 degree between adjacent vibration curves. Now the angle made by the tangent to the vibration curve at point OS and AS is β here, there is a point OS and if you draw a tangent here, then this tangent will be directed in this direction and say there is some point on the vibration curve and you draw a tangent at this point, then if you extend it by then you see that these two tangent meet here and the angle between them is β here and this is the relative phase between any two disturbances at P. And these disturbances are coming from two points on the wavefront, say, O and A.

Let us try to understand in a deeper way. Now this is a part of the spherical wavefront which we considered here in this figure, I just picked this part and then put it in this here. Now O_s is the point which is joining with P and Z_1 is at the periphery of the first Fresnel zone. Now in this Fresnel zone we randomly pick a point A, and with A chosen the question is what would be the contribution of this shaded part at in the disturbance which is observed at point of observation P, the contribution due to this shaded part at point of observation P, this is the question.

Now to answer this first we will have to draw this point on the vibration curve and say A is mapped here we just mapped this A on the vibration curve and A is mapped at this point which is A_s here. Once A_s is mapped then we draw a tangent at A_s and say the origin O_s, if we want to measure things with respect to origin then only we will have to consider origin, if there are some other points with respect to which we want to perform the measurement then we can pick any other point on this vibration curve accordingly.

But the crux of the matter is that to calculate the relative phase difference between the two we will have to draw a tangent at those two chosen points and once you draw the tangent these tangent will meet somewhere and from there you can calculate the angle between the two. And this angle will tell the relative phase between the 2 points between any 2 disturbance which is reaching a point of observation P. In this case the 2 points is O_s and A_s, here we drew 2 tangents and these tangents are crossing here and here the angle is β and this beta tells the relative phase difference or the phase difference between point O_s and point A.

Now if phase is known what would be the magnitude of the disturbance which is observed at point of observation P? The resultant at P from whole region, this all shaded region, would be equal to this vector just join O_s and A_s and the length of this vector will give you the resultant contribution from this whole region, the contribution from this whole region will be given by this vector, $\overrightarrow{O_sA_s}$, this vector we call it a phasor, and this phasor is oriented at angle δ this angle is here.

It means that once we have this vibration curve just by looking at this vibration curve we can calculate the disturbance at point of observation P, we can calculate the phase, we will have to just image the corresponding point from the Fresnel zone to this vibration curve and pick some point in the Fresnel zone and then image it here and then calculate the phase between the two, calculate directly the amplitude the resultant at P from the whole region, it will be quite easy.

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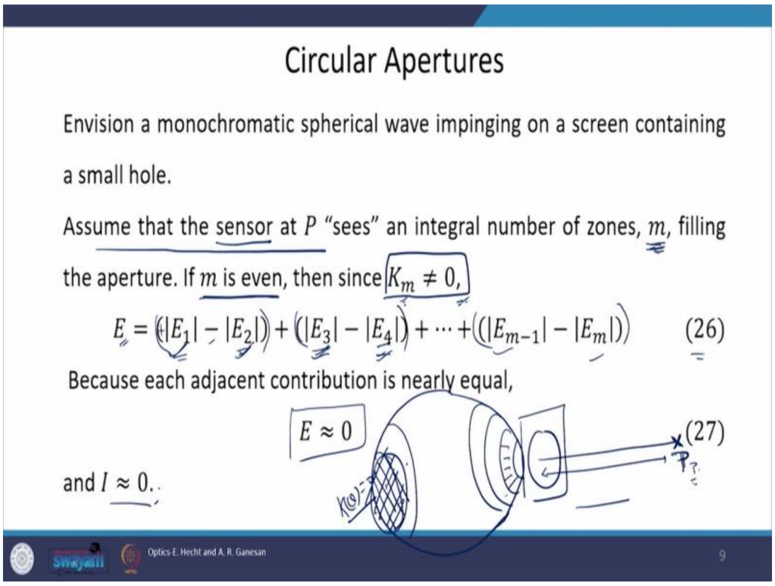

Now with this information at hand we will implement this in circular apertures. Till now, we were just considering point sources which were unobstructed and then we were measuring the field or we were measuring the disturbance at point of observation P. Now we will implement this knowledge in case of circular aperture, for this in vision that we have a monochromatic spherical wave which is impinging on screen and which has very small circular aperture or circular hole. And assume that the sensor at P, instead of detector we are calling now sensor, the sensor at P sees an integral number of zones, this is just an assumption.

We have a screen and in this screen we have a circular aperture or circular hole and the point of observation P is here and the camera here is or the sensor here is seeing integer number of zones because there is a point source here and it is emitting a spherical wavefront, and this spherical wave fronts are divided into lot many Fresnel's zones or annular rings and this annular zones are being observed from point of observation P. Now the point of observation P is situated such that it is saying integer number of zones and assume that this integer is equal to m.

Now there are 2 cases, the first case is when m is even. Now in this particular case since we are seeing a part of zones we are not seeing all zones, we are seeing a part of zones, because whole circle is like this, the whole wavefront is a spherical and the part of the wave fronts are visible from the point of observation P because aperture is only allowing a part of this wavefront letting through this opening and the rest of them are getting blocked. Therefore, the majority of course, the last zones are not visible they are not passing through this circular aperture.

Therefore, the obliquity factor for the last zone which is visible from point of observation P would not be equal to 0, because last zone in our case would be m, which are the maximum value of zone which is getting through this circular aperture. Initial m zones are only allowed to pass through this circular aperture and for this initial m zones do not include the last zones which are here in this spherical wavefront. And because for this only zone K (θ)=0 therefore, for zones which are visible from point of observation P, K_m would not be equal to 0. With this, we can for the visible zones, we can calculate the resultant disturbance at the point of observation P, for this we will add up all the contributions from visible zones.

Now we will add up the contributions from all m visible zones, this is how we added them up. Now this is also clear to us that adjacent zones are out of phase by π degree therefore, we have a positive sign before E_1, while negative sign before E_2 , positive sign before E_3, while negative sign before E_4, and we clubbed them in pairs. And in addition, we also know that these zones are adjacent and they contribute nearly equally therefore, E_1 would be almost equal to E_2, similarly E_3 would be almost equal to E_4 and therefore, we can clearly see that the resultant E would be almost equal to 0, we will not see anything at the point of observation P, if the circular aperture allows only even number of zones. And therefore, the resultant intensity would also be equal to 0.

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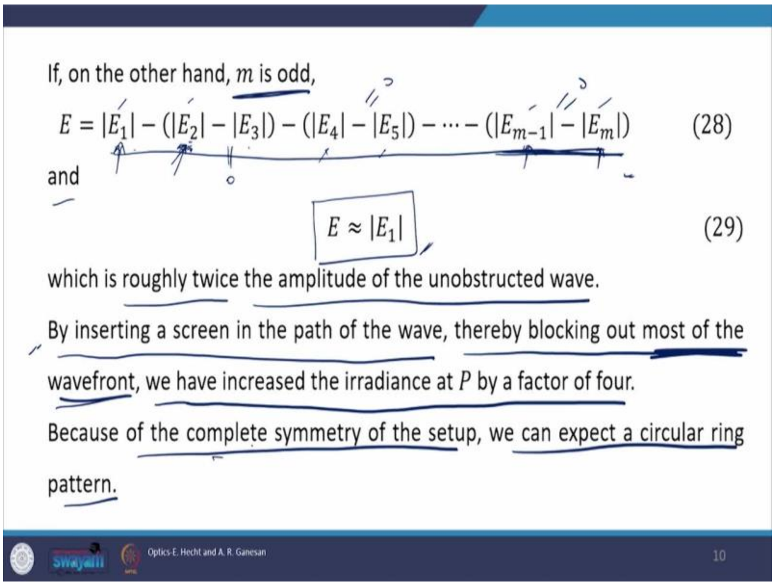

Now let us go to the next case where m is odd. Now here again we are doing the same thing, but now the clubbing is a started from the second contribution the E_2, E_2 and E_3 are here clubbed, E_4 and E_5 are clubbed and last two zones are clubbed here. Why did we that with E_2, E_1 and E_2 because as you increase θ the contributions from the successive zone reduces.

Therefore, the contribution from the last zones would be smallest and the contributions from the first two zones it would be biggest but the difference between contributions from the initial zones would also be very different, the difference between E_1 and E_2 would be larger than the difference between the last two contributions and therefore the clubbing is done this way. With this too, this term would be equal to 0, this term would be equal to 0, this term would be equal to 0, and here we get a nonzero resultant which is equal to $|E_1|$ that is equal to the contribution from the first zone.

Now this is roughly twice the amplitude of the unobstructed wave. In the last class, what we saw is that we have a point source and here it is the point of observation P, the point source is contributing a disturbance at a point of observation P in such a way that the resultant disturbance at P is equal to $E_1/2$, but now here what we are doing is that, we are covering a part of the point source, we are covering a part of the spherical wavefront which is emitted from the point source and now the resultant disturbance at point of observation P is increased, it became twice.

And this is what is written here by inserting a screen in the path of the wave thereby blocking out most of the wavefront, we have blocked most of the wavefront and still we have increased the irradiance at P by a factor of four, because in earlier case, in case of unobstructed wave, the radiance was $(E_1/2)^2$ that is $E_1^2/4$ and here the radiances $|E_1|^2$, there is a difference of a factor of 4 now.

Now because of the complete symmetry of the setup, we can expect a circular ring pattern. Now see the beauty of this vibration curve analysis, things are very much simpler now. Now since the aperture is circular we expect that there would be a circularly symmetric pattern and most probably it would be concentric ring pattern, how to know this? We will again analyze it we will see how do we get a concentric circular ring pattern using this vibration curve.

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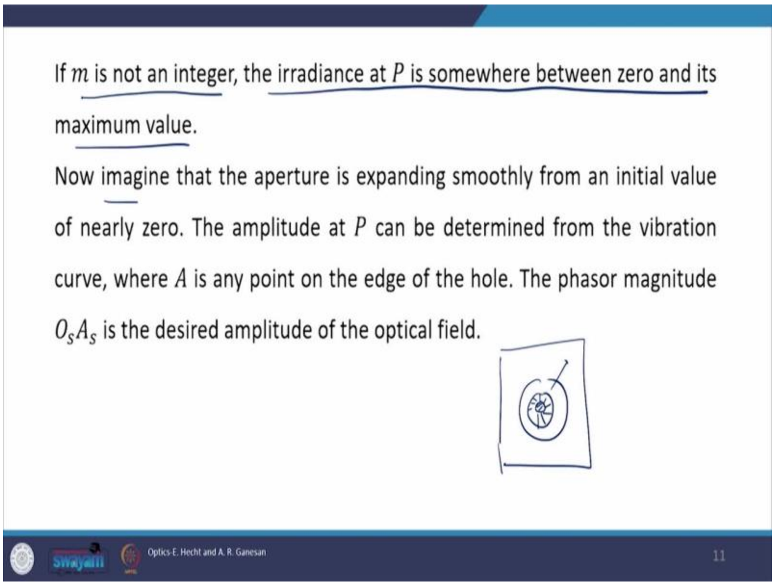

Now if we have considered the two cases, in the first m was even, in the second m is odd, but what if m is in between, if m is not an integer, the irradiance at P is somewhere between 0 and its maximum value, it would take a mid-value between 0 and its maximum value. Now consider the case where we assume that the circular aperture radius is increasing slowly it is expanding smoothly and it is assumed that initially the radius was nearly equal to 0 and then it is getting bigger and bigger slowly, then what will happen? The amplitude at point P can be determined from the vibration curve.

Now if you increase the radius, now see suppose this is the aperture and you were initially seeing, say, first Fresnel zone and the second Fresnel zone, now if you increase the radius then this Fresnel zones will keep increasing, the number of visible Fresnel zone will keep increasing. Now suppose you are just seeing the first Fresnel zone partially then you will see some intensity.

Now if you see first Fresnel zone plus second Fresnel zone then the field of one will cancel the field of the other and therefore, you will see you 0, no intensity would be there on the point of observation P. Again, increase it the third Fresnel zone will now be in the field of view again some intensity, again increase it four Fresnel zone again 0 intensity, it means with the increase of the radius of the circular aperture you will see several maxima and minima and whose amplitude slowly will fade away.

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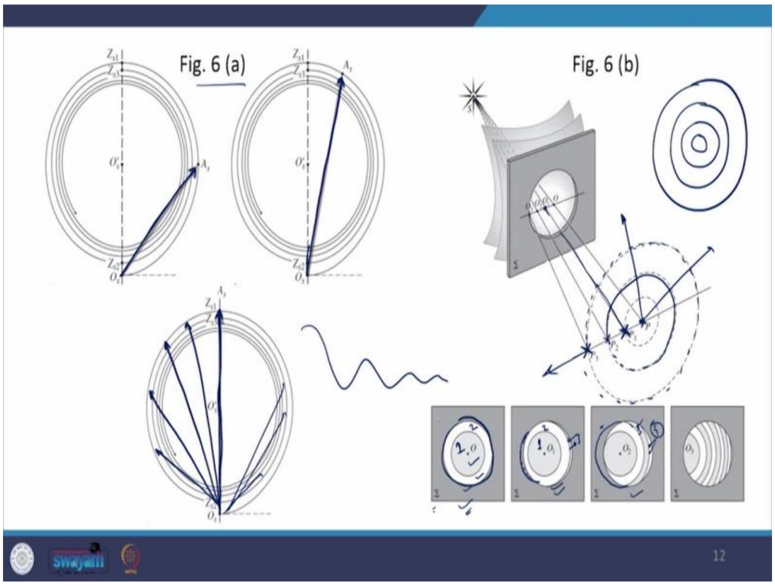

Now this is what is shown here in figure 6(a), initially when only first zone is visible partially then you will have this resultant, if you keep opening the circular aperture the resultant will increase, a full aperture is visible then this would be the resultant. And if you keep increasing the radius then this arrow will rotate anti-clockwise, this arrow will rotate in an anti-clockwise direction and you will see that sometimes the magnitude of this arrow is large and sometimes it reduces down to almost 0 and then it again increases, we will see something like this, this is the intensity pattern, which you will observe if you slowly expand the circular aperture.

Now let us see what will happen if we not only move along the OP direction along the axis, let us go in the lateral direction then what will happen. In the lateral direction, now initially we were at point P, let us suppose that we are at point P and we are observing some fringes and assume that zone 1 and zone 2 is visible as shown here in this picture, this is your aperture plane, this is the circular aperture and within this aperture we are able to see zone number 1 and number 2.

In this particular case the contribution from zone 1 will cancel the contribution of zone 2 because they are out of phase, you will not see anything at point of P. Now if you go to point P_1 with this the light is coming from O_1 and then it is going to point P_1. Now since we have shifted laterally in the transverse direction, this is zone 2 and this is zone 1, the part of zone 2 is now eclipsed, it is not visible, but a part of zone 3 is now visible from here. It means there would not be complete darkness due to partial hiding of zone 2 and partial opening of zone 3, we will not have perfect cancellation and we will see some intensity.

Similarly if you move to point P_2 the zone two will be more behind this aperture and it would not be visible and this is your zone 3 which will be here and zone 4 will also start peeking in and therefore, again that we will see some distribution which will evolve you will have minima and then more intense point thickness you will see a periodic variation in intensity if you transversely move in this direction and this is symmetric. The same thing will appear if you move either in this direction or in this direction therefore, a circle is drawn whatever intensity is measured at point P_3 it would be measured all along this circle.

Similarly, with P_2 too, you will see the intensities all along the circle the same intensity what is observed with P_2, similarly with P_1. It means that we will get a concentric ring pattern here, this type of pattern would be seen due to circular aperture. And now you can see that we can guess it very easily just by looking at the aperture and applying the knowledge of vibration curve we can draw whole diffraction pattern here without touching any mathematics.

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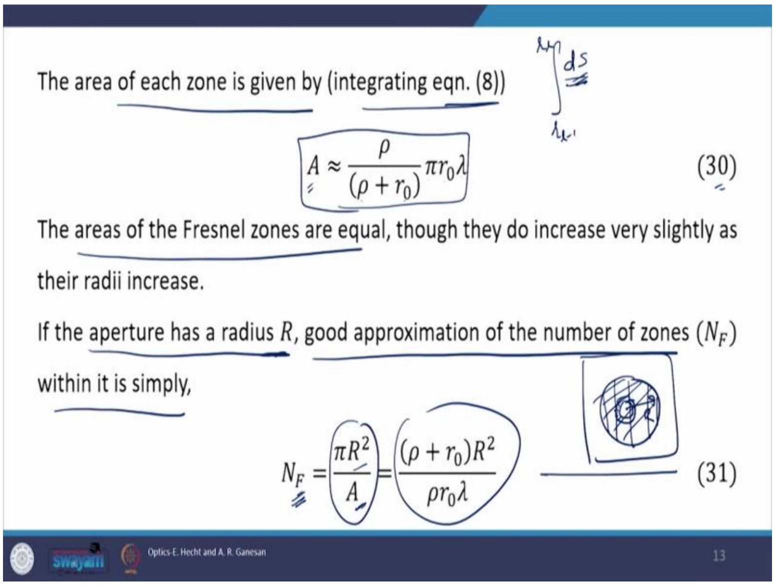

Now let us calculate the area of each zone, for this we integrate equation number 8 with biggest elementary area which we picked while analyzing the unobstructed point source and if you integrate this ds over particular zone say it is integration is from $(l-1)$ - to l, then we get the area of this zone, this is the rough expression, the integration gives us the area of a particular zone which is $(\rho/(\rho + r_0))\pi r_0 \lambda$. The areas of the Fresnel zones are almost equal, they do increase very slightly as their radii increases, but they are roughly the same.

Now if the circular aperture has radius R, good approximation of the number of zones within it is simply equal to this, this is our circular aperture which has radius R and we know from equation number 13, the area of a single Fresnel zone then once we know the area of aperture and area for single Fresnel's zone then we can calculate the total number of Fresnel zone which will fill the area of this circular aperture and this number is represented by N_F and is given by this expression, these two expression. πR^2 is the area of the circular aperture and A is the area of a single Fresnel zone if you substitute for A then you get equation number 31. And this says the total number of Fresnel zone which will approximately fall in the area of the circular aperture.

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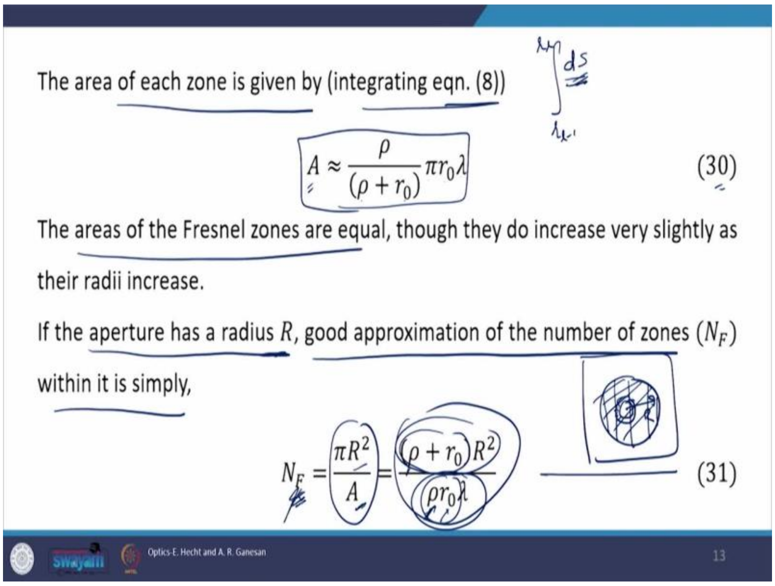

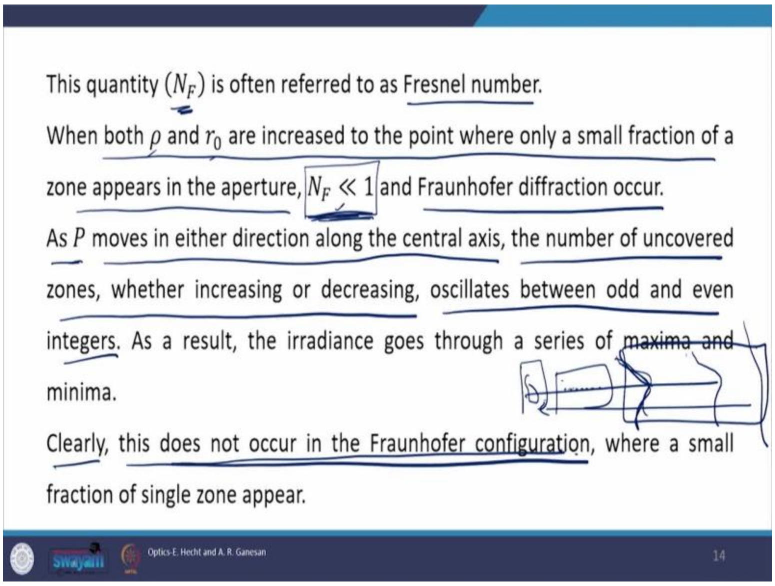

Now this quantity N_F is referred as Fresnel number. Now what will happen if we increase both ρ and r_0, if both ρ and r_0 increased to a point where only a small fraction of zone appear, small fraction of zone means within only single zone, the first zone, we are now able to see a little portion of it only. And if we are only see of the first zone partially, then only we see that ρ and r_0 are very high. And with this, you see here ρ and r_0 are increased sufficiently and in the numerator, we have $\rho + r_0$ while in the denominator we have $\rho \times r_0$ and since both ρ and r_0 are a big number, the denominator will be very huge as compared to the numerator and therefore N_F would be much-much smaller than one unity and this is what is shown here.

When both ρ and r_0 are increased to the point where only a small fraction of a zone appears in the aperture, then N_F is much-much less than 1. And then we traveled to the Fraunhofer regime and there we see Fraunhofer diffraction pattern, this the value of N_F decides whether we are in the Fresnel regime or in Fraunhofer regime, the condition from the Fraunhofer is that the Fresnel number N_F must be must be much-much smaller than unity.

Now as P, the point of observation moves in either direction along the central axis that is if you see this figure if P move along this axis, the OP line, the number of uncovered zones whether increasing or decreasing oscillates between odd and even integers, which is very much obvious, we have aperture and then we are moving along this line if we go close to this aperture we will see more number of zones, if we are away from the aperture we will see less number of zones, the number of zone will vary and it will oscillate between even and odd integer. When it is an even integer then intensity or irradiance will go down, if it is odd integer the irradians will go up, it means that irradiance goes through a series of maxima and minima it means we know what is exactly happening on the axis and we also know what is happening across the axis on the lateral or transverse direction.

Now clearly, this does not occur in the Fraunhofer configuration, because in Fraunhofer configuration, if you move along the axis, the shape remains intact, it is the size of the pattern which changes, this we have realized. If it is your aperture, then the shape of the pattern, suppose this is the some shape of your pattern, this shape will increase if you move away from the diffracting element and if this happens only when you are in the Fraunhofer regime, while in the Fresnel regime, you see that the maxima-minima condition is varying along the axis, this is characteristic of Fresnel and this does not occur in the Fraunhofer regime.

Why this does not occur in the Fraunhofer regime? Because in the Fraunhofer regime, we say that screen and the diffracting element, they are almost at infinity, and since the separation is very huge, then if you look into a circular aperture from the Fraunhofer regime, then you will see only a small part of first Fresnel zone, you will barely see a very little part of the first

Fresnel zone from that big distance. And since we are seeing only a small part of the first Fresnel zone, we will see only intensity which has certain distribution, but due to the divergence we see the increase in the size, but the shape remains intact.

Now with this, let us now move to the case where the circular aperture is illuminated with the plane wave, instead of spherical wavefront, we are assuming the wavefront which is falling on this circular aperture is plane. This happens in case when the separation between the point source and the defecting element is huge. In this particular case, we can approximate the large radius spherical wavefront with a plane wavefront.

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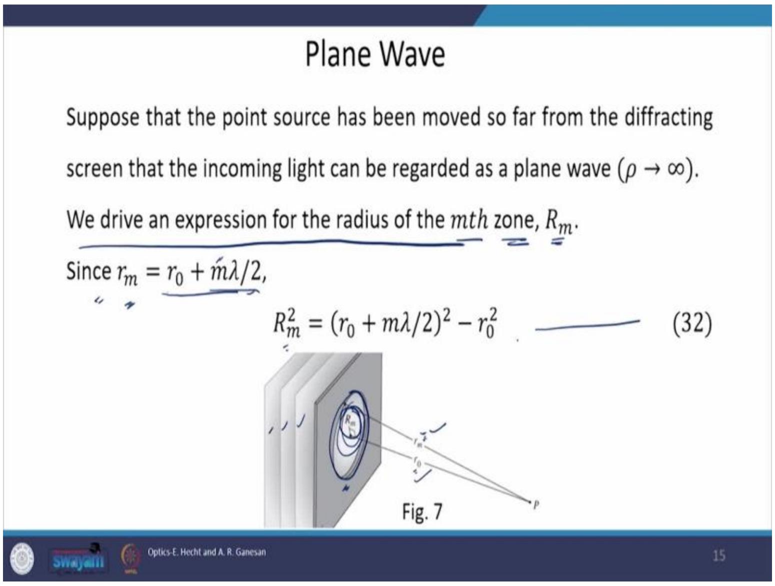

Now in this figure you see this is a circular aperture of radius R_m and these are the wave fronts which are falling on this circular aperture. Now here we derive an expression for the radius of mth zone, and we assumed this radius is R_m. Now from the figure $r_m = r_0 + m\lambda/2$. Now here we have different zones inside which are visible from point of observation P and we know that zones are defined in such a way that R_m is equal to r_0 plus integral multiple of $\lambda/2$. Now since we are talking about mth therefore, R_m in this particular case would be $r_0 + m\lambda/2$ here for mth zone, the integer values m here. Once R_m is known and r_0 is known, we can calculate R_m which would be given by equation number 32, where we have used the Pythagoras theorem.

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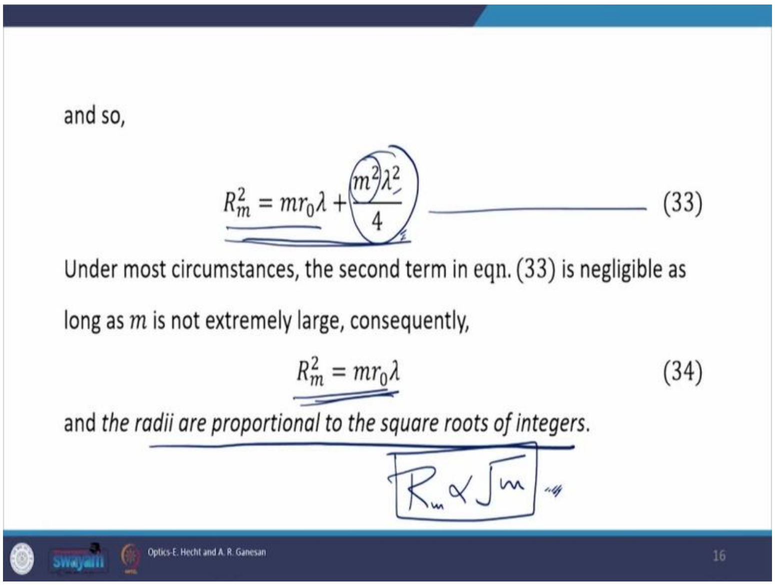

Now here in R_m can be expressed like this, why R_m can be expressed like this, you just expand it and then r_0^2 from this expression will go away and we will be left with the equation number 33, the right hand side of equation number 33. Now this term λ is very small quantity and therefore λ^2 would be even smaller and therefore, the second term on the right hand side of equation number 33 can easily be neglected, unless m is very huge.

And therefore, $R_m^2 = mr_0\lambda$. And from equation 34, we can clearly see that radii are proportional to the square root of integer, R_m is proportional to $\sqrt{m}$, which is what I wrote in my initial slides, probably second or third slide. I said that I will prove it at the end of this slide or end of this lecture. And therefore, it is proved here it is very much clear that R_m would be proportional to $\sqrt{m}$, the number of zone, where m is an integer. With this I end my lecture, thank you for joining me. See you in the next class.